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Safety and operation cost optimization of port oil terminal critical infrastructure

Keywords

safety, operation cost, optimization, port oil terminal

Abstract

The model of system safety impacted by operation process is introduced and the procedure of its safety maximization is proposed. The model of system operation total cost during the fixed operation time is introduced and the procedure of its minimization is presented. To analyse jointly the system safety and its operation cost optimization, we propose the procedure of determining the optimal values of limit transient probabilities of the system operation process at the particular operation states that allows to find maximal system safety indicators, through applying the created system safety model and linear programing. Next, to find the system conditional operation total cost during the fixed operation time, corresponding to this system maximal safety indicators, we replace the limit transient probabilities at the particular operation states, existing in the formula for the system operation total cost during the fixed operation time, by their optimal values existing in the formulae for the coordinates of the system safety function after maximization. The created models are applied to the port oil terminal critical infrastructure to maximize its safety indicators and to minimize its operation total cost during the fixed time separately. After that the port oil terminal critical infrastructure operation total cost during the fixed operation time corresponding to its maximal safety indicators is found. The evaluation of results achieved is performed and the perspective for future research in the field of the complex systems including critical infrastructures safety and their operation costs joint analysis and optimization is given.

1. Introduction

To tie the investigation of the complex technical system safety together with the investigation of its operation cost, the semi-Markov process model (Ferreira & Pacheco, 2007; Grabski, 2002, 2014; Glynn et al. 2006; Limnios & Oprisan, 2005; Mercier, 2008; Tang et al., 2007) can be used to describe this system operation process (Kołowrocki & Soszyńska-Budny, 2011/2015; Magryta, 2020).

The system operation process model, under the assumption on the system safety structure multistate model (Xue, 1985; Xue & Yang, 1995), can be used to construct the general safety model of the complex multistate system changing its safety structure and its components safety parameters during variable operation conditions (Kołowrocki, 2014; Kołowrocki & Magryta, 2020a; Kołowrocki & Soszyńska-Budny, 2011/2015; Magryta, 2020). Further, using this general model, it is possible to define the complex system main safety characteristics such as the system safety function, the mean values and standard deviations of the system lifetimes in the system safety state subsets and in the system particular

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safety states (Dąbrowska, 2020a-b; Kołowrocki, 2014, 2020; Kołowrocki & Soszyńska-Budny, 2010a-b, 2011/2015). Other system safety indicators, like the system risk function, the system fragility curve, the moment when the system risk function exceeds a permitted level, the system intensity of ageing, the coefficient of operation process impact on system intensity of ageing and the system resilience indicator to operation process impact, can be introduced as well (Gouldby et al., 2010; Kołowrocki, 2014; Kołowrocki & Soszyńska-Budny, 2018a-b, 2019a-b; Lauge et al., 2015; Szymkowiak, 2018a-b, 2019). Using the system general safety model, it is possible to change the system operation process through applying the linear programming (Klabjan & Adelman, 2006) for maximizing the system safety function (Kołowrocki & Soszyńska-Budny, 2010a) and finding the optimal limit values of the system transient probabilities in the particular operation states and determining the system optimal safety indicators.

Having the system operation process characteristics and the system conditional instantaneous operation costs in the operation states, it is possible to create the system general operation total cost model during the fixed operation time (Kołowrocki & Magryta, 2020a, 2021). Using this system operation total cost model, it is possible to change the system operation process through applying the linear programming (Klabjan & Adelman, 2006) for minimizing the system operation total cost during the fixed operation time (Kołowrocki & Magryta, 2020a, 2021) and finding the optimal limit values of the system transient probabilities in the particular operation states.

To analyse jointly the system safety and its operation cost optimization, we firstly apply the procedure of determining the optimal values of limit transient probabilities of the system operation process at the particular operation states that maximize the system safety. Next, to find the system conditional operation total cost during the fixed operation time, corresponding to this system maximal safety, we replace the limit transient probabilities at particular operation states, existing in the formula for the operation total cost during the fixed operation time, by their optimal values existing in the formula for the system maximal safety function coordinates.

The created model for maximizing the system safety is applied to the port oil terminal to find its optimal safety indicators. Next, the port oil terminal mean value of the system operation total cost during the fixed operation time of one year corresponding to its optimal safety indicators is found.

