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Maritime transportation system safety and operation cost joint optimization

Keywords

maritime transportation, safety, operation cost, optimization, safety

Abstract

The model of system safety impacted by operation process is created and the procedure of its safety in the safety state subsets not worse than the critical safety state maximization is proposed. The model of system operation total costs in the safety state subsets is introduced and the procedure of its operation total cost in the safety state subset not worse than the critical safety state minimization is presented. To analyze jointly the system safety and its operation cost optimization, we propose determining the optimal values of limit transient probabilities of the system operation process at the particular operation states that allows to find the best system safety function and other safety indicators, utilizing the created system safety model and linear programming. Next, to find the system operation total costs in the safety state subsets, corresponding to this system best safety indicators, we replace the limit transient probabilities at the particular operation states, existing in the formula for the system operation total costs in the safety state subsets, by their optimal values existing in the formulae for the coordinates of the system safety function after maximization. On the other hand, to analyze jointly the system operation cost and it's safety optimization, we propose the procedure of determining the optimal values of limit transient probabilities of the system operation process at the particular operation states that allows to find minimal system operation total cost in the safety state subset not worse than the critical safety state, using the created system operation cost model and linear programming. After that, to find the system conditional safety indicators, corresponding to this system minimal total operation cost in the safety state subset not worse than the critical safety state, we replace the limit transient probabilities at particular operation states, existing in the formula for the system safety function coordinates, and for remaining system conditional safety indicators by their optimal values existing in the formulae for the system minimal total cost in the safety state subsets not worse than the critical safety state. The created models are applied separately and jointly to the maritime transportation system. Moreover, to fulfil the obtained maritime transportation system optimal safety and operation cost results the modifications of its operation process is proposed. The evaluation of results is performed and future research in the field of the complex systems, including maritime transportation systems, safety and their operation costs joint analysis and optimization is proposed.

1. Introduction

To tie the investigation of the complex technical system safety together with the investigation of its operation cost, the semi-Markov process model (Ferreira & Pacheco, 2007; Glynn & Haas, 2006; Grabski, 2002, 2015; Limnios & Oprisan, 2005; Mercier, 2008; Tang et al., 2007),

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can be used to describe this system operation process (Kołowrocki & Soszyńska-Budny, 2011/2015; Magryta, 2020). The system operation process model, under the assumption on the system safety multistate model (Xue, 1995; Xue & Yang, 1985), can be used to construct the general safety model of the complex multistate system changing its safety structure and its components safety parameters during variable operation conditions (Kołowrocki, 2014; Kołowrocki & Magryta, 2020; Kołowrocki & Soszyńska-Budny, 2011/2015; Magryta, 2020). Further, using this general model, it is possible to define the complex system main safety characteristics such as the system safety function, the mean values and standard deviations of the system lifetimes in the system safety state subsets and in the system particular safety states (Dąbrowska, 2020, Dąbrowska & Kołowrocki, 2019a; Kołowrocki, 2014, 2020; Kołowrocki & Soszyńska, 2010a-b; Kołowrocki & Soszyńska-Budny, 2011/2015). Other system safety indicators, like the system risk function, the system fragility curve, the moment when the system risk function exceeds a permitted level, the system intensity of ageing, the coefficient of operation process impact on system intensity of ageing and the system resilience indicator to operation process impact, can be introduced as well (Gouldby et al., 2010; Kołowrocki, 2014; Kołowrocki & Soszyńska-Budny, 2018a-b, 2019a-c; Lauge et al., 2015; Szymkowiak, 2018a-b, 2019). Using the system general safety model, it is possible to change the system operation process through applying the linear programming (Klabjan & Adelman, 2006) for maximizing the system safety function (Kołowrocki & Soszyńska-Budny, 2010) and finding the optimal limit values of the system transient probabilities at the particular operation states and determining the system optimal safety indicators. Having the system operation process characteristics and the system conditional instantaneous operation costs at the operation states, it is possible to create the system general operation total costs in the safety state subsets (Kołowrocki & Kuligowska, 2018; Kołowrocki & Magryta-Mut, 2021). Using this system operation total cost model, it is possible to change the system operation process through applying the linear programming (Klabjan & Adelman, 2006) for minimizing the system operation total cost costs in the safety state subsets (Kołowrocki & Kuligowska, 2018; Kołowrocki & Magryta-Mut, 2022) and finding the optimal limit values of the system transient probabilities at the particular operation states. To analyze jointly the system safety and its operation cost optimization, in the case we prefer more the system safety maximization than the system operation cost minimization, we first apply the procedure of determining the optimal values of limit transient probabilities of the system operation process at the particular operation states that maximize the system safety. Next, to find the system conditional operation total costs in the safety state subsets, corresponding to this system maximal safety, we replace the limit transient probabilities at particular operation states, existing in the formula for the operation total costs in the safety state subsets, by their optimal values existing in the formula for the system maximal safety function coordinates. Whereas, in the case we prefer more the system operation cost minimization than the system safety maximization, then to analyze jointly the system safety and operation cost optimization, we first apply the procedure of determining the optimal values of limit transient probabilities of the system operation process at the particular operation states that minimize the system operation total costs in the safety state subsets. Next, to find the system conditional safety indicators, corresponding to this system minimal total operation costs in the safety state subsets, we replace the limit transient probabilities at the system particular operation states, existing in the formula for the system safety function coordinates, by their optimal values existing in the formula for the system minimal operation total costs in the safety state subsets in order to get the formula for the system conditional safety function related to this system minimal operation total cost. Further, applying this formula for the system conditional safety function, we find the remaining system conditional safety indicators.

The created model for maximizing the system safety is applied to the maritime transportation system to find its optimal safety indicators. Next, the maritime transportation system mean value of the system operation total costs in the safety state subsets corresponding to its optimal safety indicators is found. The created model for minimizing of the system operation total costs in the safety state subsets is applied to the maritime transportation system to find the minimal mean

value of the system operation total costs in the safety state subsets. Next, the maritime transportation system safety indicators corresponding to this minimal operation total cost are found.
The chapter is organized into 7 parts, this Introduction as Section 1, Sections 2–6 and Conclusion as Section 7. In Section 2, the model of system safety impacted by operation process is introduced and the procedure of the system safety maximization is proposed. In Section 3, the model of system operation total costs in the safety state subsets is introduced and the procedure of its minimization is presented. In Section 4, the maritime transportation system operation total costs in the safety state subsets is analyzed and its minimal value is determined. In Section 5, the maritime transportation system operation process influence on its safety indicators is examined and the best forms and values of this system safety indicators are determined. In Section 6, joint analysis of the maritime transportation system safety maximizing and its conditional operation total costs in the safety state subsets corresponding to the maritime transportation system maximal safety is performed. The maritime transportation system best safety indicators are fixed and the maritime transportation system operation total costs corresponding to this best safety indicators is determined. Joint analysis of the maritime transportation system operation total costs in the safety state subsets minimizing and its conditional safety indicators corresponding to the maritime transportation system minimal operation total cost is performed. The maritime transportation system minimal operation total cost is fixed and the system safety indicators corresponding to this minimal operation total cost are determined. In Conclusion, the evaluation of results achieved is done and the perspective for future research in the field of the complex systems, including maritime transportation systems, safety and their operation costs joint analysis and optimization is proposed.

2. System safety

2.1. System safety model

We assume that the system is operating at v, $v > 1$, operation states z_b, $b = 1,2,\ldots,v$, that have influence on the system functional structure and on the system safety. Applying semi-Markov model of the system operation process $Z(t)$, $t \geq 0$, it is possible to find this process two basic characteristics (Grabski, 2015; Kołowrocki & Magryta, 2020a; Kołowrocki & Soszyńska-Budny, 2011/2015):

- the vector of limit values

$$p_b = \lim_{t\to\infty} p_b(t) = b = 1,2,\ldots,v, \quad (1)$$

of transient probabilities

$$p_b(t) = P(Z(t) = z_b),\ t \geq 0,\ b = 1,2,\ldots,v, \quad (2)$$

of the system operation process $Z(t)$ at the particular operation states z_b, $b = 1,2,\ldots,v$,

- the vector $[\hat{M}_b]_{1xv}$ of the mean values

$$\hat{M}_b = E[\hat{\theta}_b] \cong p_b\theta,\ b = 1,2,\ldots,v, \quad (3)$$

of the total sojourn times $\hat{\theta}_b$, $b = 1,2,\ldots,v$, of the system operation process $Z(t)$, $t \geq 0$, at the particular operation states z_b, $b = 1,2,\ldots,v$, during the fixed system operation time θ, $\theta > 0$, where p_b, $b = 1,2,\ldots,v$, are defined by (1)–(2).

Considering the safety function of the system impacted by operation process

$$\boldsymbol{S}(t,\cdot) = [\boldsymbol{S}(t,1), \boldsymbol{S}(t,2),\ldots, \boldsymbol{S}(t,z)],\ t \geq 0, \quad (4)$$

coordinate given by (Kołowrocki & Soszyńska-Budny, 2011/2015)

$$\boldsymbol{S}(t,u) \cong \sum_{b=1}^{v} p_b[\boldsymbol{S}(t,u)]^{(b)},\ t \geq 0,\ u = 1,2,\ldots,z, \quad (5)$$

where p_b, $b = 1,2,\ldots,v$, are the limit transient probabilities of the system operation process at the operation states z_b, $b = 1,2,\ldots,v$, and

$$[\boldsymbol{S}(t,u)]^{(b)} = P([T(u)]^{(b)} > t),\ t \geq 0,$$

$$u = 1,2,\ldots,z,\ b = 1,2,\ldots,v,$$

at these operation states are the conditional safety functions of the system and ($[T(u)]^{(b)}$, are the system conditional lifetimes in the safety state subsets $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, at the operation states z_b, $b = 1,2,\ldots,v$, it is natural to assume that the system operation process has a significant influence on the system safety.

From the expression (5), it follows that the mean values of the system unconditional lifetimes in the safety state subsets $\{u,u+1,\ldots,z\}$, are of the form

$$\boldsymbol{\mu}(u) = \sum_{b=1}^{v} p_b [\mu(u)]^{(b)} \text{ for } u = 1,2,\ldots,z. \quad (6)$$

The values of the variances of the system unconditional lifetimes in the system safety state subsets are

$$[\boldsymbol{\sigma}(u)]^2 = 2\int_0^\infty t\boldsymbol{S}(t,u)dt - [\boldsymbol{\mu}(u)]^2, \; u = 1,2,\ldots,z, \quad (7)$$

where $\boldsymbol{\mu}(u)$ is given by (6) and $\boldsymbol{S}(t,u)$ is given by (5).

The expressions for the mean values of the system unconditional lifetimes in the particular safety states are

$$\bar{\boldsymbol{\mu}}(u) = \boldsymbol{\mu}(u) - \boldsymbol{\mu}(u+1), \; u = 1,2,\ldots,z-1,$$

$$\bar{\boldsymbol{\mu}}(z) = \boldsymbol{\mu}(z). \quad (8)$$

The system risk function and the moment when the risk exceeds a permitted level δ, respectively are given by (Kołowrocki & Soszyńska-Budny, 2011/2015):

$$\boldsymbol{r}(t) = 1 - \boldsymbol{S}(t,r), \; t \geq 0, \quad (9)$$

and

$$\tau = \boldsymbol{r}^{-1}(\delta), \quad (10)$$

where $\boldsymbol{S}(t,r)$ is given by (5) for $u = r$ and $\boldsymbol{r}^{-1}(t)$, if it exists, is the inverse function of the risk function $\boldsymbol{r}(t)$.

The mean values of the system intensities of ageing (departure from the safety state subset $\{u,u+1,\ldots,z\}$), are defined by

$$\boldsymbol{\lambda}(u) = \frac{1}{\boldsymbol{\mu}(u)}, \; u = 1,2,\ldots,z. \quad (11)$$

Considering the values of the system without operation impact intensities of ageing $\boldsymbol{\lambda}^0(u)$, defined in (Kołowrocki & Magryta-Mut, 2020c; Kołowrocki & Soszyńska-Budny, 2018b), the coefficients of the operation process impact on the system intensities of ageing are given by

$$\boldsymbol{\rho}(u) = \frac{\boldsymbol{\lambda}(u)}{\boldsymbol{\lambda}^0(u)}, \; u = 1,2,\ldots,z. \quad (12)$$

Finally, the system resilience indicators, i.e. the coefficients of the system resilience to operation process impact, are defined by

$$\boldsymbol{RI}(u) = \frac{1}{\boldsymbol{\rho}(u)}, \; u = 1,2,\ldots,z. \quad (13)$$

2.2. System safety optimization

Considering the safety function of the system impacted by operation process $\boldsymbol{S}(t,\cdot)$, $t \geq 0$, coordinate given by (5), it is natural to assume that the system operation process has a significant influence on the system safety. This influence is also clearly expressed in the equation (6) for the mean values of the system unconditional lifetimes in the safety state subsets. From the linear equation (6), we can see that the mean value of the system unconditional lifetime $\boldsymbol{\mu}(u)$, $u = 1,2,\ldots,z$, is determined by the limit values of transient probabilities p_b, $b = 1,2,\ldots,v$, of the system operation process at the operation states z_b, $b = 1,2,\ldots,v$, and the mean values $[\boldsymbol{\mu}(u)]^{(b)}$, $b = 1,2,\ldots,v$, $u = 1,2,\ldots,z$, of the system conditional lifetimes in the safety state subsets $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, at these operation states. Therefore, the system lifetime optimization approach based on the linear programming can be proposed (Klabjan & Adelman, 2006). Namely, we may look for the corresponding optimal values $\dot{p}_b$, $b = 1,2,\ldots,v$, of the transient probabilities p_b, $b = 1,2,\ldots,v$, of the system operation process at the operation states to maximize the mean value $\boldsymbol{\mu}(u)$ of the unconditional system lifetime in the safety state subset $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, under the assumption that the mean values $[\boldsymbol{\mu}(u)]^{(b)}$, $b = 1,2,\ldots,v$, $u = 1,2,\ldots,z$, of the system conditional lifetimes in the safety state subset at the particular operation states are fixed. As a special case of the above formulated system lifetime optimization problem, if r, $r \in \{1,2,\ldots,z\}$ is a system critical safety state, we may look for the optimal values $\dot{p}_b$, $b = 1,2,\ldots,v$, of the transient probabilities p_b, $b = 1,2,\ldots,v$, of the system operation process at the system operation states to maximize the mean value $\boldsymbol{\mu}(r)$ of the unconditional system lifetime in the safety state subset $\{r,r+1,\ldots,z\}$, $r = 1,2,\ldots,z$, under the assumption that the mean values $[\boldsymbol{\mu}(r)]^{(b)}$, $b = 1,2,\ldots,v$, $u = 1,2,\ldots,z$, of the system conditional lifetimes in this safety state subset not worse than the critical safety state at the particular operation states are fixed. More exactly, we formulate the optimization problem as a linear programming model with the objective function of the following form

$$\boldsymbol{\mu}(r) = \sum_{b=1}^{v} p_b [\mu(r)]^{(b)}, \tag{14}$$

for a fixed $r \in \{1,2,\ldots,z\}$ and with the following bound constraints

$$\breve{p}_b \le p_b \le \hat{p}_b, \; b = 1,2,\ldots,v, \tag{15}$$

$$\sum_{b=1}^{v} p_b = 1, \tag{16}$$

where

$$[\mu(r)]^{(b)}, \; [\mu(r)]^{(b)} \ge 0, \; b = 1,2,\ldots,v, \tag{17}$$

are fixed mean values of the system conditional lifetimes in the safety state subset $\{r,r+1,\ldots,z\}$ and

$$\breve{p}_b, \; 0 \le \breve{p}_b \le 1 \text{ and } \hat{p}_b, \; 0 \le \hat{p}_b \le 1, \; \breve{p}_b \le \hat{p}_b,$$

$$b = 1,2,\ldots,v, \tag{18}$$

are lower and upper bounds of the unknown transient probabilities at the particular operation states p_b, $b = 1,2,\ldots,v$, respectively.

Now, we can obtain the optimal solution of the formulated by (14)–(18) the optimization problem, i.e. we can find the optimal values $\dot{p}_b$ of the transient probabilities p_b, $b = 1,2,\ldots,v$, that maximize the objective function given by (14). The maximizing procedure is described in (Kołowrocki & Magryta, 2020b; Magryta-Mut, 2020).

