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General stochastic models for series multicomponent system reliability (short communication)

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stochastic dependence, multivariate reliability function, two of its universal representations, marginal representation, base-line representation

Abstract

Multicomponent systems with known reliabilities (survival functions of the life-times) of their components are considered. Based on known either baseline or marginal survival functions of the component life-times we construct joint survival functions of random vectors of the life-times. These multivariate survival functions (here considered as system reliability models) are to be given in their general (universal) forms by means of joiners or system functions (instead of copula methodology) as a stochastic dependences determination.

Given multicomponent system with the components $c_1,\ldots,c_n$ and with series reliability structure. The components life-times, as statistically estimated in laboratory conditions (in physical separation of each other), are described by *independent* nonnegative random variables $T_1,\ldots,T_n$.
Their probability distributions are *base-line* reliability functions $R_1(t_1),\ldots,R_n(t_n)$.
Once the components are installed to the system physical interactions between them occur.
The interactions cause transformation of the random vector $(T_1,\ldots,T_n)$ into random vector, say $(X_1,\ldots,X_n)$ whose coordinates are stochastically dependent. The *marginal* survival (reliability) functions $S_1(x_1),\ldots,S_n(x_n)$ are, in general (but not always), different than $R_1(t_1),\ldots,R_n(t_n)$, respectively (see, for example, the Freund bivariate exponential). The stochastic model for the system reliability is, finally, the joint survival function $S(x_1,\ldots,x_n)$ of the random vector $(X_1,\ldots,X_n)$. As we will show, there are two, in general different, universal representations of the model $S(x_1,\ldots,x_n)$. In general uniform theory we rather encounter the *marginal representation* of the model:

$$S(x_1,\ldots,x_n) = S_1(x_1),\ldots,S_n(x_n)\ J(x_1,\ldots,x_n),$$

where the dependence function $J(x_1,\ldots,x_n)$ we call the *joiner*. At this point, the problem (for which we find solutions) arises as follows: given, the marginals $S_1(x_1),\ldots,S_n(x_n)$, find sufficient and necessary conditions for a continuous functions, say $J(x_1, \ldots ,x_n)$, such that the product

$$S_1(x_1),\ldots,S_n(x_n)\ J(x_1,\ldots,x_n)$$

is a legitimate n-variate survival function. Such conditions will be given together with examples of particular models $S(x_1,\ldots,x_n)$. The, relatively recently invented, joiner theory can be considered as competitive to the *copula* theory.
However in applications, especially in reliability problems, instead of marginal representation we rather need to apply other form of the same model $S(x_1,\ldots,x_n)$, namely the *base-line representation*. That is because we usually start with initial data $R_1(t_1),\ldots,R_n(t_n)$ obtained in laboratory conditions and not with, yet unknown, marginal distributions. Thus, we have other, equivalent, universal

representation of the model:

$$S(x_1,\ldots,x_n) = R_1(x_1) \ldots R_n(x_n)\, K(x_1,\ldots,x_n),$$

where $K(x_1,\ldots,x_n)$ is called *system function*.
Nice analytical relationship between system function and the corresponding joiner as well as between the base-lines $R_1(t_1),\ldots,R_n(t_n)$ and the marginals $S_1(t_1),\ldots,S_n(t_n)$ will be given. Some other examples of various stochastic models will be given too.

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