The chapter is organized into 7 parts, this Introduction as Section 1, Sections 2–6 and Conclusion as Section 7. In Section 2, the model of system safety impacted by operation process is introduced and the procedure of the system safety maximization is proposed. In Section 3, the model of system operation total cost during the fixed operation time is introduced and the procedure of its minimization is presented. In Section 4, the port oil terminal operation process influence on its safety indicators is examined and the maximal values of the this system safety indicators are determined. In Section 5, the port oil terminal operation total cost during one year is analysed and its minimal is determined. In Section 6, joint analysis of the port oil terminal safety maximizing and its conditional operation total cost during one year corresponding to this system maximal safety is performed. The maximal safety indicators are fixed and the system conditional operation total cost corresponding to this maximal safety indicators is determined. In Conclusion, the evaluation of results achieved is done and the perspective for future research in the field of the complex systems including critical infrastructures safety and their operation costs joint analysis and optimization is proposed.

2. System safety

2.1. System safety model

We assume that the system is operating at v, $v > 1$, operation states z_b, $b = 1,2,\ldots,v$, that have influence on the system functional structure and on the system safety. Applying semi-Markov model of the system operation process $Z(t)$, $t \geq 0$, it is possible to find this process two basic characteristics (Grabski, 2014; Kołowrocki & Magryta, 2020a; Kołowrocki & Soszyńska-Budny, 2011/2015):

- the vector of limit values

$$p_b = \lim_{t\to\infty} p_b(t) = \; b = 1,2,\ldots,v, \quad (1)$$

of transient probabilities

$$p_b(t) = \mathrm{P}(\mathrm{Z(t)} = z_b),\ t \geq 0,\ b = 1,2,\ldots,v, \quad (2)$$

of the system operation process $Z(t)$ at the particular operation states z_b, $b = 1,2,\ldots,v$,

- the vector $[\hat{M}_b]_{1xn}$ of the mean values

$$\hat{M}_b = E[\hat{q}_b] \cong p_b q,\ b = 1,2,\ldots,v, \quad (3)$$

of the total sojourn times $\hat{q}_b$, $b = 1,2,\ldots,v$, of the system operation process $Z(t)$, $t \geq 0$, at the particular operation states z_b, b = 1, 2,…,v, during the fixed system operation time q, $q > 0$, where p_b, $b = 1,2,\ldots,v$, are defined by (1)–(2).

Considering the safety function of the system impacted by operation process

$$\boldsymbol{S}(t,\cdot) = [\boldsymbol{S}(t,1), \boldsymbol{S}(t,2),\ldots, \boldsymbol{S}(t,z)],\ t \geq 0, \quad (4)$$

coordinate given by (Kołowrocki & Soszyńska-Budny, 2011/2015)

$$\boldsymbol{S}(t,u) \cong \sum_{b=1}^{n} p_b [\boldsymbol{S}(t,u)]^{(b)},\ t \geq 0, u = 1,2,\ldots,z, \quad (5)$$

where p_b, $b = 1,2,\ldots,v$, are the limit transient probabilities of the system operation process at the operation states z_b, $b = 1,2,\ldots,v$, and

$$[\boldsymbol{S}(t,u)]^{(b)} = P([T(u)]^{(b)} > t),\ t \geq 0,$$

$$u = 1,2,\ldots,z,\ b = 1,2,\ldots,v,$$

at these operation states are the conditional safety functions of the system and ($[T(u)]^{(b)}$, are the system conditional lifetimes in the safety state subsets $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, at the operation states z_b, $b = 1,2,\ldots,v$, it is natural to assume that the system operation process has a significant influence on the system safety.

From the expression (5), it folloews that the mean values of the system unconditional lifetimes in the safety state subsets $\{u,u+1,\ldots,z\}$, are of the form

$$\boldsymbol{\mu}(u) = \sum_{b=1}^{n} p_b [\mu(u)]^{(b)} \text{ for } u = 1,2,\ldots,z. \quad (6)$$

The values of the variances of the system unconditional lifetimes in the system safety state subsets are

$$[\boldsymbol{\sigma}(u)]^2 = 2\int_0^\infty t\boldsymbol{S}(t,u)\mathrm{d}t - [\boldsymbol{\mu}(u)]^2,\ u = 1,2,\ldots,z, \quad (7)$$

where $\boldsymbol{\mu}(u)$ is given by (6) and $\boldsymbol{S}(t,u)$ is given by (5).