Finally, after applying this procedure, we can get the maximum value of the system total mean lifetime in the safety state subset $\{r,r+1,\ldots,z\}$ defined by the linear form (14), in the following form

$$\dot{\boldsymbol{\mu}}(r) = \sum_{b=1}^{v} \dot{p}_b [\mu(r)]^{(b)} \tag{19}$$

for a fixed $r \in \{1,2,\ldots,z\}$.

Further, by replacing the limit transient probabilities p_b, $b = 1,2,\ldots,v$, existing in the formulae (4)–(5) by their optimal values $\dot{p}_b$, $b = 1,2,\ldots,v$, we get the optimal form of the system safety and the expressions for all remaining safety indicators considered in Section 2.1.

3. System operation cost with system safety impact

3.1. System operation cost model with system safety impact

Similarly to safety analysis of the system impacted by its operation process, we may investigate the system operation total costs in the safety state subsets. Namely, we define the instantaneous system operation cost in the form of the vector

$$\boldsymbol{C}(t,\cdot) = [\boldsymbol{C}(t,1),\ldots,\boldsymbol{C}(t,z)], \; t \ge 0, \tag{20}$$

with the coordinates given by

$$\boldsymbol{C}(t,u) \cong \sum_{b=1}^{v} p_b [\boldsymbol{C}(t,u)]^{(b)}, \tag{21}$$

$$t \ge 0, \; u = 1,2,\ldots,z,$$

where $[\boldsymbol{C}(t,u)]^{(b)}$, $u = 1,2,\ldots,z$, $b = 1,2,\ldots,v$, are the coordinates of the vector

$$[\boldsymbol{C}(t,\cdot)]^{(b)} = [[\boldsymbol{C}(t,1)]^{(b)},\ldots,[\boldsymbol{C}(t,z)]^{(b)}],$$

$$t \ge 0, \; b = 1,2,\ldots,v,$$

representing the system conditional instantaneous operation costs in the safety state subsets $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, at the system operation states z_b, $b = 1,2,\ldots,v$, and p_b, $b = 1,2,\ldots,v$, are the system operation process limit transient probabilities in the particular operation states (Grabski, 2015). Thus, it is naturally to assume that the system instantaneous operation cost depends significantly on the system operation state and the system operation cost at the operation state as well. This dependency is also clearly expressed in mean value of the system total operation cost

$$\boldsymbol{C}(\cdot) = [\boldsymbol{C}(1), \boldsymbol{C}(2), ..., \boldsymbol{C}(z)] \quad (22)$$

with coordinates given by the linear equations

$$\boldsymbol{C}(u) \cong \sum_{b=1}^{v} p_b [\boldsymbol{C}(u)]^{(b)}, \ u = 1, 2, \ldots, z, \quad (23)$$

for the mean values of the system total unconditional operation costs in the safety state subsets $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, where $\boldsymbol{C}(t,u)]^{(b)}$, $u = 1,2,\ldots,z$, $b = 1,2,\ldots,v$, are the mean values of the system total conditional operation costs in the safety state subsets $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, at the particular system operation states z_b, $b = 1,2,\ldots,v$, determined by

$$[\boldsymbol{C}(u)]^{(b)} \cong \int_0^{[\mu(u)]^{(b)}} [\boldsymbol{C}(t,u)]^{(b)} dt, \quad (24)$$

$u = 1,2,\ldots,z$, $b = 1,2,\ldots,v$,

where

$$[\boldsymbol{\mu}(u)]^{(b)} = E[[T(u)]^{(b)}], u = 1,2,\ldots,z, \quad (25)$$

are the mean values of the system conditional lifetimes $[T(u)]^{(b)}$ in the safety state subset $\{u,u+1,\ldots,z\}$ at the operation state z_b, $b = 1,2,\ldots,v$, given by (Kołowrocki & Magryta, 2020c; Kołowrocki & Soszyńska-Budny, 2011/2015):

$$[\boldsymbol{\mu}(u)]^{(b)} = \int_0^{\infty} [\boldsymbol{S}(t,u)]^{(b)} dt, \ u = 1,2,\ldots,z, \quad (26)$$

and $[\boldsymbol{S}(t,u)]^{(b)}$, $u = 1,2,\ldots,z$, $b = 1,2,\ldots,v$, are the system safety function defined above and p_b are limit transient probabilities defined in (Kołowrocki & Soszyńska-Budny, 2011/2015).

3.2. System operation cost optimization model with system safety impact

From the linear equations (23), we can see that the mean value of the system total unconditional operation cost $\boldsymbol{C}(u)$, $u = 1,2,\ldots,z$, is determined by the limit values of transient probabilities p_b, $b = 1,2,\ldots,v$, of the system operation process at the operation states z_b, $b = 1,2,\ldots,v$, and by the mean values $[\boldsymbol{C}(u)]^{(b)}$ of the system total conditional operation costs in the safety state subsets $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, at the particular system operation states z_b, $b = 1,2,\ldots,v$, that by (24) are dependent on the mean values $[\boldsymbol{\mu}(u)]^{(b)}$, $b = 1,2,\ldots,v$, $u = 1,2,\ldots,z$, of the system conditional lifetimes and by the system conditional instantaneous operation costs $[\boldsymbol{C}(t,u)]^{(b)}$ in the safety state subsets $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, at the system operation state z_b, $b = 1,2,\ldots,v$. Therefore, the system operations cost optimization based on the linear programming (Klabjan & Adelman, 2006), can be proposed. Namely, we may look for the corresponding optimal values $\dot{p}_b$, $b = 1,2,\ldots,v$, of the transient probabilities p_b, $b = 1,2,\ldots,v$, of the system operation process at the operation states to minimize the mean value $\boldsymbol{C}(u)$ of the system total unconditional operation costs in the safety state subsets $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, under the assumption that the mean values $[\boldsymbol{C}(u)]^{(b)}$, $b = 1,2,\ldots,v$, $u = 1,2,\ldots,z$, of the system total conditional operation costs in the safety state subsets $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, at the particular system operation states z_b, $b = 1,2,\ldots,v$, are fixed. As a special and practically important case of the above formulated system operation cost optimization problem for $u = r$, where if r, $r = 1,2,\ldots,z$, is a system critical safety state, we may look for the optimal values $\dot{p}_b$, $b = 1,2,\ldots,v$, of the transient probabilities p_b, $b = 1,2,\ldots,v$, of the system operation process at the system operation states to minimize the mean value $\boldsymbol{C}(r)$ of the system total unconditional operation costs in the safety state subset $\{r,r+1,\ldots,z\}$, $r = 1,2,\ldots,z$, under the assumption that the mean values $[\boldsymbol{C}(r)]^{(b)}$, $b = 1,2,\ldots,v$, $r = 1,2,\ldots,z$, of the system total conditional operation costs in this safety state subsets are fixed. More exactly, we may formulate the optimization problem as a linear programming model (Lauge et al., 2015) with the objective function of the following form

$$\boldsymbol{C}(r) \cong \sum_{b=1}^{v} p_b [\boldsymbol{C}(r)]^{(b)}, \quad (27)$$

for a fixed $r \in \{1,2,\ldots,z\}$ and with the following bound constraints

$$\breve{p}_b \le p_b \le \hat{p}_b,\ b = 1,2,\ldots,v,\ \sum_{b=1}^{v} p_b = 1, \tag{28}$$

where

$[\boldsymbol{C}(r)]^{(b)}$, $[\boldsymbol{C}(r)]^{(b)} \ge 0$, $b = 1, 2,\ldots, v$,

are fixed mean values of the system conditional operation costs in the safety state subset $\{r,r+1,\ldots,z\}$, and

$$\breve{p}_b,\ 0 \le \breve{p}_b \le 1 \text{ and } \hat{p}_b,\ 0 \le \hat{p}_b \le 1,\ \breve{p}_b \le \hat{p}_b,$$

$$b = 1,2,\ldots,v, \tag{29}$$

are lower and upper bounds of the unknown transient probabilities p_b, $b = 1,2,\ldots,v$, respectively.
Now, we can obtain the optimal solution of the formulated by (27)–(28) the linear programming problem, i.e. we can find the optimal values $\dot{p}_b$, of the transient probabilities p_b, $b = 1,2,\ldots,v$, that minimize the objective function given by (27).
We arrange the mean values of the system total conditional operation costs $[\boldsymbol{C}(r)]^{(b)}$, $b = 1,2,\ldots,v$, in non-decreasing order

$$[C(r)]^{(b_1)} \le [C(r)]^{(b_2)} \le \ldots \le [C(r)]^{(b_i)},$$

where $b_i \in \{1,2,\ldots,v\}$ for $i = 1,2,\ldots,v$.
Next, we substitute

$$x_i = p_{b_i},\ \breve{x}_i = \breve{p}_{b_i},\ \hat{x}_i = \hat{p}_{b_i} \text{ for } i = 1,2,\ldots,v, \tag{30}$$

and we minimize with respect to $x_i, i = 1, 2,\ldots,v$, the linear form (27) that after this transformation takes the form

$$\boldsymbol{C}(r) \cong \sum_{b=1}^{v} x_i [\boldsymbol{C}(r)]^{(b_i)}, \tag{31}$$

for a fixed $r \in \{1,2,\ldots,z\}$ with

$$\breve{x}_i \le x_i \le \hat{x}_i,\ i = 1,\ 2,\ldots,\ v,\ \sum_{i=1}^{v} x_i = 1, \tag{32}$$

where

$[C(r)]^{(b_1)}$, $[C(r)]^{(b_1)} \ge 0$, $i = 1, 2,\ldots, v$,

are fixed mean values of the system conditional operation costs in the safety state subset $\{r,r+1,\ldots,z\}$ arranged in non-decreasing order and

$$\breve{x}_i,\ 0 \le \breve{x}_i \le 1 \text{ and } \hat{x}_i,\ 0 \le \hat{x}_i \le 1,\ \breve{x}_i \le \hat{x}_i,$$

$$i = 1, 2,\ldots, v, \tag{33}$$

are lower and upper bounds of the unknown probabilities x_i, $i = 1,2,\ldots,v$, respectively.
To find the optimal values of x_i, $i = 1,2,\ldots,v$, we define

$$\breve{x} = \sum_{i=1}^{v} \breve{x}_i,\ \hat{y} = 1 - \breve{x} \tag{34}$$

and

$$\breve{x}_i^{\,0} = 0,\ \hat{x}_i^{\,0} = 0 \text{ and } \breve{x}^I = \sum_{i=1}^{I} \breve{x}_i,\ \hat{x}^I = \sum_{i=1}^{I} \hat{x}_i$$

for $I = 1,2,\ldots,v$, (35)

Next, we find the largest value $I \in \{0,1,\ldots,v\}$ such that

$$\hat{x}^I - \breve{x}^I < \hat{y} \tag{36}$$

and we fix the optimal solution that minimize (31) in the following way:
i) if $I = 0$, the optimal solution is

$$\dot{x}_1 = \hat{y} + \breve{x}_1 \text{ and } \dot{x}_i = \breve{x}_i \text{ for } i = 2,3,\ldots,v, \tag{37}$$

ii) if $0 < I < v$, the optimal solution is

$$\dot{x}_i = \hat{x}_i \text{ for } i = 1, 2,\ldots, I,$$

$$\dot{x}_{I+1} = \hat{y} - \hat{x}^I + \breve{x}^I + \breve{x}_{I+1}$$

$$\text{and } \dot{x}_i = \breve{x}_i \text{ for } i = I+2, I+3,\ldots,v, \tag{38}$$

iii) if $I = v$, the optimal solution is

$$\dot{x}_i = \hat{x}_i \text{ for } i = 1, 2,\ldots, v. \tag{39}$$

Finally, after making the inverse to (30) substitution, we get the optimal limit transient probabilities

$$\dot{p}_{b_i} = \dot{x}_i \text{ for } i = 1,2,\ldots,\nu, \quad (40)$$

that minimize the mean value of the system total unconditional operation costs in the safety state subset $\{r,r+1,\ldots,z\}$, defined by the linear form (27), giving its minimum value in the following form

$$\dot{\boldsymbol{C}}(r) = \sum_{b=1}^{\nu} \dot{p}_b [\boldsymbol{C}(r)]^{(b)} \quad (41)$$

for a fixed $r \in \{1,2,\ldots,z\}$.
From the expression (41) for the minimum mean value $\dot{\boldsymbol{C}}(r)$ of the system unconditional operation cost in the safety state subset $\{r,r+1,\ldots,z\}$, replacing in it the critical safety state r by the safety state u, $u = 1,2,\ldots,z$, we obtain the corresponding optimal solutions for the mean values of the system unconditional operation costs in the safety state subsets $\{u,u+1,\ldots,z\}$ of the form

$$\dot{\boldsymbol{C}}(u) = \sum_{b=1}^{\nu} \dot{p}_b [\boldsymbol{C}(u)]^{(b)}, \; u = 1,2,\ldots,z. \quad (42)$$

According to (22)–(23), the mean value of the system optimal total operation cost can by expressed by

$$\dot{\boldsymbol{C}}(\cdot) = [\dot{\boldsymbol{C}}(1),\ldots,\dot{\boldsymbol{C}}(z)], \quad (43)$$

with coordinates given by the linear equations (41)

$$\dot{\boldsymbol{C}}(u) \cong \sum_{b=1}^{\nu} \dot{p}_b [\boldsymbol{C}(u)]^{(b)}, \; u = 1,2,\ldots,z. \quad (44)$$

for the mean values of the system optimal total unconditional operation costs in the safety state subsets $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, where $[\boldsymbol{C}(u)]^{(b)}$, $b = 1,2,\ldots,\nu$, $u = 1,2,\ldots,z$, are the mean values of the system total conditional operation costs in the safety state subsets $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, at the particular system operation states z_b, $b = 1,2,\ldots,\nu$, and $\dot{p}_b$, $b = 1,2,\ldots,\nu$, are the system operation process optimal limit transient probabilities at these operation states given by (40).
The expressions for the optimal mean values of the system total operation costs in the particular safety states are

$$\dot{\bar{\boldsymbol{C}}}(u) = \dot{\boldsymbol{C}}(u) - \dot{\boldsymbol{C}}(u+1), \; u = 1,2,\ldots,z-1,$$

$$\dot{\bar{\boldsymbol{C}}}(z) = \dot{\boldsymbol{C}}(z), \quad (45)$$

where $\dot{\boldsymbol{C}}(u)$, $u = 1,2,\ldots,z$, are the optimal mean values of the system total unconditional operation costs in the safety state subsets $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, given by (42).

4. Maritime ferry technical system operation cost

4.1. Operation process

We will examine the operation cost of a selected maritime ferry technical system that is a member of the shipping critical infrastructure. The considered maritime ferry is described in (Kołowrocki & Magryta-Mut, 2020c; Kołowrocki et al., 2016; Magryta-Mut, 2020).
The maritime ferry operation process $Z(t)$, $t \geq 0$, was identified and specified in (Kołowrocki et al., 2016). Having regards to the opinions of experts on the varying in time operation process of the pondered maritime ferry system, we identify the eighteen operation states:

- an operation state z_1 – loading at Gdynia Port,
- an operation state z_2 – unmooring operations at Gdynia Port,
- an operation state z_3 – leaving Gdynia Port and navigation to "GD" buoy,
- an operation state z_4 – navigation at restricted waters from "GD" buoy to the end of Traffic Separation Scheme,
- an operation state z_5 – navigation at open waters from the end of Traffic Separation Scheme to "Angoering" buoy,
- an operation state z_6 – navigation at restricted waters from "Angoering" buoy to "Verko" berth at Karlskrona,
- an operation state z_7 – mooring operations at Karlskrona Port,
- an operation state z_8 – unloading at Karlskrona Port,
- an operation state z_9 – loading at Karlskrona Port,
- an operation state z_{10} – unmooring operations at Karlskrona Port,
- an operation state z_{11} – ferry turning at Karlskrona Port,

- an operation state z_{12} – leaving Karlskrona Port and navigation at restricted waters to “Angoering” buoy,
- an operation state z_{13} – navigation at open waters from “Angoering” buoy to the entering Traffic Separation Scheme,
- an operation state z_{14} – navigation at restricted waters from the entering Traffic Separation Scheme to “GD” buoy,
- an operation state z_{15} – navigation from “GD” buoy to turning area,
- an operation state z_{16} – ferry turning at Gdynia Port,
- an operation state z_{17} – mooring operations at Gdynia Port,
- an operation state z_{18} – unloading at Gdynia Port.

To identify the unknown parameters of the ferry technical system operation process the suitable statistical data coming from its real realizations should be collected. It is possible to collect these data because of the high frequency of the ferry voyages that result in a large number of its technical system operation process realizations. The ferry technical system operation process is very regular in the sense that the operation state changes are from the particular state z_b, $b = 1,2,\dots,17$, to the neighboring state z_{b+1}, $b = 1,2,\dots,17$, and from z_{18} to z_1 only.

The ferry technical system operation process $Z(t)$ characteristics are:

- the limit values of transients probabilities p_b, of the operation process $Z(t)$ at the particular operation states z_b, $b = 1,2,\dots,18$, (Kołowrocki & Magryta, 2020a, 2021; Kołowrocki & Soszyńska-Budny, 2011/2015):

$$p_1 = 0.038,\ p_2 = 0.002,\ p_3 = 0.026,$$
$$p_4 = 0.036,\ p_5 = 0.363,\ p_6 = 0.026,$$
$$p_7 = 0.005,\ p_8 = 0.016,\ p_9 = 0.037,$$
$$p_{10} = 0.002,\ p_{11} = 0.003,\ p_{12} = 0.016,$$
$$p_{13} = 0.351,\ p_{14} = 0.034,\ p_{15} = 0.024,$$
$$p_{16} = 0.003,\ p_{17} = 0.005,\ p_{18} = 0.013. \quad (46)$$

4.2. Operation cost

The subsystems S_1, S_2, S_3, S_4, S_5 use at particular operation states imply that the system at the particular operation states conditional instantaneous operation costs $[\boldsymbol{C}(t,u)]^{(b)}$, $u = 1,2,3,4$, $b = 1,2,\dots,18$, in the safety state subsets $\{1,2,3,4\}$, $\{2,3,4\}$, $\{3,4\}$, $\{4\}$ for $t \geq 0$, $b = 1,2,\dots,18$, are:

$$[\boldsymbol{C}(t,1)]^{(1)} = [\boldsymbol{C}(t,2)]^{(1)} = [\boldsymbol{C}(t,3)]^{(1)} = [\boldsymbol{C}(t,4)]^{(1)} = 93c,$$
$$[\boldsymbol{C}(t,1)]^{(2)} = [\boldsymbol{C}(t,2)]^{(2)} = [\boldsymbol{C}(t,3)]^{(2)} = [\boldsymbol{C}(t,4)]^{(2)} = 145c,$$
$$[\boldsymbol{C}(t,1)]^{(3)} = [\boldsymbol{C}(t,2)]^{(3)} = [\boldsymbol{C}(t,3)]^{(3)} = [\boldsymbol{C}(t,4)]^{(3)} = 120c,$$
$$[\boldsymbol{C}(t,1)]^{(4)} = [\boldsymbol{C}(t,2)]^{(4)} = [\boldsymbol{C}(t,3)]^{(4)} = [\boldsymbol{C}(t,4)]^{(4)} = 103c,$$
$$[\boldsymbol{C}(t,1)]^{(5)} = [\boldsymbol{C}(t,2)]^{(5)} = [\boldsymbol{C}(t,3)]^{(5)} = [\boldsymbol{C}(t,4)]^{(5)} = 103c,$$
$$[\boldsymbol{C}(t,1)]^{(6)} = [\boldsymbol{C}(t,2)]^{(6)} = [\boldsymbol{C}(t,3)]^{(6)} = [\boldsymbol{C}(t,4)]^{(6)} = 123c,$$
$$[\boldsymbol{C}(t,1)]^{(7)} = [\boldsymbol{C}(t,2)]^{(7)} = [\boldsymbol{C}(t,3)]^{(7)} = [\boldsymbol{C}(t,4)]^{(7)} = 145c,$$
$$[\boldsymbol{C}(t,1)]^{(8)} = [\boldsymbol{C}(t,2)]^{(8)} = [\boldsymbol{C}(t,3)]^{(8)} = [\boldsymbol{C}(t,4)]^{(8)} = 83c,$$
$$[\boldsymbol{C}(t,1)]^{(9)} = [\boldsymbol{C}(t,2)]^{(9)} = [\boldsymbol{C}(t,3)]^{(9)} = [\boldsymbol{C}(t,4)]^{(9)} = 83c,$$
$$[\boldsymbol{C}(t,1)]^{(10)} = [\boldsymbol{C}(t,2)]^{(10)} = [\boldsymbol{C}(t,3)]^{(10)} = [\boldsymbol{C}(t,4)]^{(10)} = 145c,$$
$$[\boldsymbol{C}(t,1)]^{(11)} = [\boldsymbol{C}(t,2)]^{(11)} = [\boldsymbol{C}(t,3)]^{(11)} = [\boldsymbol{C}(t,4)]^{(11)} = 120c,$$
$$[\boldsymbol{C}(t,1)]^{(12)} = [\boldsymbol{C}(t,2)]^{(12)} = [\boldsymbol{C}(t,3)]^{(12)} = [\boldsymbol{C}(t,4)]^{(12)} = 103c,$$
$$[\boldsymbol{C}(t,1)]^{(13)} = [\boldsymbol{C}(t,2)]^{(13)} = [\boldsymbol{C}(t,3)]^{(13)} = [\boldsymbol{C}(t,4)]^{(13)} = 103c,$$
$$[\boldsymbol{C}(t,1)]^{(14)} = [\boldsymbol{C}(t,2)]^{(14)} = [\boldsymbol{C}(t,3)]^{(14)} = [\boldsymbol{C}(t,4)]^{(14)} = 103c,$$
$$[\boldsymbol{C}(t,1)]^{(15)} = [\boldsymbol{C}(t,2)]^{(15)} = [\boldsymbol{C}(t,3)]^{(15)} = [\boldsymbol{C}(t,4)]^{(15)} = 120c,$$
$$[\boldsymbol{C}(t,1)]^{(16)} = [\boldsymbol{C}(t,2)]^{(16)} = [\boldsymbol{C}(t,3)]^{(16)} = [\boldsymbol{C}(t,4)]^{(16)} = 120c,$$
$$[\boldsymbol{C}(t,1)]^{(17)} = [\boldsymbol{C}(t,2)]^{(17)} = [\boldsymbol{C}(t,3)]^{(17)} = [\boldsymbol{C}(t,4)]^{(17)} = 145c,$$
$$[\boldsymbol{C}(t,1)]^{(18)} = [\boldsymbol{C}(t,2)]^{(18)} = [\boldsymbol{C}(t,3)]^{(18)} = [\boldsymbol{C}(t,4)]^{(18)} = 93c. \quad (47)$$

The mean values $[\boldsymbol{\mu}(u)]^{(b)}$, $u = 1,2,3,4$, of the ferry conditional lifetimes $[T(u)]^{(b)}$, $u = 1,2,3,4$, in the safety state subset $\{1,2,3,4\}$, $\{2,3,4\}$, $\{3,4\}$ and $\{4\}$ at the operation state z_b, $b = 1,2,\dots,18$, determined in Section2, respectively (expressed in years) are:

$$[\boldsymbol{\mu}(1)]^{(1)} \cong 1.70476,\ [\boldsymbol{\mu}(1)]^{(2)} \cong 1.60772,$$
$$[\boldsymbol{\mu}(1)]^{(3)} = 1.68087,\ [\boldsymbol{\mu}(1)]^{(4)} = 1.6956,$$
$$[\boldsymbol{\mu}(1)]^{(5)} \cong 1.69547,\ [\boldsymbol{\mu}(1)]^{(6)} \cong 1.67434,$$
$$[\boldsymbol{\mu}(1)]^{(7)} \cong 1.54736,\ [\boldsymbol{\mu}(1)]^{(8)} \cong 1.72871,$$
$$[\boldsymbol{\mu}(1)]^{(9)} \cong 1.72871,\ [\boldsymbol{\mu}(1)]^{(10)} \cong 1.60772,$$
$$[\boldsymbol{\mu}(1)]^{(11)} \cong 1.6102,\ [\boldsymbol{\mu}(1)]^{(12)} \cong 1.70148,$$

$[\boldsymbol{\mu}(1)]^{(13)} \cong 1.69547$, $[\boldsymbol{\mu}(1)]^{(14)} \cong 1.6863$,
$[\boldsymbol{\mu}(1)]^{(15)} \cong 1.68087$, $[\boldsymbol{\mu}(1)]^{(16)} \cong 1.61025$,
$[\boldsymbol{\mu}(1)]^{(17)} \cong 1.54736$, $[\boldsymbol{\mu}(1)]^{(18)} \cong 1.70476$,

$[\boldsymbol{\mu}(2)]^{(1)} \cong 1.41708$, $[\boldsymbol{\mu}(2)]^{(2)} \cong 1.32879$,
$[\boldsymbol{\mu}(2)]^{(3)} = 1.3912$, $[\boldsymbol{\mu}(2)]^{(4)} = 1.39303$,
$[\boldsymbol{\mu}(2)]^{(5)} \cong 1.39292$, $[\boldsymbol{\mu}(2)]^{(6)} \cong 1.37699$,
$[\boldsymbol{\mu}(2)]^{(7)} \cong 1.27865$, $[\boldsymbol{\mu}(2)]^{(8)} \cong 1.43719$,
$[\boldsymbol{\mu}(2)]^{(9)} \cong 1.43719$, $[\boldsymbol{\mu}(2)]^{(10)} \cong 1.32879$,
$[\boldsymbol{\mu}(2)]^{(11)} \cong 1.3336$, $[\boldsymbol{\mu}(2)]^{(12)} \cong 1.39692$,
$[\boldsymbol{\mu}(2)]^{(13)} \cong 1.39292$, $[\boldsymbol{\mu}(2)]^{(14)} \cong 1.3854$,
$[\boldsymbol{\mu}(2)]^{(15)} \cong 1.3912$, $[\boldsymbol{\mu}(2)]^{(16)} \cong 1.3336$,
$[\boldsymbol{\mu}(2)]^{(17)} \cong 1.27865$, $[\boldsymbol{\mu}(2)]^{(18)} \cong 1.41708$,

$[\boldsymbol{\mu}(3)]^{(1)} \cong 1.22861$, $[\boldsymbol{\mu}(3)]^{(2)} \cong 1.18936$,
$[\boldsymbol{\mu}(3)]^{(3)} = 1.24553$, $[\boldsymbol{\mu}(3)]^{(4)} = 1.24632$,
$[\boldsymbol{\mu}(3)]^{(5)} \cong 1.24619$, $[\boldsymbol{\mu}(3)]^{(6)} \cong 1.23228$,
$[\boldsymbol{\mu}(3)]^{(7)} \cong 1.15851$, $[\boldsymbol{\mu}(3)]^{(8)} \cong 1.26722$,
$[\boldsymbol{\mu}(3)]^{(9)} \cong 1.26722$, $[\boldsymbol{\mu}(3)]^{(10)} \cong 1.18936$,
$[\boldsymbol{\mu}(3)]^{(11)} \cong 1.19593$, $[\boldsymbol{\mu}(3)]^{(12)} \cong 1.24985$,
$[\boldsymbol{\mu}(3)]^{(13)} \cong 1.24619$, $[\boldsymbol{\mu}(3)]^{(14)} \cong 1.23945$,
$[\boldsymbol{\mu}(3)]^{(15)} \cong 1.24553$, $[\boldsymbol{\mu}(3)]^{(16)} \cong 1.19593$,
$[\boldsymbol{\mu}(3)]^{(17)} \cong 1.15851$, $[\boldsymbol{\mu}(3)]^{(18)} \cong 1.22861$,

$[\boldsymbol{\mu}(4)]^{(1)} \cong 1.11601$, $[\boldsymbol{\mu}(4)]^{(2)} \cong 1.06574$,
$[\boldsymbol{\mu}(4)]^{(3)} = 1.11512$, $[\boldsymbol{\mu}(4)]^{(4)} = 1.11522$,
$[\boldsymbol{\mu}(4)]^{(5)} \cong 1.1151$, $[\boldsymbol{\mu}(4)]^{(6)} \cong 1.10301$,
$[\boldsymbol{\mu}(4)]^{(7)} \cong 1.02847$, $[\boldsymbol{\mu}(4)]^{(8)} \cong 1.13163$,
$[\boldsymbol{\mu}(4)]^{(9)} \cong 1.13163$, $[\boldsymbol{\mu}(4)]^{(10)} \cong 1.06574$,
$[\boldsymbol{\mu}(4)]^{(11)} \cong 1.07262$, $[\boldsymbol{\mu}(4)]^{(12)} \cong 1.11836$,
$[\boldsymbol{\mu}(4)]^{(13)} \cong 1.1151$, $[\boldsymbol{\mu}(4)]^{(14)} \cong 1.1091$,
$[\boldsymbol{\mu}(4)]^{(15)} \cong 1.11512$, $[\boldsymbol{\mu}(4)]^{(16)} \cong 1.07262$,
$[\boldsymbol{\mu}(4)]^{(17)} \cong 1.02847$, $[\boldsymbol{\mu}(4)]^{(18)} \cong 1.11601$. (48)

Applying the formula (24) to (47) and (48) we get the approximate mean values $[\boldsymbol{C}_1(1)]^{(b)}$, $b = 1,2,\ldots,18$, of the total costs of the entire maritime ferry technical system at the particular operation states given by:

$[\boldsymbol{C}(1)]^{(1)} = 1.70476 \cdot 93c \cong 158.54268c$ PLN,
$[\boldsymbol{C}(1)]^{(2)} = 1.60772 \cdot 145c \cong 233.11940c$ PLN,
$[\boldsymbol{C}(1)]^{(3)} = 1.68087 \cdot 120c \cong 201.70440c$ PLN,
$[\boldsymbol{C}(1)]^{(4)} = 1.6956 \cdot 103c \cong 174.64680c$ PLN,
$[\boldsymbol{C}(1)]^{(5)} = 1.69547 \cdot 103c \cong 174.63341c$ PLN,
$[\boldsymbol{C}(1)]^{(6)} = 1.67434 \cdot 123c \cong 205.94382c$ PLN,
$[\boldsymbol{C}(1)]^{(7)} = 1.54736 \cdot 145c \cong 224.36720c$ PLN,
$[\boldsymbol{C}(1)]^{(8)} = 1.72871 \cdot 83c \cong 143.48293c$ PLN,
$[\boldsymbol{C}(1)]^{(9)} = 1.72871 \cdot 83c \cong 143.48293c$ PLN,
$[\boldsymbol{C}(1)]^{(10)} = 1.60772 \cdot 145c \cong 233.11940c$ PLN,
$[\boldsymbol{C}(1)]^{(11)} = 1.6102 \cdot 120c \cong 193.22400c$ PLN,
$[\boldsymbol{C}(1)]^{(12)} = 1.70148 \cdot 103c \cong 175.25244c$ PLN,
$[\boldsymbol{C}(1)]^{(13)} = 1.69547 \cdot 103c \cong 174.63341c$ PLN,
$[\boldsymbol{C}(1)]^{(14)} = 1.6863 \cdot 103c \cong 173.68890c$ PLN,
$[\boldsymbol{C}(1)]^{(15)} = 1.68087 \cdot 120c \cong 201.70440c$ PLN,
$[\boldsymbol{C}(1)]^{(16)} = 1.61025 \cdot 120c \cong 193.23000c$ PLN,
$[\boldsymbol{C}(1)]^{(17)} = 1.54736 \cdot 145c \cong 224.36720c$ PLN,
$[\boldsymbol{C}(1)]^{(18)} = 1.70476 \cdot 93c \cong 158.54268c$ PLN, (49)