The expressions for the mean values of the system unconditional lifetimes in the particular safety states are

$$\bar{\boldsymbol{\mu}}(u) = \boldsymbol{\mu}(u) - \boldsymbol{\mu}(u+1),\ u = 1,2,\ldots,z-1,$$

$$\bar{\boldsymbol{\mu}}(z) = \boldsymbol{\mu}(z). \quad (8)$$

The system risk function and the moment when the risk exceeds a permitted level d, respectively are given by (Kołowrocki & Soszyńska-Budny, 2011/2015):

$$\boldsymbol{r}(t) = 1 - \boldsymbol{S}(t,r),\ t \geq 0, \quad (9)$$

and

$$t = \boldsymbol{r}^{-1}(d), \quad (10)$$

where $\boldsymbol{S}(t,r)$ is given by (5) for $u = r$ and $\boldsymbol{r}^{-1}(t)$, if it exists, is the inverse function of the risk function $\boldsymbol{r}(t)$.

The mean values of the system intensities of ageing (departure from the safety state subset $\{u,u+1,\ldots,z\}$), are defined by

$$\lambda(u) = \frac{1}{\boldsymbol{\mu}(u)},\ u = 1,2,\ldots,z. \quad (11)$$

Considering the values of the system without operation impact intensities of ageing $\lambda^0(u)$, defined in (Kołowrocki & Magryta-Mut, 2020c; Kołowrocki & Soszyńska-Budny, 2018b), the coefficients of the operation process impact on the system intensities of ageing are given by

$$\boldsymbol{\rho}(u) = \frac{\lambda(u)}{\lambda^{0}(u)},\ u = 1,2,\ldots,z. \quad (12)$$

Finally, the system resilience indicators, i.e. the coefficients of the system resilience to operation process impact, are defined by

$$\boldsymbol{RI}(u) = \frac{1}{\boldsymbol{\rho}(u)}, \ \ u = 1,2,...,z. \tag{13}$$

2.2. System safety optimization

Considering the safety function of the system impacted by operation process $\boldsymbol{S}(t,\cdot)$, $t \geq 0$, coordinate given by (5), it is natural to assume that the system operation process has a significant influence on the system safety. This influence is also clearly expressed in the equation (6) for the mean values of the system unconditional lifetimes in the safety state subsets. From the linear equation (6), we can see that the mean value of the system unconditional lifetime $\boldsymbol{\mu}(u)$, $u = 1,2,\ldots,z$, is determined by the limit values of transient probabilities p_b, $b = 1,2,\ldots,v$, of the system operation process at the operation states z_b, $b = 1,2,\ldots,v$, and the mean values $[\boldsymbol{\mu}(u)]^{(b)}$, $b = 1,2,\ldots,v$, $u = 1,2,\ldots,z$, of the system conditional lifetimes in the safety state subsets $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, at these operation states. Therefore, the system lifetime optimization approach based on the linear programming can be proposed (Klabjan & Adelman, 2006). Namely, we may look for the corresponding optimal values $\dot{p}_b$, $b = 1,2,\ldots,v$, of the transient probabilities p_b, $b = 1,2,\ldots,v$, of the system operation process at the operation states to maximize the mean value $\boldsymbol{\mu}(u)$ of the unconditional system lifetime in the safety state subset $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$,
under the assumption that the mean values $[\boldsymbol{\mu}(u)]^{(b)}$, $b = 1,2,\ldots,v$, $u = 1,2,\ldots,z$, of the system conditional lifetimes in the safety state subset at the particular operation states are fixed. As a special case of the above formulated system lifetime optimization problem, if r, $r \in \{1,2,\ldots,z\}$ is a system critical safety state, we may look for the optimal values $\dot{p}_b$, $b = 1,2,\ldots,v$, of the transient probabilities p_b, $b = 1,2,\ldots,v$, of the system operation process at the system operation states to maximize the mean value $\boldsymbol{\mu}(r)$ of the unconditional system lifetime in the safety state subset $\{r,r+1,\ldots,z\}$, $r = 1,2,\ldots,z$, under the assumption that the mean values $[\boldsymbol{\mu}(r)]^{(b)}$, $b = 1,2,\ldots,v$, $u = 1,2,\ldots,z$, of the system conditional lifetimes in this safety state subset not worse then the criticakl safety state at the particular operation states are fixed. More exactly, we formulate the optimization problem as a linear programming model with the objective function of the following form

$$\boldsymbol{\mu}(r) = \sum_{b=1}^{n} p_b [\mu(r)]^{(b)}, \tag{14}$$

for a fixed $r \in \{1,2,\ldots,z\}$ and with the following bound constraints

$$\check{p}_b \leq p_b \leq \hat{p}_b, \ b = 1,2,\ldots,v, \tag{15}$$

$$\sum_{b=1}^{n} p_b = 1, \tag{16}$$

where

$$[m(r)]^{(b)}, \ [m(\boldsymbol{r})]^{(b)} \geq 0, \ b = 1,2,\ldots,v, \tag{17}$$

are fixed mean values of the system conditional lifetimes in the safety state subset $\{r,r+1,\ldots,z\}$ and

$$\check{p}_b, \ 0 \leq \check{p}_b \leq 1 \text{ and } \hat{p}_b, \ 0 \leq \hat{p}_b \leq 1, \ \check{p}_b \leq \hat{p}_b,$$

$$b = 1,2,\ldots,v, \tag{18}$$

are lower and upper bounds of the unknown transient probabilities at the particular operation states p_b, $b = 1,2,\ldots,v$, respectively.
Now, we can obtain the optimal solution of the formulated by (14)–(18) the optimization problem, i.e. we can find the optimal values $\dot{p}_b$ of the transient probabilities p_b, $b = 1,2,\ldots,v$, that maximize the objective function given by (14). The maximizing procedure is described in (Kołowrocki & Magryta, 2020b, Magryta-Mut, 2020).
Finally, after applying this procedure, we can get the maximum value of the system total mean lifetime in the safety state subset $\{r,r+1,\ldots,z\}$ defined by the linear form (14), in the following form

$$\dot{\mu}(r) = \sum_{b=1}^{n} \dot{p}_b [m(r)]^{(b)} \tag{19}$$

for a fixed $r \in \{1,2,\ldots,z\}$.
Further, by replacing the limit transient probabilities p_b, $b = 1,2,\ldots,v$, existing in the formulae

(4)–(5) by their optimal values $\dot{p}_b$, $b = 1,2,\ldots,v$, we get the optimal form of the system safety and the expressions for all remaining safety indicators considered in Section 2.1.

3. System operation cost

3.1. System operation cost model

Similarly to safety analysis of the system impacted by its operation process, we may investigate the system operation total cost during the fixed operation time. Namely, we firstly define the instantaneous system operation cost in the form of the vector

$$\boldsymbol{C}(t) = [[C(t)]^{(1)}, C(t)]^{(2)}, \ldots, [C(t)]^{(n)}], \; t \geq 0, \quad (20)$$

with the coordinates

$$[C(t)]^{(b)}, \; t \geq 0, \; b = 1,2,\ldots,v, \quad (21)$$

that are the system conditional instantaneous operation costs at the system operation states z_b, $b = 1,2,\ldots,v$.

Further, it is natural to assume that the system operation total cost during the fixed operation time depends significantly on the system operation total costs at the operation states. This dependency is clearly expressed in mean value of the system operation total cost during the system operation time q, given by

$$\boldsymbol{C}(q) = \sum_{b=1}^{n} p_b [C(q)]^{(b)}, \; q > 0, \quad (22)$$

where p_b, $b = 1,2,\ldots,v$, are limit transient probabilities at operation states defined by (1)–(2), and $[C(\theta)]^{(b)}$, $b = 1,2,\ldots,v$, are the mean values of the system conditional operation total costs at the particular system operation states z_b, $b = 1,2,\ldots,v$, given by

$$[C(q)]^{(b)} = \int_0^{\hat{M}_b} C[(t)]^{(b)} dt, \; q > 0, b = 1,2,\ldots,v, \quad (23)$$

where $\hat{M}_b$, $b = 1,2,\ldots,v$, are the mean values of the system operation process total sojourn times at the operation states during the fixed system operatkion time q, given by (3), and $[C(t)]^{(b)}$, $t \geq 0$, $b = 1,2,\ldots,v$, are the system conditional instantaneous operation costs at the system particular operation states defined by (22).