in the safety state subset {1,2,3,4} and

$[\boldsymbol{C}(2)]^{(1)} = 1.41708 \cdot 93c \cong 131.78844c$ PLN,
$[\boldsymbol{C}(2)]^{(2)} = 1.32879 \cdot 145c \cong 192.67455c$ PLN,
$[\boldsymbol{C}(2)]^{(3)} = 1.3912 \cdot 120c \cong 166.94400c$ PLN,
$[\boldsymbol{C}(2)]^{(4)} = 1.39303 \cdot 103c \cong 143.48209c$ PLN,
$[\boldsymbol{C}(2)]^{(5)} = 1.39292 \cdot 103c \cong 143.47076c$ PLN,
$[\boldsymbol{C}(2)]^{(6)} = 1.37699 \cdot 123c \cong 169.36977c$ PLN,
$[\boldsymbol{C}(2)]^{(7)} = 1.27865 \cdot 145c \cong 185.40425c$ PLN,
$[\boldsymbol{C}(2)]^{(8)} = 1.43719 \cdot 83c \cong 119.286777c$ PLN,
$[\boldsymbol{C}(2)]^{(9)} = 1.43719 \cdot 83c \cong 119.286777c$ PLN,
$[\boldsymbol{C}(2)]^{(10)} = 1.32879 \cdot 145c \cong 192.67455c$ PLN,
$[\boldsymbol{C}(2)]^{(11)} = 1.3336 \cdot 120c \cong 160.03200c$ PLN,
$[\boldsymbol{C}(2)]^{(12)} = 1.39692 \cdot 103c \cong 143.88276c$ PLN,
$[\boldsymbol{C}(2)]^{(13)} = 1.39292 \cdot 103c \cong 143.47076c$ PLN,
$[\boldsymbol{C}(2)]^{(14)} = 1.3854 \cdot 103c \cong 142.69620c$ PLN,
$[\boldsymbol{C}(2)]^{(15)} = 1.3912 \cdot 120c \cong 166.94400c$ PLN,
$[\boldsymbol{C}(2)]^{(16)} = 1.3336 \cdot 120c \cong 160.03200c$ PLN,
$[\boldsymbol{C}(2)]^{(17)} = 1.27865 \cdot 145c \cong 185.40425c$ PLN,
$[\boldsymbol{C}(2)]^{(18)} = 1.41708 \cdot 93c \cong 131.78844c$ PLN, (50)

in the safety state subset {2,3,4} and

$[\boldsymbol{C}(3)]^{(1)} = 1.22861 \cdot 93c \cong 114.26073c$ PLN,
$[\boldsymbol{C}(3)]^{(2)} = 1.18936 \cdot 145c \cong 172.45720c$ PLN,
$[\boldsymbol{C}(3)]^{(3)} = 1.24553 \cdot 120c \cong 149.46360c$ PLN,
$[\boldsymbol{C}(3)]^{(4)} = 1.24632 \cdot 103c \cong 128.37096c$ PLN,
$[\boldsymbol{C}(3)]^{(5)} = 1.24619 \cdot 103c \cong 128.35757c$ PLN,
$[\boldsymbol{C}(3)]^{(6)} = 1.23228 \cdot 123c \cong 151.57044c$ PLN,
$[\boldsymbol{C}(3)]^{(7)} = 1.15851 \cdot 145c \cong 167.98395c$ PLN,
$[\boldsymbol{C}(3)]^{(8)} = 1.26722 \cdot 83c \cong 105.17926c$ PLN,
$[\boldsymbol{C}(3)]^{(9)} = 1.26722 \cdot 83c \cong 105.17926c$ PLN,
$[\boldsymbol{C}(3)]^{(10)} = 1.18936 \cdot 145c \cong 172.45720c$ PLN,
$[\boldsymbol{C}(3)]^{(11)} = 1.19593 \cdot 120c \cong 143.51160c$ PLN,
$[\boldsymbol{C}(3)]^{(12)} = 1.24985 \cdot 103c \cong 128.73455c$ PLN,
$[\boldsymbol{C}(3)]^{(13)} = 1.24619 \cdot 103c \cong 128.35757c$ PLN,
$[\boldsymbol{C}(3)]^{(14)} = 1.23945 \cdot 103c \cong 127.66335c$ PLN,
$[\boldsymbol{C}(3)]^{(15)} = 1.24553 \cdot 120c \cong 149.46360c$ PLN,
$[\boldsymbol{C}(3)]^{(16)} = 1.19593 \cdot 120c \cong 143.51160c$ PLN,
$[\boldsymbol{C}(3)]^{(17)} = 1.15851 \cdot 145c \cong 167.98395c$ PLN,

$[\boldsymbol{C}(3)]^{(18)} = 1.22861 \cdot 93c \cong 114.26073c$ PLN, (51)

in the safety state subset {3,4} and

$[\boldsymbol{C}(4)]^{(1)} = 1.11601 \cdot 93c \cong 103.78893c$ PLN,
$[\boldsymbol{C}(4)]^{(2)} = 1.06574 \cdot 145c \cong 154.53230c$ PLN,
$[\boldsymbol{C}(4)]^{(3)} = 1.11512 \cdot 120c \cong 133.81440c$ PLN,
$[\boldsymbol{C}(4)]^{(4)} = 1.11522 \cdot 103c \cong 114.86766c$ PLN,
$[\boldsymbol{C}(4)]^{(5)} = 1.1151 \cdot 103c \cong 114.85530c$ PLN,
$[\boldsymbol{C}(4)]^{(6)} = 1.10301 \cdot 123c \cong 135.67023c$ PLN,
$[\boldsymbol{C}(4)]^{(7)} = 1.02847 \cdot 145c \cong 149.12815c$ PLN,
$[\boldsymbol{C}(4)]^{(8)} = 1.13163 \cdot 83c \cong 93.92529c$ PLN,
$[\boldsymbol{C}(4)]^{(9)} = 1.13163 \cdot 83c \cong 93.92529c$ PLN,
$[\boldsymbol{C}(4)]^{(10)} = 1.06574 \cdot 145c \cong 154.53230c$ PLN,
$[\boldsymbol{C}(4)]^{(11)} = 1.07262 \cdot 120c \cong 128.71440c$ PLN,
$[\boldsymbol{C}(4)]^{(12)} = 1.11836 \cdot 103c \cong 115.19108c$ PLN,
$[\boldsymbol{C}(4)]^{(13)} = 1.1151 \cdot 103c \cong 114.85530c$ PLN,
$[\boldsymbol{C}(4)]^{(14)} = 1.1091 \cdot 103c \cong 114.23730c$ PLN,
$[\boldsymbol{C}(4)]^{(15)} = 1.11512 \cdot 120c \cong 133.81440c$ PLN,
$[\boldsymbol{C}(4)]^{(16)} = 1.07262 \cdot 120c \cong 128.71440c$ PLN,
$[\boldsymbol{C}(4)]^{(17)} = 1.02847 \cdot 145c \cong 149.12815c$ PLN,
$[\boldsymbol{C}(4)]^{(18)} = 1.11601 \cdot 93c \cong 103.78893c$ PLN, (52)

in the safety state subset {4}.
Considering the values of $[\boldsymbol{C}(u)]^{(b)}$, $u = 1,2$, $b = 1,2,\ldots,18$, from (49)–(52) and the values of transient probabilities p_b, $b = 1,2,\ldots,18$, the maritime ferry technical system total unconditional operation cost, according to (23), is given by

$$\begin{aligned}\boldsymbol{C}(1) \cong\ & p_1[\boldsymbol{C}(1)]^{(1)} + p_2[\boldsymbol{C}(1)]^{(2)} + p_3[\boldsymbol{C}(1)]^{(3)} \\ & + p_4[\boldsymbol{C}(1)]^{(4)} + p_5[\boldsymbol{C}(1)]^{(5)} + p_6[\boldsymbol{C}(1)]^{(6)} \\ & + p_7[\boldsymbol{C}(1)]^{(7)} + p_8[\boldsymbol{C}(1)]^{(8)} + p_9[\boldsymbol{C}(1)]^{(9)} \\ & + p_{10}[\boldsymbol{C}(1)]^{(10)} + p_{11}[\boldsymbol{C}(1)]^{(11)} + p_{12}[\boldsymbol{C}(1)]^{(12)} \\ & + p_{13}[\boldsymbol{C}(1)]^{(13)} + p_{14}[\boldsymbol{C}(1)]^{(14)} + p_{15}[\boldsymbol{C}(1)]^{(15)} \\ & + p_{16}[\boldsymbol{C}(1)]^{(16)} + p_{17}[\boldsymbol{C}(1)]^{(17)} + p_{18}[\boldsymbol{C}(1)]^{(18)} \\ & \cong 175.15054c, \qquad (53)\end{aligned}$$

in the safety state subset {1,2,3,4} and

$$\begin{aligned}\boldsymbol{C}(2) \cong\ & p_1[\boldsymbol{C}(2)]^{(1)} + p_2[\boldsymbol{C}(2)]^{(2)} + p_3[\boldsymbol{C}(2)]^{(3)} \\ & + p_4[\boldsymbol{C}(2)]^{(4)} + p_5[\boldsymbol{C}(2)]^{(5)} + p_6[\boldsymbol{C}(2)]^{(6)} \\ & + p_7[\boldsymbol{C}(2)]^{(7)} + p_8[\boldsymbol{C}(2)]^{(8)} + p_9[\boldsymbol{C}(2)]^{(9)} \\ & + p_{10}[\boldsymbol{C}(2)]^{(10)} + p_{11}[\boldsymbol{C}(2)]^{(11)} + p_{12}[\boldsymbol{C}(2)]^{(12)} \\ & + p_{13}[\boldsymbol{C}(2)]^{(13)} + p_{14}[\boldsymbol{C}(2)]^{(14)} + p_{15}[\boldsymbol{C}(2)]^{(15)} \\ & + p_{16}[\boldsymbol{C}(2)]^{(16)} + p_{17}[\boldsymbol{C}(2)]^{(17)} + p_{18}[\boldsymbol{C}(2)]^{(18)} \\ & \cong 144.13643c, \qquad (54)\end{aligned}$$

in the safety state subset {2,3,4} and

$$\begin{aligned}\boldsymbol{C}(3) \cong\ & p_1[\boldsymbol{C}(3)]^{(1)} + p_2[\boldsymbol{C}(3)]^{(2)} + p_3[\boldsymbol{C}(3)]^{(3)} \\ & + p_4[\boldsymbol{C}(3)]^{(4)} + p_5[\boldsymbol{C}(3)]^{(5)} + p_6[\boldsymbol{C}(3)]^{(6)} \\ & + p_7[\boldsymbol{C}(3)]^{(7)} + p_8[\boldsymbol{C}(3)]^{(8)} + p_9[\boldsymbol{C}(3)]^{(9)} \\ & + p_{10}[\boldsymbol{C}(3)]^{(10)} + p_{11}[\boldsymbol{C}(3)]^{(11)} + p_{12}[\boldsymbol{C}(3)]^{(12)} \\ & + p_{13}[\boldsymbol{C}(3)]^{(13)} + p_{14}[\boldsymbol{C}(3)]^{(14)} + p_{15}[\boldsymbol{C}(3)]^{(15)} \\ & + p_{16}[\boldsymbol{C}(3)]^{(16)} + p_{17}[\boldsymbol{C}(3)]^{(17)} + p_{18}[\boldsymbol{C}(3)]^{(18)} \\ & \cong 128.71551c, \qquad (55)\end{aligned}$$

in the safety state subset {3,4} and

$$\begin{aligned}\boldsymbol{C}(4) \cong\ & p_1[\boldsymbol{C}(4)]^{(1)} + p_2[\boldsymbol{C}(4)]^{(2)} + p_3[\boldsymbol{C}(4)]^{(3)} \\ & + p_4[\boldsymbol{C}(4)]^{(4)} + p_5[\boldsymbol{C}(4)]^{(5)} + p_6[\boldsymbol{C}(4)]^{(6)} \\ & + p_7[\boldsymbol{C}(4)]^{(7)} + p_8[\boldsymbol{C}(4)]^{(8)} + p_9[\boldsymbol{C}(4)]^{(9)} \\ & + p_{10}[\boldsymbol{C}(4)]^{(10)} + p_{11}[\boldsymbol{C}(4)]^{(11)} + p_{12}[\boldsymbol{C}(4)]^{(12)} \\ & + p_{13}[\boldsymbol{C}(4)]^{(13)} + p_{14}[\boldsymbol{C}(4)]^{(14)} + p_{15}[\boldsymbol{C}(4)]^{(15)} \\ & + p_{16}[\boldsymbol{C}(4)]^{(16)} + p_{17}[\boldsymbol{C}(4)]^{(17)} + p_{18}[\boldsymbol{C}(4)]^{(18)} \\ & \cong 115.24016c, \qquad (56)\end{aligned}$$

in the safety state subset {4}.

4.3. Cost optimization

Assuming the critical safety state $r = 2$ and considering (50) to find the minimum value of this cost, we define the objective function given by (27), in the following form

$$\begin{aligned}\boldsymbol{C}(2) =\ & p_1 \cdot 131.78844c + p_2 \cdot 192.67455c \\ & + p_3 \cdot 166.94400c + p_4 \cdot 143.48209c \\ & + p_5 \cdot 143.47076c + p_6 \cdot 169.36977c \\ & + p_7 \cdot 185.40425c + p_8 \cdot 119.28677c \\ & + p_9 \cdot 119.28677c + p_{10} \cdot 192.67455c \\ & + p_{11} \cdot 160.03200c + p_{12} \cdot 143.88276c \\ & + p_{13} \cdot 143.47076c + p_{14} \cdot 142.69620c \\ & + p_{15} \cdot 166.94400c + p_{16} \cdot 160.03200c \\ & + p_{17} \cdot 185.40425c + p_{18} \cdot 131.78844c. \qquad (57)\end{aligned}$$

The lower $\breve{p}_b$, and upper $\hat{p}_b$, bounds of the unknown optimal values of transient probabilities p_b, $b = 1,2,\ldots,18$, respectively are (Kołowrocki & Soszyńska-Budny, 2011/2015):

$\breve{p}_1 = 0.0006$, $\breve{p}_2 = 0.001$, $\breve{p}_3 = 0.018$,
$\breve{p}_4 = 0.027$, $\breve{p}_5 = 0.286$, $\breve{p}_6 = 0.018$,
$\breve{p}_7 = 0.002$, $\breve{p}_8 = 0.001$, $\breve{p}_9 = 0.001$,
$\breve{p}_{10} = 0.001$, $\breve{p}_{11} = 0.002$, $\breve{p}_{12} = 0.013$,
$\breve{p}_{13} = 0.286$, $\breve{p}_{14} = 0.025$, $\breve{p}_{15} = 0.018$,
$\breve{p}_{16} = 0.002$, $\breve{p}_{17} = 0.002$, $\breve{p}_{18} = 0.001$,

$\hat{p}_1 = 0.056,\ \hat{p}_2 = 0.002,\ \hat{p}_3 = 0.027,$
$\hat{p}_4 = 0.056,\ \hat{p}_5 = 0.780,\ \hat{p}_6 = 0.024,$
$\hat{p}_7 = 0.018,\ \hat{p}_8 = 0.018,\ \hat{p}_9 = 0.056,$
$\hat{p}_{10} = 0.003, \hat{p}_{11} = 0.004,\ \hat{p}_{12} = 0.024,$
$\hat{p}_{13} = 0.780,\ \hat{p}_{14} = 0.043,\ \hat{p}_{15} = 0.024,$
$\hat{p}_{16} = 0.004,\ \hat{p}_{17} = 0.007,\ \hat{p}_{18} = 0.018.$ (58)

Therefore, according to (28)–(29), we assume the following bound constraints

$0.0006 \le p_1 \le 0.056,\ 0.001 \le p_2 \le 0.002,$
$0.018 \le p_3 \le 0.027,\ 0.027 \le p_4 \le 0.056,$
$0.286 \le p_5 \le 0.780,\ 0.018 \le p_6 \le 0.024,$
$0.002 \le p_7 \le 0.018,\ 0.001 \le p_8 \le 0.018,$
$0.001 \le p_9 \le 0.056,\ 0.001 \le p_{10} \le 0.003,$
$0.002 \le p_{11} \le 0.004,\ 0.013 \le p_{12} \le 0.024,$
$0.286 \le p_{13} \le 0.780,\ 0.025 \le p_{14} \le 0.043,$
$0.018 \le p_{15} \le 0.024,\ 0.002 \le p_{16} \le 0.004,$
$0.002 \le p_{17} \le 0.007,\ 0.001 \le p_{18} \le 0.018,$
$\sum_{b=1}^{18} p_b = 1.$ (59)

Now, before we find optimal values $\dot{p}_b$ of the transient probabilities p_b, $b = 1,2,\ldots,18$, that minimize the objective function (57), we arrange the mean values of the ferry technical system conditional operation costs $[C(2)]^{(b)}$, $b = 1,2,\ldots,18$, determined by (50), in non-decreasing order

$119.28677c \le 119.28677c \le 131.78844c$
$\le 131.78844c \le 142.69620c \le 143.47076c$
$\le 143.47076c \le 143.48209c \le 143.88276c$
$\le 160.03200c \le 160.03200c \le 166.94400c$
$\le 166.94400c \le 169.36977c \le 185.40425c$
$\le 185.40425c \le 192.67455c \le 192.67455c,$

i.e.