3.2. System operation cost optimization model

From the linear equation (22) of the system operation cost model introduced in Section 3.1, we can see that the mean value of the system total unconditional operation cost $\boldsymbol{C}(q), q > 0,$ is determined by the limit values of transient probabilities p_b, $b = 1,2,\ldots,v$, of the system operation process at the operation states z_b, $b = 1,2,\ldots,v$, defined by (1)–(2) and by the mean values $[C(q)]^{(b)}$, $q > 0$, $b = 1,2,\ldots,v$, of the system conditional operation total costs at the particular system operation states z_b, $b = 1,2,\ldots,v$, determined by (23). Therefore, the system operation cost optimization based on the linear programming (Klabjan & Adelman, 2006), can be proposed. Namely, we may look for the corresponding optimal values $\dot{p}_b$, $b = 1,2,\ldots,v$, of the limit transient probabilities p_b, $b = 1,2,\ldots,v$, of the system operation process at the operation states to minimize the mean value $\boldsymbol{C}(\theta)$ of the system unconditional operation yotal cost under the assumption that the mean values $[C(q)]^{(b)}$, $b = 1,2,\ldots,v$, of the system conditional operation totsl costs at the particular system operation states z_b, $b = 1, 2,\ldots,v$, are fixed.

Thus, we may formulate the optimization problem as a linear programming model with the objective function of the form given by (22) with the bound constraints

$$\breve{p}_b \leq p_b \leq \hat{p}_b, \; b = 1,2,\ldots,v, \; \sum_{b=1}^{n} p_b = 1, \quad (24)$$

where

$$[C(q)]^{(b)}, \; [C(q)]^{(b)} \geq 0, \; b = 1,2,\ldots,v, \quad (25)$$

are fixed mean values of the system conditional operation total costs at the operation states z_b, $b = 1,2,\ldots,v$, determined according to (23) and

$$\breve{p}_b, \; 0 \leq \breve{p}_b \leq 1 \text{ and } \hat{p}_b, \; 0 \leq \hat{p}_b \leq 1, \; \breve{p}_b \leq \hat{p}_b,$$

$$b = 1,2,\ldots,v, \quad (26)$$

are lower and upper bounds of the unknown transient probabilities p_b, $b = 1,2,\ldots,v$, respectively.

Now, we can find the optimal solution of the formulated by (22)–(26) the optimization problem, i.e. we can determine the optimal values $\dot{p}_b$, of the transient probabilities p_b, $b = 1,2,\ldots,v$, that minimize the objective function given by (22). The minimizing procedure is described in (Magryta, 2021).

Finally, after applying this procedure, we can get the minimum value of the system unconditional operation total cost, defined by the linear equation (22), in the following form

$$\dot{C}(q) = \sum_{i=1}^{i} \dot{p}_b [\boldsymbol{C}(q)]^{(b)}. \qquad (27)$$

4. Port oil terminal system safety

4.1. Terminal description

The port oil terminal placed at the Baltic seaside is designated for receiving oil products from ships, storage and sending them by carriages or trucks to inland. The terminal can operate in reverse way as well. The terminal is described in details in (Kołowrocki & Soszyńska-Budny, 2019a).

The considered terminal is composed of three parts A, B and C, linked by the piping transportation system with the pier. The approximate length of the port oil piping transportation system is equal to around 25 km.

The main technical assets (components) of the port oil terminal critical infrastructure are:

A_1 – port oil piping transportation system,
A_2 – internal pipeline technological system,
A_3 – supporting pump station,
A_4 – internal pump system,
A_5 – port oil tanker shipment terminal,
A_6 – loading railway carriage station,
A_7 – loading road carriage station,
A_8 – unloading railway carriage station,
A_9 – oil storage reservoir system.

The asset A_1, the port oil piping transportation system operating at the port oil terminal critical infrastructure consists of three subsystems:

- the subsystem S_1 composed of two pipelines, each composed of 176 pipe segments and 2 valves,
- the subsystem S_2 composed of two pipelines, each composed of 717 pipe segments and 2 valves,
- the subsystem S_3 composed of three pipelines, each composed of 360 pipe segments and 2 valves.

Its operation is the main activity of the port oil terminal involving the remaining assets A_2 – A_9.

The port oil transportation system is a series system composed of two series-parallel subsystems S_1, S_2, each containing two pipelines (assets) and one series-“2 out of 3” subsystem S_3 containing 3 pipelines (assets).

The subsystems S_1, S_2 and S_3 are forming a general series port oil transportation system safety structure presented in Figure 1.