$$[\boldsymbol{C}(2)]^{(8)} \le [\boldsymbol{C}(2)]^{(9)} \le [\boldsymbol{C}(2)]^{(1)} \le [\boldsymbol{C}(2)]^{(18)} \le [\boldsymbol{C}(2)]^{(14)} \le [\boldsymbol{C}(2)]^{(5)} \le [\boldsymbol{C}(2)]^{(13)} \le [\boldsymbol{C}(2)]^{(4)} \le [\boldsymbol{C}(2)]^{(12)} \le [\boldsymbol{C}(2)]^{(11)} \le [\boldsymbol{C}(2)]^{(16)} \le [\boldsymbol{C}(2)]^{(3)} \le [\boldsymbol{C}(2)]^{(15)} \le [\boldsymbol{C}(2)]^{(6)} \le [\boldsymbol{C}(2)]^{(7)} \le [\boldsymbol{C}(2)]^{(17)} \le [\boldsymbol{C}(2)]^{(2)} \le [\boldsymbol{C}(2)]^{(10)}. \quad (60)$$

Further, according to (30), we substitute

$x_1 = p_8,\ x_2 = p_9,\ x_3 = p_1,\ x_4 = p_{18},$
$x_5 = p_{14},\ x_6 = p_5,\ x_7 = p_{13},\ x_8 = p_4,$
$x_9 = p_{12},\ x_{10} = p_{11},\ x_{11} = p_{16},\ x_{12} = p_3,$
$x_{13} = p_{15},\ x_{14} = p_6,\ x_{15} = p_7,\ x_{16} = p_{17},$
$x_{17} = p_2,\ x_{18} = p_{10},$ (61)

and

$\breve{x}_1 = \breve{p}_8 = 0.001,\ \breve{x}_2 = \breve{p}_9 = 0.001,$
$\breve{x}_3 = \breve{p}_1 = 0.0006,\ \breve{x}_4 = \breve{p}_{18} = 0.001,$
$\breve{x}_5 = \breve{p}_{14} = 0.025,\ \breve{x}_6 = \breve{p}_5 = 0.286,$
$\breve{x}_7 = \breve{p}_{13} = 0.286,\ \breve{x}_8 = \breve{p}_4 = 0.027,$
$\breve{x}_9 = \breve{p}_{12} = 0.013,\ \breve{x}_{10} = \breve{p}_{11} = 0.002,$
$\breve{x}_{11} = \breve{p}_{16} = 0.002,\ \breve{x}_{12} = \breve{p}_3 = 0.018,$
$\breve{x}_{13} = \breve{p}_{15} = 0.018,\ \breve{x}_{14} = \breve{p}_6 = 0.018,$
$\breve{x}_{15} = \breve{p}_7 = 0.002,\ \breve{x}_{16} = \breve{p}_{17} = 0.002,$
$\breve{x}_{17} = \breve{p}_2 = 0.001,\ \breve{x}_{18} = \breve{p}_{10} = 0.001,$

$\hat{x}_1 = \hat{p}_8 = 0.018,\ \hat{x}_2 = \hat{p}_9 = 0.056,$
$\hat{x}_3 = \hat{p}_1 = 0.056,\ \hat{x}_4 = \hat{p}_{18} = 0.018,$
$\hat{x}_5 = \hat{p}_{14} = 0.043,\ \hat{x}_6 = \hat{p}_5 = 0.780,$
$\hat{x}_7 = \hat{p}_{13} = 0.780,\ \hat{x}_8 = \hat{p}_4 = 0.056,$
$\hat{x}_9 = \hat{p}_{12} = 0.024,\ \hat{x}_{10} = \hat{p}_{11} = 0.004,$
$\hat{x}_{11} = \hat{p}_{16} = 0.004,\ \hat{x}_{12} = \hat{p}_3 = 0.027,$
$\hat{x}_{13} = \hat{p}_{15} = 0.024,\ \hat{x}_{14} = \hat{p}_6 = 0.024,$
$\hat{x}_{15} = \hat{p}_7 = 0.018,\ \hat{x}_{16} = \hat{p}_{17} = 0.007,$
$\hat{x}_{17} = \hat{p}_2 = 0.002,\ \hat{x}_{18} = \hat{p}_{10} = 0.003$ (62)

and we minimize with respect to x_i, $i = 1,2,\ldots,18$, the linear form (57) that according to (30)–(32) and (60)–(61) takes the form

$$\begin{aligned}\boldsymbol{C}(2) = {} & x_1 \cdot 119.28677c + x_2 \cdot 119.28677c \\ & + x_3 \cdot 131.78844c + x_4 \cdot 131.78844c \\ & + x_5 \cdot 142.69620c + x_6 \cdot 143.47076c \\ & + x_7 \cdot 143.47076c + x_8 \cdot 143.48209c \\ & + x_9 \cdot 143.88276c + x_{10} \cdot 160.03200c \\ & + x_{11} \cdot 160.03200c + x_{12} \cdot 166.94400c \\ & + x_{13} \cdot 166.94400c + x_{14} \cdot 169.36977c \\ & + x_{15} \cdot 185.40425c + x_{16} \cdot 185.40425c \\ & + x_{17} \cdot 192.67455c + x_{18} \cdot 192.67455c, \quad (63)\end{aligned}$$

with the following bound constraints

$0.001 \le x_1 \le 0.018,\ 0.001 \le x_2 \le 0.056,$

$0.0006 \le x_3 \le 0.056,\ 0.001 \le x_4 \le 0.018,$
$0.025 \le x_5 \le 0.043,\ 0.286 \le x_6 \le 0.780,$
$0.286 \le x_7 \le 0.780,\ 0.027 \le x_8 \le 0.056,$
$0.013 \le x_9 \le 0.024,\ 0.002 \le x_{10} \le 0.004,$
$0.002 \le x_{11} \le 0.004,\ 0.018 \le x_{12} \le 0.027,$
$0.018 \le x_{13} \le 0.027,\ 0.018 \le x_{14} \le 0.024,$
$0.002 \le x_{15} \le 0.018,\ 0.002 \le x_{16} \le 0.007,$
$0.001 \le x_{17} \le 0.002,\ 0.001 \le x_{18} \le 0.003,$

$$\sum_{i=1}^{18} x_i = 1. \tag{64}$$

According to (34), we calculate

$$\breve{x} = \sum_{i=1}^{18} \breve{x}_i = 0.7046,$$

$$\hat{y} = 1 - \breve{x} = 1 - 0.7046 = 0.2954 \tag{65}$$

and according to (35), we find

$$\begin{aligned}
&\breve{x}^0 = 0,\ \widehat{x}^0 = 0,\ \widehat{x}^0 - \breve{x}^0 = 0,\\
&\breve{x}^1 = 0.001,\ \widehat{x}^1 = 0.018,\ \widehat{x}^1 - \breve{x}^1 = 0.017,\\
&\breve{x}^2 = 0.002,\ \widehat{x}^2 = 0.074,\ \widehat{x}^2 - \breve{x}^2 = 0.072,\\
&\breve{x}^3 = 0.0026,\ \widehat{x}^3 = 0.13,\ \widehat{x}^3 - \breve{x}^3 = 0.1274,\\
&\breve{x}^4 = 0.0036,\ \widehat{x}^4 = 0.148,\ \widehat{x}^4 - \breve{x}^4 = 0.1444,\\
&\breve{x}^5 = 0.0286,\ \widehat{x}^5 = 0.191,\ \widehat{x}^5 - \breve{x}^5 = 0.1624,\\
&\breve{x}^6 = 0.3146,\ \widehat{x}^6 = 0.971,\ \widehat{x}^6 - \breve{x}^6 = 0.6564,\\
&\breve{x}^7 = 0.6006,\ \widehat{x}^7 = 1.751,\ \widehat{x}^7 - \breve{x}^7 = 1.1504,\\
&\breve{x}^8 = 0.6276,\ \widehat{x}^8 = 1.807,\ \widehat{x}^8 - \breve{x}^8 = 1.1794,\\
&\breve{x}^9 = 0.6406,\ \widehat{x}^9 = 1.831,\ \widehat{x}^9 - \breve{x}^9 = 1.1904,\\
&\breve{x}^{10} = 0.6426,\ \widehat{x}^{10} = 1.835,\ \widehat{x}^{10} - \breve{x}^{10} = 1.1924,\\
&\breve{x}^{11} = 0.6446,\ \widehat{x}^{11} = 1.839,\ \widehat{x}^{11} - \breve{x}^{11} = 1.1944,\\
&\breve{x}^{12} = 0.6626,\ \widehat{x}^{12} = 1.866,\ \widehat{x}^{12} - \breve{x}^{12} = 1.2034,\\
&\breve{x}^{13} = 0.6806,\ \widehat{x}^{13} = 1.89,\ \widehat{x}^{13} - \breve{x}^{13} = 1.2094,\\
&\breve{x}^{14} = 0.6986,\ \widehat{x}^{14} = 1.914,\ \widehat{x}^{14} - \breve{x}^{14} = 1.2154,\\
&\breve{x}^{15} = 0.7006,\ \widehat{x}^{15} = 1.932,\ \widehat{x}^{15} - \breve{x}^{15} = 1.2314,\\
&\breve{x}^{16} = 0.7026,\ \widehat{x}^{16} = 1.939,\ \widehat{x}^{16} - \breve{x}^{16} = 1.2364,\\
&\breve{x}^{17} = 0.7036,\ \widehat{x}^{17} = 1.941,\ \widehat{x}^{17} - \breve{x}^{17} = 1.2374,\\
&\breve{x}^{18} = 0.7046,\ \widehat{x}^{18} = 1.944,\ \widehat{x}^{18} - \breve{x}^{18} = 1.2394,
\end{aligned} \tag{66}$$

From the above, since the expression (36) takes the form

$$\widehat{x}^I - \breve{x}^I < 0.2954, \tag{67}$$

then it follows that the largest value $I \in \{0,1,\ldots,18\}$ such that this inequality holds is $I = 5$. Therefore, we fix the optimal solution that minimize linear function (57) according to the rule (38). Namely, we get

$$\begin{aligned}
&\dot{x}_1 = \widehat{x}_1 = 0.018,\ \dot{x}_2 = \widehat{x}_2 = 0.056,\\
&\dot{x}_3 = \widehat{x}_3 = 0.056,\ \dot{x}_4 = \widehat{x}_4 = 0.018,\\
&\dot{x}_5 = \widehat{x}_5 = 0.043,\\
&\dot{x}_6 = \hat{y} - \widehat{x}^5 + \breve{x}^5 + \breve{x}_6\\
&= 0.2954 - 0.191 + 0.0286 + 0.286 = 0.419,\\
&\dot{x}_7 = \breve{x}_7 = 0.286,\ \dot{x}_8 = \breve{x}_8 = 0.027,\\
&\dot{x}_9 = \breve{x}_9 = 0.013,\ \dot{x}_{10} = \breve{x}_{10} = 0.002,\\
&\dot{x}_{11} = \breve{x}_{11} = 0.002,\ \dot{x}_{12} = \breve{x}_{12} = 0.018,\\
&\dot{x}_{13} = \breve{x}_{13} = 0.018,\ \dot{x}_{14} = \breve{x}_{14} = 0.018,\\
&\dot{x}_{15} = \breve{x}_{15} = 0.002,\ \dot{x}_{16} = \breve{x}_{16} = 0.002,\\
&\dot{x}_{17} = \breve{x}_{17} = 0.001,\ \dot{x}_{18} = \breve{x}_{18} = 0.001.
\end{aligned} \tag{68}$$

Finally, after making the substitution inverse to (30), we get the optimal transient probabilities

$$\begin{aligned}
&\dot{p}_8 = \dot{x}_1 = 0.018,\ \dot{p}_9 = \dot{x}_2 = 0.056,\\
&\dot{p}_1 = \dot{x}_3 = 0.056,\ \dot{p}_{18} = \dot{x}_4 = 0.018,\\
&\dot{p}_{14} = \dot{x}_5 = 0.043,\ \dot{p}_5 = \dot{x}_6 = 0.419,\\
&\dot{p}_{13} = \dot{x}_7 = 0.286,\ \dot{p}_4 = \dot{x}_8 = 0.027,\\
&\dot{p}_{12} = \dot{x}_9 = 0.013,\ \dot{p}_{11} = \dot{x}_{10} = 0.002,\\
&\dot{p}_{16} = \dot{x}_{11} = 0.002,\ \dot{p}_3 = \dot{x}_{12} = 0.018,\\
&\dot{p}_{15} = \dot{x}_{13} = 0.018,\ \dot{p}_6 = \dot{x}_{14} = 0.018,\\
&\dot{p}_7 = \dot{x}_{15} = 0.002,\ \dot{p}_{17} = \dot{x}_{16} = 0.002,\\
&\dot{p}_2 = \dot{x}_{17} = 0.001,\ \dot{p}_{10} = \dot{x}_{18} = 0.001,
\end{aligned} \tag{69}$$

that minimize the mean value of the ferry technical system total operation cost $\boldsymbol{C}(2)$ expressed by the linear form (57) and according to (41) and (69), its optimal value in the safety state subset $\{2,3,4\}$ is

$$\begin{aligned}
\dot{\boldsymbol{C}}(2) \cong\ & 0.056 \cdot 131.78844c + 0.001 \cdot 192.67455c\\
&+ 0.018 \cdot 166.94400c + 0.027 \cdot 143.48209c\\
&+ 0.419 \cdot 143.47076c + 0.018 \cdot 169.36977c\\
&+ 0.002 \cdot 185.40425c + 0.018 \cdot 119.28677c\\
&+ 0.056 \cdot 119.28677c + 0.001 \cdot 192.67455c\\
&+ 0.002 \cdot 160.03200c + 0.013 \cdot 143.88276c\\
&+ 0.286 \cdot 143.47076c + 0.043 \cdot 142.69620c\\
&+ 0.018 \cdot 166.94400c + 0.002 \cdot 160.03200c\\
&+ 0.002 \cdot 185.40425c + 0.018 \cdot 131.78844c\\
\cong\ & 142.43261c
\end{aligned} \tag{70}$$

and further, considering (49), the optimal mean value of the ferry technical system operation total cost in the safety state subset {1,2,3,4} is

$$\begin{aligned}\dot{C}(1) \cong\ & 0.056 \cdot 158.54268c + 0.001 \cdot 233.11940c \\ & + 0.018 \cdot 201.70440c + 0.027 \cdot 174.64680c \\ & + 0.419 \cdot 174.63341c + 0.018 \cdot 205.94382c \\ & + 0.002 \cdot 224.36720c + 0.018 \cdot 143.48293c \\ & + 0.056 \cdot 143.48293c + 0.001 \cdot 233.11940c \\ & + 0.002 \cdot 193.22400c + 0.013 \cdot 175.25244c \\ & + 0.286 \cdot 174.63341c + 0.043 \cdot 173.6889c \\ & + 0.018 \cdot 201.70440c + 0.002 \cdot 193.23000c \\ & + 0.002 \cdot 224.36720c + 0.018 \cdot 158.54268c \\ & \cong 173.03378c \end{aligned} \quad (71)$$

and further, considering (51), the optimal mean value of the ferry technical system operation total cost in the safety state subset {3,4} is

$$\begin{aligned}\dot{C}(3) \cong\ & .0056 \cdot 114.26073c + 0.001 \cdot 172.45720c \\ & + 0.018 \cdot 149.46360c + 0.027 \cdot 128.37096c \\ & + 0.419 \cdot 128.35757c + 0.018 \cdot 151.57044c \\ & + 0.002 \cdot 167.98395c + 0.018 \cdot 105.17926c \\ & + 0.056 \cdot 105.17926c + 0.001 \cdot 172.45720c \\ & + 0.002 \cdot 143.51160c + 0.013 \cdot 128.73455c \\ & + 0.286 \cdot 128.35757c + 0.043 \cdot 127.66335c \\ & + 0.018 \cdot 149.46360c + 0.002 \cdot 143.51160c \\ & + 0.002 \cdot 167.98395c + 0.018 \cdot 114.26073c \\ & \cong 127.05959c \end{aligned} \quad (72)$$

and further, considering (52), the optimal mean value of the ferry technical system operation total cost in the safety state subset {4} is

$$\begin{aligned}\dot{C}(4) \cong\ & 0.056 \cdot 103.78893c + 0.001 \cdot 154.53230c \\ & + 0.018 \cdot 133.81440c + 0.027 \cdot 114.86766c \\ & + 0.419 \cdot 114.85530c + 0.018 \cdot 135.67023c \\ & + 0.002 \cdot 149.12815c + 0.018 \cdot 93.92529c \\ & + 0.056 \cdot 93.92529c + 0.001 \cdot 154.53230c \\ & + 0.002 \cdot 128.71440c + 0.013 \cdot 115.19108c \\ & + 0.286 \cdot 114.85530c + 0.043 \cdot 114.23730c \\ & + 0.018 \cdot 133.81440c + 0.002 \cdot 128.71440c \\ & + 0.002 \cdot 149.12815c + 0.018 \cdot 103.78893c \\ & \cong 113.79477c \end{aligned} \quad (73)$$

Hence, and according to (45), the optimal values of the ferry technical system total operation costs in the particular safety states 1, 2, 3 and 4, respectively are:

$$\begin{aligned} & \dot{\bar{C}}(1) \cong 173.03378c - 142.43261c \cong 30.60117c, \\ & \dot{\bar{C}}(2) \cong 142.43261c - 127.05959c \cong 15.37302c \\ & \dot{\bar{C}}(3) \cong 127.05959c - 113.79477c \cong 13.26482c, \\ & \dot{\bar{C}}(4) \cong 113.79477c. \end{aligned} \quad (74)$$

The analyzed costs after optimization are lower than before it, what is respectively expressed by comparison of the results before the optimization given in (53)–(56) and the results after the optimization given in (70)–(74).

5. Maritime ferry technical system safety

5.1. Safety characteristics

We assume, that the maritime ferry incorporates a number of main technical subsystems having an crucial impact on its safety, further termed the ferry technical system:

- S_1 – a navigational subsystem,
- S_2 – a propulsion and controlling subsystem,
- S_3 – a loading and unloading subsystem,
- S_4 – a stability control subsystem,
- S_5 – an anchoring and mooring subsystem.

The subsystems S_1, S_2, S_3, S_4, S_5, are forming a general series safety structure of the ferry technical system shown in Figure 1.

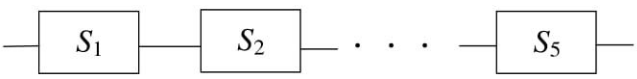

Figure 1. The general structure of the ferry technical system safety.

After analyzing the matter with help of experts and taking into consideration the safety of the operation of the ferry, we identify the five safety states of the ferry technical system and its components:

- a safety state 4 – the ferry operation is fully safe,
- a safety state 3 – the ferry operation is less safe and more dangerous because of the possibility of environment pollution,
- a safety state 2 called a critical safety state – the ferry operation is less safe and more dangerous because of the possibility of environment pollution and causing small accidents,

- a safety state 1 – the ferry operation is much less safe and much more dangerous because of the possibility of serious environment pollution and causing extensive accidents,
- a safety state 0 – the ferry technical system is destroyed (Kołowrocki & Soszyńska-Budny, 2011/2015).

Furthermore, we presume that only possible transitions between the components' safety states are those from better to worse and that that critical safety state for the system and its components is $r = 2$.

Applying (4)–(5) and using (46) the safety function of maritime ferry technical system is given by

$$\boldsymbol{S}(t,\cdot) = [\boldsymbol{S}(t,1), \boldsymbol{S}(t,2), \boldsymbol{S}(t,3), \boldsymbol{S}(t,4)],\ t \geq 0, \quad (75)$$

and

$$\begin{aligned}\boldsymbol{S}(t,u) = {} & 0.038\cdot[\boldsymbol{S}(t,u)]^{(1)} + 0.002\cdot[\boldsymbol{S}(t,u)]^{(2)} \\ & + 0.026\cdot[\boldsymbol{S}(t,u)]^{(3)} + 0.036\cdot[\boldsymbol{S}(t,u)]^{(4)} \\ & + 0.363\cdot[\boldsymbol{S}(t,u)]^{(5)} + 0.026\cdot[\boldsymbol{S}(t,u)]^{(6)} \\ & + 0.005\cdot[\boldsymbol{S}(t,u)]^{(7)} + 0.016\cdot[\boldsymbol{S}(t,u)]^{(8)} \\ & + 0.037\cdot[\boldsymbol{S}(t,u)]^{(9)} + 0.002\cdot[\boldsymbol{S}(t,u)]^{(10)} \\ & + 0.003\cdot[\boldsymbol{S}(t,u)]^{(11)} + 0.0016\cdot[\boldsymbol{S}(t,u)]^{(12)} \\ & + 0.351\cdot[\boldsymbol{S}(t,u)]^{(13)} + 0.034\cdot[\boldsymbol{S}(t,u)]^{(14)} \\ & + 0.024\cdot[\boldsymbol{S}(t,u)]^{(15)} + 0.003\cdot[\boldsymbol{S}(t,u)]^{(16)} \\ & + 0.005\cdot[\boldsymbol{S}(t,u)]^{(17)} + 0.013\cdot[\boldsymbol{S}(t,u)]^{(18)}, \end{aligned}$$
$$t \geq 0, \text{ for } u = 1,2,\ldots,4, \quad (76)$$

where $[\boldsymbol{S}(t,u)]^{(b)}$, $b = 1,2,\ldots,18$, are the system conditional safety functions at the operation state z_b, $b = 1,2,\ldots,18$, determined in (Kołowrocki & Magryta-Mut, 2020c, 2021; Magryta-Mut, 2020, 2021).

Hence, in particular for $u = 2$, we have

$$\begin{aligned}\boldsymbol{S}(t,2) = {} & 0.038\cdot[\boldsymbol{S}(t,2)]^{(1)} + 0.002\cdot[\boldsymbol{S}(t,2)]^{(2)} \\ & + 0.026\cdot[\boldsymbol{S}(t,2)]^{(3)} + 0.036\cdot[\boldsymbol{S}(t,2)]^{(4)} \\ & + 0.363\cdot[\boldsymbol{S}(t,2)]^{(5)} + 0.026\cdot[\boldsymbol{S}(t,2)]^{(6)} \\ & + 0.005\cdot[\boldsymbol{S}(t,2)]^{(7)} + 0.016\cdot[\boldsymbol{S}(t,2)]^{(8)} \\ & + 0.037\cdot[\boldsymbol{S}(t,2)]^{(9)} + 0.002\cdot[\boldsymbol{S}(t,2)]^{(10)} \\ & + 0.003\cdot[\boldsymbol{S}(t,2)]^{(11)} + 0.0016\cdot[\boldsymbol{S}(t,2)]^{(12)} \\ & + 0.351\cdot[\boldsymbol{S}(t,2)]^{(13)} + 0.034\cdot[\boldsymbol{S}(t,2)]^{(14)} \\ & + 0.024\cdot[\boldsymbol{S}(t,2)]^{(15)} + 0.003\cdot[\boldsymbol{S}(t,2)]^{(16)} \\ & + 0.005\cdot[\boldsymbol{S}(t,2)]^{(17)} + 0.013\cdot[\boldsymbol{S}(t,2)]^{(18)}, \end{aligned}$$
$$t \geq 0, \quad (77)$$

where $[\boldsymbol{S}(t,2)]^{(b)}$, $t \geq 0$, $b = 1,2,\ldots,18$, are the system conditional safety functions at the operation state z_b, $b = 1,2,\ldots,18$, determined in (Kołowrocki & Magryta-Mut, 2020c, 2021; Magryta-Mut, 2020, 2021).

Further, the expected values of the analyzed system conditional lifetimes in the safety state subset not worse than the critical safety state {2,3,4} at the operation states b, $b = 1,2,\ldots,18$, respectively (expressed in years) are (Kołowrocki & Magryta-Mut, 2020c, 2021; Kołowrocki & Soszyńska-Budny, 2018a; Magryta-Mut, 2020):

$$\begin{aligned}&[\boldsymbol{\mu}(2)]^{(1)} \cong 1.47,\ [\boldsymbol{\mu}(2)]^{(2)} \cong 1.33,\ [\boldsymbol{\mu}(2)]^{(3)} \cong 1.40, \\ &[\boldsymbol{\mu}(2)]^{(4)} \cong 1.39,\ [\boldsymbol{\mu}(2)]^{(5)} \cong 1.39,\ [\boldsymbol{\mu}(2)]^{(6)} \cong 1.38, \\ &[\boldsymbol{\mu}(2)]^{(7)} \cong 1.28,\ [\boldsymbol{\mu}(2)]^{(8)} \cong 1.44,\ [\boldsymbol{\mu}(2)]^{(9)} \cong 1.44, \\ &[\boldsymbol{\mu}(2)]^{(10)} \cong 1.33,\ [\boldsymbol{\mu}(2)]^{(11)} \cong 1.34,\ [\boldsymbol{\mu}(2)]^{(12)} \cong 1.40, \\ &[\boldsymbol{\mu}(2)]^{(13)} \cong 1.39,\ [\boldsymbol{\mu}(2)]^{(14)} \cong 1.39,\ [\boldsymbol{\mu}(2)]^{(15)} \cong 1.40, \\ &[\boldsymbol{\mu}(2)]^{(16)} \cong 1.34,\ [\boldsymbol{\mu}(2)]^{(17)} \cong 1.28,\ [\boldsymbol{\mu}(2)]^{(18)} \cong 1.46. \end{aligned} \quad (78)$$

The standard deviation of the considered system conditional lifetimes in the safety state subset {2,3,4} at the operation state z_b, $b = 1,2,\ldots,18$, respectively (expressed in years) are (Kołowrocki & Magryta, 2020a; Kołowrocki & Magryta-Mut, 2020c; Kołowrocki & Soszyńska-Budny, 2018a; Magryta-Mut, 2020):

$$\begin{aligned}&[\boldsymbol{\sigma}(2)]^{(1)} \cong 1.45,\ [\boldsymbol{\sigma}(2)]^{(2)} \cong 1.31,\ [\boldsymbol{\sigma}(2)]^{(3)} \cong 1.38, \\ &[\boldsymbol{\sigma}(2)]^{(4)} \cong 1.38,\ [\boldsymbol{\sigma}(2)]^{(5)} \cong 1.37,\ [\boldsymbol{\sigma}(2)]^{(6)} \cong 1.37, \\ &[\boldsymbol{\sigma}(2)]^{(7)} \cong 1.26,\ [\boldsymbol{\sigma}(2)]^{(8)} \cong 1.42,\ [\boldsymbol{\sigma}(2)]^{(9)} \cong 1.42, \\ &[\boldsymbol{\sigma}(2)]^{(10)} \cong 1.31,\ [\boldsymbol{\sigma}(2)]^{(11)} \cong 1.31,\ [\boldsymbol{\sigma}(2)]^{(12)} \cong 1.38, \\ &[\boldsymbol{\sigma}(2)]^{(13)} \cong 1.39,\ [\boldsymbol{\sigma}(2)]^{(14)} \cong 1.38,\ [\boldsymbol{\sigma}(2)]^{(15)} \cong 1.37, \\ &[\boldsymbol{\sigma}(2)]^{(16)} \cong 1.31,\ [\boldsymbol{\sigma}(2)]^{(17)} \cong 1.26, \\ &[\boldsymbol{\sigma}(2)]^{(18)} \cong 1.45. \end{aligned} \quad (79)$$

Thus, applying (26) and considering (46) and (77), the value of the ferry technical system unconditional lifetime in the safety state subset not worse than this critical safety state {2,3,4} is

$$\begin{aligned}\boldsymbol{\mu}(2) = {} & p_1\cdot 1.47 + p_2\cdot 1.33 + p_3\cdot 1.40 + p_4\cdot 1.39 \\ & + p_5\cdot 1.39 + p_6\cdot 1.38 + p_7\cdot 1.28 + p_8\cdot 1.44 \\ & + p_9\cdot 1.44 + p_{10}\cdot 1.33 + p_{11}\cdot 1.34 + p_{12}\cdot 1.40 \\ & + p_{13}\cdot 1.39 + p_{14}\cdot 1.39 + p_{15}\cdot 1.40 + p_{16}\cdot 1.34 \\ & + p_{17}\cdot 1.28 + p_{18}\cdot 1.46 \cong 1.395. \end{aligned} \quad (80)$$

Further, considering (79)–(80), the corresponding standard deviation of the analyzed system unconditional lifetime in the state subset {2,3,4} is (Magryta-Mut, 2021)

$$\boldsymbol{\sigma}(2) \cong 1.383 \text{ years.} \quad (81)$$

As the ferry technical system's critical safety state is $r = 2$, then its system risk function is given by (Kołowrocki & Magryta, 2020a; Kołowrocki & Magryta-Mut, 2020c, 2021; Magryta-Mut, 2021):

$$\boldsymbol{r}(t) \cong 1 - \boldsymbol{S}(t,2), \ t \geq 0, \quad (82)$$

where $\boldsymbol{S}(t,2)$, $t \geq 0$, is given by (77).
Hence, and considering (80), the moment when the system risk function exceeds a permitted level, for instance $\delta = 0.05$, is

$$\tau = \boldsymbol{r}^{-1}(\delta) \cong 0.0727 \text{ year.} \quad (83)$$

By 11, considering (79), the analyzed system's mean value of the intensity of ageing is

$$\boldsymbol{\lambda}(2) = \frac{1}{\boldsymbol{\mu}(2)} = \frac{1}{1.395} \cong 0.717. \quad (84)$$

By (12), considering (84) and the values of the ferry technical system without operation impact intensity of ageing $\lambda^0(2) = 0.678$, determined in (Kołowrocki & Magryta-Mut, 2020c; Kołowrocki & Soszyńska-Budny, 2018b; Magryta-Mut, 2021), the coefficient of the operation process impact on the ferry technical system intensity of ageing is

$$\boldsymbol{\rho}(2) = \frac{\lambda\ (2)}{\lambda^0(2)} = \frac{0.717}{0.678} \cong 1.058. \quad (85)$$

Hence, applying (13), the ferry technical system resilience indicator, i.e. the coefficient of the ferry technical system resilience to operation process impact, is

$$\boldsymbol{RI}(2) = \frac{1}{\boldsymbol{\rho}(2)} = \frac{1}{1.058} \cong 0.945 = 94.5\%. \quad (86)$$

5.2. Optimal safety characteristics

Applying the optimization procedure from Section 2.2, we obtain the optimal mean value of the ferry technical system lifetime is (Magryta-Mut, 2020)

$$\begin{aligned}\dot{\mu}(2) = {} & \dot{p}_1 \cdot 1.47 + \dot{p}_2 \cdot 1.33 + \dot{p}_3 \cdot 1.40 \\ & + \dot{p}_4 \cdot 1.39 + \dot{p}_5 \cdot 1.39 + \dot{p}_6 \cdot 1.38 \\ & + \dot{p}_7 \cdot 1.28 + \dot{p}_8 \cdot 1.44 + \dot{p}_9 \cdot 1.44 \\ & + \dot{p}_{10} \cdot 1.33 + \dot{p}_{11} \cdot 1.34 + \dot{p}_{12} \cdot 1.40 \\ & + \dot{p}_{13} \cdot 1.39 + \dot{p}_{14} \cdot 1.39 + \dot{p}_{15} \cdot 1.40 \\ & + \dot{p}_{16} \cdot 1.34 + \dot{p}_{17} \cdot 1.28 + \dot{p}_{18} \cdot 1.46 \\ & \cong 1.399 \text{ years,} \end{aligned} \quad (87)$$

where

$$\begin{aligned} & \dot{p}_1 = 0.056, \dot{p}_2 = 0.001, \dot{p}_3 = 0.027, \\ & \dot{p}_4 = 0.056, \dot{p}_5 = 0.382, \dot{p}_6 = 0.018, \\ & \dot{p}_7 = 0.002, \dot{p}_8 = 0.018, \dot{p}_9 = 0.056, \\ & \dot{p}_{10} = 0.001, \dot{p}_{11} = 0.002, \dot{p}_{12} = 0.024, \\ & \dot{p}_{13} = 0.286, \dot{p}_{14} = 0.025, \dot{p}_{15} = 0.024, \\ & \dot{p}_{16} = 0.002, \dot{p}_{17} = 0.002, \dot{p}_{18} = 0.018. \end{aligned}$$

Moreover, the corresponding optimal unconditional safety function of the ferry technical system takes the form

$$\begin{aligned}\dot{\boldsymbol{S}}(t,2) = {} & 0.056 \cdot [\boldsymbol{S}(t,2)]^{(1)} + 0.001 \cdot [\boldsymbol{S}(t,2)]^{(2)} \\ & + 0.027 \cdot [\boldsymbol{S}(t,2)]^{(3)} + 0.056 \cdot [\boldsymbol{S}(t,2)]^{(4)} \\ & + 0.382 \cdot [\boldsymbol{S}(t,2)]^{(5)} + 0.018 \cdot [\boldsymbol{S}(t,2)]^{(6)} \\ & + 0.002 \cdot [\boldsymbol{S}(t,2)]^{(7)} + 0.018 \cdot [\boldsymbol{S}(t,2)]^{(8)} \\ & + 0.056 \cdot [\boldsymbol{S}(t,2)]^{(9)} + 0.001 \cdot [\boldsymbol{S}(t,2)]^{(10)} \\ & + 0.002 \cdot [\boldsymbol{S}(t,2)]^{(11)} + 0.024 \cdot [\boldsymbol{S}(t,2)]^{(12)} \\ & + 0.286 \cdot [\boldsymbol{S}(t,2)]^{(13)} + 0.025 \cdot [\boldsymbol{S}(t,2)]^{(14)} \\ & + 0.024 \cdot [\boldsymbol{S}(t,2)]^{(15)} + 0.002 \cdot [\boldsymbol{S}(t,2)]^{(16)} \\ & + 0.002 \cdot [\boldsymbol{S}(t,2)]^{(17)} + 0.018 \cdot [\boldsymbol{S}(t,2)]^{(18)}, \\ t \geq 0, & \end{aligned} \quad (88)$$

where $[\boldsymbol{S}(t,2)]^{(b)}$, $t \geq 0$, $b = 1,2,\ldots,18$, are determined in (Kołowrocki & Magryta-Mut, 2020c; Magryta-Mut, 2020, 2021).
Moreover, considering (87) and (88), the corresponding optimal standard deviations of the ferry technical system unconditional lifetime in the state subset not worse than the critical safety state is (Kołowrocki & Magryta-Mut, 2020c; Magryta-Mut, 2020, 2021)

$$\dot{\boldsymbol{\sigma}}(2) \cong 1.386 \text{ years.} \quad (89)$$

As the ferry technical system critical safety state is $r = 2$, then considering (9) and (88), its optimal system risk function, is given by

$$\dot{\boldsymbol{r}}(t) \cong 1 - \dot{\boldsymbol{S}}(t,2), \ t \geq 0. \quad (90)$$

Considering (10) and (88) the moment when the optimal system risk function exceeds a permitted level, for instance $\delta = 0.05$, is

$$\dot{\tau} = \dot{\boldsymbol{r}}^{-1}(\delta) \cong 0.0729 \text{ year}. \quad (91)$$

By (11) and (87) the ferry technical system mean value of the optimal intensity of ageing is

$$\dot{\boldsymbol{\lambda}}(2) = \frac{1}{\dot{\boldsymbol{\mu}}(2)} = \frac{1}{1.399} \cong 0.715. \quad (92)$$

Considering (12) and (92) and the values of the analyzed system without operation impact intensity of ageing $\boldsymbol{\lambda}^0(t) = 0.678$, determined in (Kołowrocki & Magryta, 2020a; Kołowrocki & Magryta-Mut, 2020c; Magryta-Mut, 2021), the optimal coefficient of the operation process impact on the ferry technical system intensity of ageing is

$$\dot{\boldsymbol{\rho}}(2) = \frac{\lambda\ (2)}{\lambda^0(2)} = \frac{0.715}{0.678} \cong 1.055. \quad (93)$$

Hence, applying (13), the ferry technical system optimal resilience indicator, i.e. the optimal coefficient of the ferry technical system resilience to operation process impact is

$$\dot{\boldsymbol{RI}}(2) = \frac{1}{\dot{\boldsymbol{\rho}}(2)} = \frac{1}{1.055} \cong 0.948 = 94.8\%. \quad (94)$$

6. Joint system safety optimization and operation cost analysis

6.1. System operation cost with system safety impact corresponding to its maximal safety

To analyze jointly the system safety and its operation cost optimization, it is possible to propose the procedure of determining the optimal values of limit transient probabilities of the system operation process at the particular operation states that allows to find the best values of the system safety indicators, through applying the proposed system safety general model from Section 2.1 and safety optimization described in Section 2.2. Next, to find the system total unconditional operation costs in the safety state subsets, corresponding to this system best safety indicators, we replace the limit transient probabilities at the system particular operation states, existing in the formula for the system operation total cost during the fixed operation time by their optimal values existing in the formula for the system best safety function coordinates.

Thus, in Section 2.2, there is presented the procedure of determining the optimal values $\dot{p}_b$, $b = 1,2,\ldots,\nu$, of the limit transient probabilities of the system operation process $Z(t)$ at the particular operation states z_b, $b = 1,2,\ldots,\nu$, that allows to find the system maximal safety indicators, through applying the general system safety model and linear programming and determining their values. More exactly, to find the system total unconditional operation costs in the safety state subsets, corresponding to the system maximal safety indicators, we replace p_b, $b = 1,2,\ldots,\nu$, existing in the formula (23) for the system operation total cost by $\dot{p}_b$, $b = 1,2,\ldots,\nu$, for its maximal safety indicator, the system maximal mean lifetime in the system safety state subset not worse than the system critical safety state.

6.1.1. Maritime ferry technical system operation cost with system safety impact corresponding to its maximal safety

In Section 4.3, we get the optimal limit transient probabilities of the ferry operation process $Z(t)$ at the particular operation states z_b, $b = 1,2,\ldots,18$,:

$$\dot{p}_1 = 0.056,\ \dot{p}_2 = 0.001,\ \dot{p}_3 = 0.018,\ \dot{p}_4 = 0.027,$$
$$\dot{p}_5 = 0.419,\ \dot{p}_6 = 0.018,\ \dot{p}_7 = 0.002,\ \dot{p}_8 = 0.018,$$
$$\dot{p}_9 = 0.056,\ \dot{p}_{10} = 0.001,\ \dot{p}_{11} = 0.002,$$
$$\dot{p}_{12} = 0.013,\ \dot{p}_{13} = 0.286,\ \dot{p}_{14} = 0.043,$$
$$\dot{p}_{15} = 0.018,\ \dot{p}_{16} = 0.002,\ \dot{p}_{17} = 0.002,$$
$$\dot{p}_{18} = 0.018, \quad (95)$$

that maximize the maritime ferry technical system safety and determine its optimal values given by (46).

To find the system conditional total operation costs in the safety state subsets, determined by the cost model with considering safety impact, corresponding to this system maximal safety, we replace p_b, $b = 1,2,\ldots,18$, existing in the formulae (53)–(56) for the system operation cost in the safety state subsets {1,2,3,4}, {2,3,4} {3,4} and {4} respectively, by $\dot{p}_b$, $b = 1,2,\ldots,18$, defined by (95). This way, we get the ferry technical system conditional total operation costs in the safety state subsets, determined by the cost model with safety impact, corresponding to this system maximal safety, given by

$$\dot{\boldsymbol{C}}(1) \cong \dot{p}_1[\boldsymbol{C}(1)]^{(1)} + \dot{p}_2[\boldsymbol{C}(1)]^{(2)} + \dot{p}_3[\boldsymbol{C}(1)]^{(3)} + \dot{p}_4[\boldsymbol{C}(1)]^{(4)} + \dot{p}_5[\boldsymbol{C}(1)]^{(5)} + \dot{p}_6[\boldsymbol{C}(1)]^{(6)} + \dot{p}_7[\boldsymbol{C}(1)]^{(7)} + \dot{p}_8[\boldsymbol{C}(1)]^{(8)} + \dot{p}_9[\boldsymbol{C}(1)]^{(9)} + \dot{p}_{10}[\boldsymbol{C}(1)]^{(10)} + \dot{p}_{11}[\boldsymbol{C}(1)]^{(11)} + \dot{p}_{12}[\boldsymbol{C}(1)]^{(12)} + \dot{p}_{13}[\boldsymbol{C}(1)]^{(13)} + \dot{p}_{14}[\boldsymbol{C}(1)]^{(14)} + \dot{p}_{15}[\boldsymbol{C}(1)]^{(15)} + \dot{p}_{16}[\boldsymbol{C}(1)]^{(16)} + \dot{p}_{17}[\boldsymbol{C}(1)]^{(17)} + \dot{p}_{18}[\boldsymbol{C}(1)]^{(18)} \cong 173.034c, \qquad (96)$$

in the safety state subset {1,2,3,4} and

$$\dot{\boldsymbol{C}}(2) \cong \dot{p}_1[\boldsymbol{C}(2)]^{(1)} + \dot{p}_2[\boldsymbol{C}(2)]^{(2)} + \dot{p}_3[\boldsymbol{C}(2)]^{(3)} + \dot{p}_4[\boldsymbol{C}(2)]^{(4)} + \dot{p}_5[\boldsymbol{C}(2)]^{(5)} + \dot{p}_6[\boldsymbol{C}(2)]^{(6)} + \dot{p}_7[\boldsymbol{C}(2)]^{(7)} + \dot{p}_8[\boldsymbol{C}(2)]^{(8)} + \dot{p}_9[\boldsymbol{C}(2)]^{(9)} + \dot{p}_{10}[\boldsymbol{C}(2)]^{(10)} + \dot{p}_{11}[\boldsymbol{C}(2)]^{(11)} + \dot{p}_{12}[\boldsymbol{C}(2)]^{(12)} + \dot{p}_{13}[\boldsymbol{C}(2)]^{(13)} + \dot{p}_{14}[\boldsymbol{C}(2)]^{(14)} + \dot{p}_{15}[\boldsymbol{C}(2)]^{(15)} + \dot{p}_{16}[\boldsymbol{C}(2)]^{(16)} + \dot{p}_{17}[\boldsymbol{C}(2)]^{(17)} + \dot{p}_{18}[\boldsymbol{C}(2)]^{(18)} \cong 142.433c, \qquad (97)$$

in the safety state subset {2,3,4} and

$$\dot{\boldsymbol{C}}(3) \cong \dot{p}_1[\boldsymbol{C}(3)]^{(1)} + \dot{p}_2[\boldsymbol{C}(3)]^{(2)} + \dot{p}_3[\boldsymbol{C}(3)]^{(3)} + \dot{p}_4[\boldsymbol{C}(3)]^{(4)} + \dot{p}_5[\boldsymbol{C}(3)]^{(5)} + \dot{p}_6[\boldsymbol{C}(3)]^{(6)} + \dot{p}_7[\boldsymbol{C}(3)]^{(7)} + \dot{p}_8[\boldsymbol{C}(3)]^{(8)} + \dot{p}_9[\boldsymbol{C}(3)]^{(9)} + \dot{p}_{10}[\boldsymbol{C}(3)]^{(10)} + \dot{p}_{11}[\boldsymbol{C}(3)]^{(11)} + \dot{p}_{12}[\boldsymbol{C}(3)]^{(12)} + \dot{p}_{13}[\boldsymbol{C}(3)]^{(13)} + \dot{p}_{14}[\boldsymbol{C}(3)]^{(14)} + \dot{p}_{15}[\boldsymbol{C}(3)]^{(15)} + \dot{p}_{16}[\boldsymbol{C}(3)]^{(16)} + \dot{p}_{17}[\boldsymbol{C}(3)]^{(17)} + \dot{p}_{18}[\boldsymbol{C}(3)]^{(18} \cong 127.06c, \qquad (98)$$

in the safety state subset {3,4} and

$$\dot{\boldsymbol{C}}(4) \cong \dot{p}_1[\boldsymbol{C}(4)]^{(1)} + \dot{p}_2[\boldsymbol{C}(4)]^{(2)} + \dot{p}_3[\boldsymbol{C}(4)]^{(3)} + \dot{p}_4[\boldsymbol{C}(4)]^{(4)} + \dot{p}_5[\boldsymbol{C}(4)]^{(5)} + \dot{p}_6[\boldsymbol{C}(4)]^{(6)} + \dot{p}_7[\boldsymbol{C}(4)]^{(7)} + \dot{p}_8[\boldsymbol{C}(4)]^{(8)} + \dot{p}_9[\boldsymbol{C}(4)]^{(9)} + \dot{p}_{10}[\boldsymbol{C}(4)]^{(10)} + \dot{p}_{11}[\boldsymbol{C}(4)]^{(11)} + \dot{p}_{12}[\boldsymbol{C}(4)]^{(12)} + \dot{p}_{13}[\boldsymbol{C}(4)]^{(13)} + \dot{p}_{14}[\boldsymbol{C}(4)]^{(14)} + \dot{p}_{15}[\boldsymbol{C}(4)]^{(15)} + \dot{p}_{16}[\boldsymbol{C}(4)]^{(16)} + \dot{p}_{17}[\boldsymbol{C}(4)]^{(17)} + \dot{p}_{18}[\boldsymbol{C}(4)]^{(18)} \cong 113.795c, \qquad (99)$$

in the safety state subset {4}.

6.2. System safety corresponding to its minimal operation cost with safety impact

To analyze jointly the system operation cost and its safety optimization it is possible to propose the procedure of determining the optimal values of limit transient probabilities of the system operation process at the particular operation states that allows to find the system minimal operation total costs in the safety state subsets, through applying the proposed in Section 4.2 system operation cost model and linear programming described in Section 4.3. Next, to find the system conditional safety indicators, corresponding to this system minimal total operation costs in the safety state subsets, we replace the limit transient probabilities at particular operation states, existing in the formula for the system safety function coordinates, by their optimal values existing in the formula for the system minimal operation total costs in the safety state subsets.

Thus, in Section 4.3, there is presented the procedure of determining the optimal values $\dot{p}_b$, $b = 1,2,\ldots,v$, of the limit transient probabilities of the system operation process $Z(t)$ at the particular operation states z_b, $b = 1,2,\ldots,v$, that allows to find the minimal operation total costs in the safety state subsets, through applying the system operation cost model from Section 4.2 and determining its mean value. More exactly, to find the system conditional safety indicators, corresponding to this system minimal operation total costs in the safety state subsets, we replace p_b, $b = 1,2,\ldots,v$, existing in the formula (5) for the system safety function coordinates, by $\dot{p}_b$, $b = 1,2,\ldots,v$, existing in the formula (41) for its minimal operation total cost in the safety state subsets.

6.2.1. Maritime ferry technical system safety corresponding to its minimal operation cost with safety impact

In Section 4.3, we get the optimal limit transient probabilities of the ferry operation process $Z(t)$ at the particular operation states z_b, $b = 1,2,\ldots,18$:

$$\dot{p}_1 = 0.056,\ \dot{p}_2 = 0.001,\ \dot{p}_3 = 0.018,$$
$$\dot{p}_4 = 0.027,\ \dot{p}_5 = 0.419,\ \dot{p}_6 = 0.018,$$
$$\dot{p}_7 = 0.002,\ \dot{p}_8 = 0.018,\ \dot{p}_9 = 0.056,$$
$$\dot{p}_{10} = 0.001,\ \dot{p}_{11} = 0.002,\ \dot{p}_{12} = 0.013,$$
$$\dot{p}_{13} = 0.286,\ \dot{p}_{14} = 0.043,\ \dot{p}_{15} = 0.018,$$
$$\dot{p}_{16} = 0.002,\ \dot{p}_{17} = 0.002,\ \dot{p}_{18} = 0.018, \quad (100)$$

that minimize the maritime ferry technical system operation cost through applying the system operation cost model with considering safety impact and determining its optimal values in the safety state subsets given by (70)–(73) and in the particular safety state given by (74).

To find the system conditional safety indicators, corresponding to this system optimal total operation costs in the system safety subsets, we replace p_b, $b = 1,2,\ldots,7$, existing in the formula (46) for the system safety function, by $\dot{p}_b$, $b = 1,2,\ldots,7$, defined by (100). This way, we get the conditional maritime ferry technical system safety function, corresponding to this system optimal total operation costs in the system safety subsets, given by the vector

$$\boldsymbol{S}(t,\cdot) = [\boldsymbol{S}(t,1), \boldsymbol{S}(t,2), \boldsymbol{S}(t,3), \boldsymbol{S}(t,4)],\ t \geq 0, \quad (101)$$

where considering the ferry technical system operation process optimal limit transient probabilities at the operation states determined by (100), the vector coordinates are given respectively by

$$\boldsymbol{S}(t,1) = 0.056\cdot[\boldsymbol{S}(t,1)]^{(1)} + 0.001\cdot[\boldsymbol{S}(t,1)]^{(2)} + 0.018\cdot[\boldsymbol{S}(t,1)]^{(3)} + 0.027\cdot[\boldsymbol{S}(t,1)]^{(4)} + 0.419\cdot[\boldsymbol{S}(t,1)]^{(5)} + 0.018\cdot[\boldsymbol{S}(t,1)]^{(6)} + 0.002\cdot[\boldsymbol{S}(t,1)]^{(7)} + 0.018\cdot[\boldsymbol{S}(t,1)]^{(8)} + 0.056\cdot[\boldsymbol{S}(t,1)]^{(9)} + 0.001\cdot[\boldsymbol{S}(t,1)]^{(10)} + 0.002\cdot[\boldsymbol{S}(t,1)]^{(11)} + 0.013\cdot[\boldsymbol{S}(t,1)]^{(12)} + 0.286\cdot[\boldsymbol{S}(t,1)]^{(13)} + 0.043\cdot[\boldsymbol{S}(t,1)]^{(14)} + 0.018\cdot[\boldsymbol{S}(t,1)]^{(15)} + 0.002\cdot[\boldsymbol{S}(t,1)]^{(16)} + 0.002\cdot[\boldsymbol{S}(t,1)]^{(17)} + 0.018\cdot[\boldsymbol{S}(t,1)]^{(18)}, \quad (102)$$

$$\boldsymbol{S}(t,2) = 0.056\cdot[\boldsymbol{S}(t,2)]^{(1)} + 0.001\cdot[\boldsymbol{S}(t,2)]^{(2)} + 0.018\cdot[\boldsymbol{S}(t,2)]^{(3)} + 0.027\cdot[\boldsymbol{S}(t,2)]^{(4)} + 0.419\cdot[\boldsymbol{S}(t,2)]^{(5)} + 0.018\cdot[\boldsymbol{S}(t,2)]^{(6)} + 0.002\cdot[\boldsymbol{S}(t,2)]^{(7)} + 0.018\cdot[\boldsymbol{S}(t,2)]^{(8)} + 0.056\cdot[\boldsymbol{S}(t,2)]^{(9)} + 0.001\cdot[\boldsymbol{S}(t,2)]^{(10)} + 0.002\cdot[\boldsymbol{S}(t,2)]^{(11)} + 0.013\cdot[\boldsymbol{S}(t,2)]^{(12)} + 0.286\cdot[\boldsymbol{S}(t,2)]^{(13)} + 0.043\cdot[\boldsymbol{S}(t,2)]^{(14)} + 0.018\cdot[\boldsymbol{S}(t,2)]^{(15)} + 0.002\cdot[\boldsymbol{S}(t,2)]^{(16)} + 0.002\cdot[\boldsymbol{S}(t,2)]^{(17)} + 0.018\cdot[\boldsymbol{S}(t,2)]^{(18)}, \quad (103)$$

$$\boldsymbol{S}(t,3) = 0.056\cdot[\boldsymbol{S}(t,3)]^{(1)} + 0.001\cdot[\boldsymbol{S}(t,3)]^{(2)} + 0.018\cdot[\boldsymbol{S}(t,3)]^{(3)} + 0.027\cdot[\boldsymbol{S}(t,3)]^{(4)} + 0.419\cdot[\boldsymbol{S}(t,3)]^{(5)} + 0.018\cdot[\boldsymbol{S}(t,3)]^{(6)} + 0.002\cdot[\boldsymbol{S}(t,3)]^{(7)} + 0.018\cdot[\boldsymbol{S}(t,3)]^{(8)} + 0.056\cdot[\boldsymbol{S}(t,3)]^{(9)} + 0.001\cdot[\boldsymbol{S}(t,3)]^{(10)} + 0.002\cdot[\boldsymbol{S}(t,3)]^{(11)} + 0.013\cdot[\boldsymbol{S}(t,3)]^{(12)} + 0.286\cdot[\boldsymbol{S}(t,3)]^{(13)} + 0.043\cdot[\boldsymbol{S}(t,3)]^{(14)} + 0.018\cdot[\boldsymbol{S}(t,3)]^{(15)} + 0.002\cdot[\boldsymbol{S}(t,3)]^{(16)} + 0.002\cdot[\boldsymbol{S}(t,3)]^{(17)} + 0.018\cdot[\boldsymbol{S}(t,3)]^{(18)}, \quad (104)$$

$$\boldsymbol{S}(t,4) = 0.056\cdot[\boldsymbol{S}(t,4)]^{(1)} + 0.001\cdot[\boldsymbol{S}(t,4)]^{(2)} + 0.018\cdot[\boldsymbol{S}(t,4)]^{(3)} + 0.027\cdot[\boldsymbol{S}(t,4)]^{(4)} + 0.419\cdot[\boldsymbol{S}(t,4)]^{(5)} + 0.018\cdot[\boldsymbol{S}(t,4)]^{(6)} + 0.002\cdot[\boldsymbol{S}(t,4)]^{(7)} + 0.018\cdot[\boldsymbol{S}(t,4)]^{(8)} + 0.056\cdot[\boldsymbol{S}(t,4)]^{(9)} + 0.001\cdot[\boldsymbol{S}(t,4)]^{(10)} + 0.002\cdot[\boldsymbol{S}(t,4)]^{(11)} + 0.013\cdot[\boldsymbol{S}(t,4)]^{(12)} + 0.286\cdot[\boldsymbol{S}(t,4)]^{(13)} + 0.043\cdot[\boldsymbol{S}(t,4)]^{(14)} + 0.018\cdot[\boldsymbol{S}(t,4)]^{(15)} + 0.002\cdot[\boldsymbol{S}(t,4)]^{(16)} + 0.002\cdot[\boldsymbol{S}(t,4)]^{(17)} + 0.018\cdot[\boldsymbol{S}(t,4)]^{(18)}, \quad (105)$$

where $[\boldsymbol{S}(t,u)]^{(b)}$, $t \geq 0$, $u = 1,2,3,4$, $b = 1,2,\ldots,18$, are given in (Kołowrocki & Magryta-Mut, 2020c)

The conditional expected values and standard deviations of the ferry technical system lifetimes in the safety state subsets {1,2,3,4}, {2,3,4}, {3,4}, {4} calculated from the results given by (102)–(105), given in (Kołowrocki & Magryta-Mut, 2020c) corresponding to this system optimal total operation costs in the system safety subsets, respectively are:

$$\boldsymbol{\mu}(1) \cong 0.056\cdot 1.70476 + 0.001\cdot 1.60772 + 0.018\cdot 1.68087 + 0.027\cdot 1.6956 + 0.419\cdot 1.69547 + 0.018\cdot 1.67434 + 0.002\cdot 1.54736 + 0.018\cdot 1.72871 + 0.056\cdot 1.72871 + 0.001\cdot 1.60772 + 0.002\cdot 1.6102 + 0.013\cdot 1.70148 + 0.286\cdot 1.69547 + 0.043\cdot 1.6863$$

$$+ 0.018 \cdot 1.68087 + 0.002 \cdot 1.61025$$
$$+ 0.002 \cdot 1.54736 + 0.018 \cdot 1.70476$$
$$\cong 1.69629 \text{ years}, \quad (106)$$

$$\boldsymbol{\sigma}(1) \cong 1.67052 \text{ years}, \quad (107)$$

$$\boldsymbol{\mu}(2) \cong 0.056 \cdot 1.41708 + 0.001 \cdot 1.32879$$
$$+ 0.018 \cdot 1.3912 + 0.027 \cdot 1.39303$$
$$+ 0.419 \cdot 1.39292 + 0.018 \cdot 1.37699$$
$$+ 0.002 \cdot 1.27865 + 0.018 \cdot 1.43719$$
$$+ 0.056 \cdot 1.43719 + 0.001 \cdot 1.32879$$
$$+ 0.002 \cdot 1.3336 + 0.013 \cdot 1.39692$$
$$+ 0.286 \cdot 1.39292 + 0.043 \cdot 1.3854$$
$$+ 0.018 \cdot 1.3912 + 0.002 \cdot 1.3336$$
$$+ 0.002 \cdot 1.27865 + 0.018 \cdot 1.41708$$
$$\cong 1.39654 \text{ years}, \quad (108)$$

$$\boldsymbol{\sigma}(2) \cong 1.37901 \text{ years}, \quad (109)$$

$$\boldsymbol{\mu}(3) \cong 0.056 \cdot 1.22861 + 0.001 \cdot 1.18936$$
$$+ 0.018 \cdot 1.24553 + 0.027 \cdot 1.24632$$
$$+ 0.419 \cdot 1.24619 + 0.018 \cdot 1.23228$$
$$+ 0.002 \cdot 1.15851 + 0.018 \cdot 1.26722$$
$$+ 0.056 \cdot 1.26722 + 0.001 \cdot 1.18936$$
$$+ 0.002 \cdot 1.19593 + 0.013 \cdot 1.24985$$
$$+ 0.286 \cdot 1.24619 + 0.043 \cdot 1.23945$$
$$+ 0.018 \cdot 1.24553 + 0.002 \cdot 1.19593$$
$$+ 0.002 \cdot 1.15851 + 0.018 \cdot 1.22861$$
$$\cong 1.24527 \text{ years} \quad (110)$$

$$\boldsymbol{\sigma}(3) \cong 1.23182 \text{ years}, \quad (111)$$

$$\boldsymbol{\mu}(4) \cong 0.056 \cdot 1.11601 + 0.001 \cdot 1.06574$$
$$+ 0.018 \cdot 1.11512 + 0.027 \cdot 1.11522$$
$$+ 0.419 \cdot 1.1151 + 0.018 \cdot 1.10301$$
$$+ 0.002 \cdot 1.02847 + 0.018 \cdot 1.13163$$
$$+ 0.056 \cdot 1.13163 + 0.001 \cdot 1.06574$$
$$+ 0.002 \cdot 1.07262 + 0.013 \cdot 1.11836$$
$$+ 0.3286 \cdot 1.1151 + 0.043 \cdot 1.1091$$
$$+ 0.018 \cdot 1.11512 + 0.002 \cdot 1.07262$$
$$+ 0.002 \cdot 1.02847 + 0.018 \cdot 1.11601$$
$$\cong 1.11535 \text{ years}, \quad (112)$$

$$\boldsymbol{\sigma}(4) \cong 1.10288 \text{ years}. \quad (113)$$

And further. considering (8) and (106), (108), (110) and (112), the conditional mean values of the system lifetimes in the particular safety states 1, 2, 3, 4, corresponding to this system optimal total operation costs in the system safety subsets, respectively are:

$$\bar{\boldsymbol{\mu}}(1) = \boldsymbol{\mu}(1) - \boldsymbol{\mu}(2) = 0.299816,$$
$$\bar{\boldsymbol{\mu}}(2) = \boldsymbol{\mu}(2) - \boldsymbol{\mu}(3) = 0.149614,$$
$$\bar{\boldsymbol{\mu}}(3) = \boldsymbol{\mu}(3) - \boldsymbol{\mu}(4) = 0.130199,$$
$$\bar{\boldsymbol{\mu}}(4) = \boldsymbol{\mu}(4) = 1.114243. \quad (114)$$

Since the critical safety state is $r = 2$. then the conditional system risk function, corresponding to this system optimal total operation costs in the system safety subsets, according to (9) is given by

$$\boldsymbol{r}(t) = 1 - \boldsymbol{S}(t, 2), \quad (115)$$

where $\boldsymbol{S}(t,2)$ is given by (102).
Hence, according to (10) the moment when the system conditional risk function exceeds a permitted level, for instance $\delta = 0.05$, corresponding to this system optimal total operation costs in the system safety subsets, is

$$\tau = \boldsymbol{r}^{-1}(\delta) \cong 0.06467 \text{ year}. \quad (116)$$

Applying (11), the conditional ferry technical system conditional intensities of ageing, corresponding to this system optimal total operation costs in the system safety subsets, are:

$$\lambda(1) \cong 0.58952, \ \lambda(2) \cong 0.71605,$$
$$\lambda(3) \cong 0.80304, \ \lambda(4) \cong 0.89658. \quad (117)$$

Considering (12) and (7.75), the conditional coefficients of the operation process impact on the ferry technical system intensities of ageing, corresponding to this system optimal total operation costs in the system safety subsets, are:

$$\boldsymbol{\rho}(t,1) \cong 1.04345, \ \boldsymbol{\rho}(t,2) \cong 1.05689,$$
$$\boldsymbol{\rho}(t,3) \cong 1.04395, \ \boldsymbol{\rho}(t,4) \cong 1.04362. \quad (118)$$

Finally, by (13) and (118), the ferry technical system conditional resilience indicator, corresponding to this system optimal total operation costs in the system safety subsets, i.e. the conditional coefficient of the ferry technical system resilience to the operation process impact, corresponding to this system optimal total operation costs in the system safety subsets, is

$$\boldsymbol{RI}(t) = 1/\boldsymbol{\rho}(t,2) \cong 0.9462 = 94{,}62\%. \quad (119)$$

7. Conclusion

The procedures of using the general safety analytical model and the operation cost model of complex multistate technical system related to its operation process (Kołowrocki, 2014) and the linear programming (Klabjan, 2006) were presented and proposed to separate and joint analysis of the system safety maximization and its operation cost minimization.

The separate system safety maximization was depended on the mean value of the complex multistate system in the system safety state subset not worse than the system critical safety state maximization through the system operation process modification. This operation process modification allowed to determine the corresponding the best forms and values of the system safety indicators.

The separate system operation cost minimization was depended on the complex system mean value of the operation total costs in the safety state subsets minimization through the system operation process modification. This operation process modification allowed to determine the corresponding minimal system operation total costs in the safety state subsets.

The procedure of joint system safety and its operation cost optimization allowed us to use firstly the system safety maximization and next determining its conditional operation total costs in the safety state subsets corresponding to this system maximal safety. In this case, the operation process modification allowed us to find the complex system conditional operation total costs in the safety state subsets corresponding to the system best safety indicators. The proposed system safety optimization procedure and corresponding system operation total cost finding delivered practically important possibility of the system safety indicators maximization and keeping fixed the corresponding system operation total costs in the safety state subsets through the system new operation strategy. The procedure of joint system safety and its operation cost optimization allowed us also to use firstly the system operation total costs in the safety state subsets minimization and next determining its conditional safety function and remaining safety indicators corresponding to this system minimal operation total cost. In this case, the operation process modification allowed us to find the complex system conditional safety indicators corresponding to the system minimal operation total costs in the safety state subsets. The proposed cost optimization procedure and finding corresponding system safety indicators delivered practically important possibility of the system total operation cost minimizing and keeping fixed the corresponding conditional safety indicators during the operation through the system new operation strategy.

The proposed system safety and system operation cost optimization models and procedures were applied to the maritime ferry technical system examination. These procedures can be used in safety and operation cost optimization of various real complex systems and critical infrastructures (Gouldby et al., 2010; Habibullah et al., 2009; Kołowrocki & Magryta, 2020a-c; Kołowrocki et al., 2016, Lauge et al., 2015; Magryta-Mut, 2020). Further research can be related to considering other impacts on the system safety and its operation cost, for instance a very important impact related to climate-weather factors (Kołowrocki & Kuligowska, 2018, Kołowrocki & Soszyńska-Budny, 2017; Torbicki, 2019a-b; Torbicki & Drabiński, 2020) and resolving the issues of critical infrastructure (Lauge et al., 2015) safety and operation cost optimization and discovering optimal values of safety, operation cost and resilience indicators of system impacted by the operation and climate-weather conditions (Kołowrocki & Soszyńska-Budny, 2017). These developments can also benefit the mitigation of critical infrastructure accident consequences (Bogalecka, 2020; Dąbrowska & Kołowrocki, 2019a-b, 2020a-b) and inside and outside dependences (Kołowrocki, 2021, 2022) and to minimize the system operation cost and to improve critical infrastructure resilience to operation and climate-weather conditions (Kołowrocki & Kuligowska, 2018, Kołowrocki & Soszyńska-Budny, 2017; Torbicki, 2019a-b; Torbicki & Drabiński, 2020).

The proposed optimization procedures and perspective of future research applied to system operation cost and to safety and resilience optimization of the complex systems and critical infrastructures can give practically important possibility of these systems effectiveness improvement through the proposing their new operation strategy application.

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