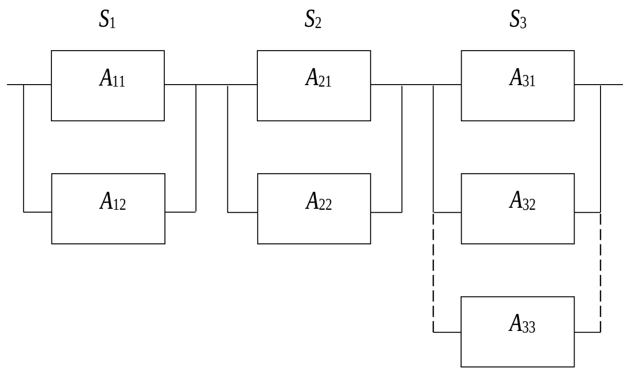

Figure 1. The port oil piping transportation system safety structure.

4.2. Operation process

We consider the port oil terminal critical infrastructure impacted by its operation process.

On the basis of the statistical data and expert opinions, it is possible to fix and to evaluate the following unknown basic parameters of the oil terminal critical infrastructure operation process: the number of operation process states $v = 7$ and the operation process states:

- the operation state z_1 – transport of one kind of medium from the terminal part B to part C using two out of three pipelines of the subsystem S_3 of the asset A_1 and assets A_2, A_4, A_6, A_7, A_9,
- the operation state z_2 – transport of one kind of medium from the terminal part C to part B using one out of three pipelines of the subsystem S_3 of the asset A_1 and assets A_2, A_4, A_8, A_9,
- the operation state z_3 – transport of one kind of medium from the terminal part B through

part A to pier using one out of two pipelines of the subsystem S_1 and one out of two pipelines of the subsystem S_2 of the asset A_1 and assets A_2, A_4, A_5, A_9,

- the operation state z_4 – transport of one kind of medium from the pier through parts A and B to part C using one out of two pipelines of the subsystem S_1, one out of two pipelines in subsystem S_2 and two out of three pipelines of the subsystem S_3 of the asset A_1 and assets A_2, A_3, A_4, A_5, A_6, A_7, A_9,
- the operation state z_5 – transport of one kind of medium from the pier through part A to B using one out of two pipelines of the subsystem S_1 and one out of two pipelines of the subsystem S_2 of the asset A_1 and assets A_2, A_3, A_4, A_5, A_9,
- the operation state z_6 – transport of one kind of medium from the terminal part B to C using two out of three pipelines of the subsystem S_3, and simultaneously transport one kind of medium from the pier through part A to B using one out of two pipelines of the subsystem S_1 and one out of two pipelines of the subsystem S_2 of the asset A_1 and assets A_2, A_3, A_4, A_5, A_6, A_7, A_9,
- the operation state z_7 – transport of one kind of medium from the terminal part B to C using one out of three pipelines of the subsystem S_3, and simultaneously transport second kind of medium from the terminal part C to B using one out of three pipelines of the subsystem S_3 of the asset A_1 and assets A_2, A_4, A_6, A_7, A_8, A_9.

To identify the unknown parameters of the port oil piping transportation system operation process the suitable statistical data coming from its real realizations should be collected. On the basic of this data (GMU, 2018), it is possible to estimate these parameters and to fix the port oil terminal characteristics (Kołowrocki & Soszyńska-Budny, 2011/2015):

- the limit values of transient probabilities of the operation process $Z(t)$ at the particular operation states z_b, $b = 1,2,\ldots, 7$:

$$p_1 = 0.395,\ p_2 = 0.060,\ p_3 = 0.003,\ p_4 = 0.002,$$
$$p_5 = 0.20,\ p_6 = 0.058,\ p_7 = 0.282, \tag{28}$$

- the expected values of the total sojourn times $\hat{q}_b$, $b = 1,2,\ldots,7$, of the system operation process at the particular operation states z_b, $b = 1,2,\ldots,7$, during the fixed operation time $q = 1$ year = 365 days:

$$M_1 = E[\hat{q}_1] = 0.395 \text{ year} = 144.175 \text{ days},$$
$$M_2 = E[\hat{q}_2] = 0.060 \text{ year} = 21.9 \text{ days},$$
$$M_3 = E[\hat{q}_3] = 0.003 \text{ year} = 1.095 \text{ day},$$
$$M_4 = E[\hat{q}_4] = 0.002 \text{ year} = 0.73 \text{ day},$$
$$M_5 = E[\hat{q}_5] = 0.20 \text{ year} = 73 \text{ days},$$
$$M_6 = E[\hat{q}_6] = 0.058 \text{ year} = 21.17 \text{ days},$$
$$M_7 = E[\hat{q}_7] = 0.282 \text{ year} = 102.93 \text{ days}. \tag{29}$$

4.3. Safety parameters

After considering the comments and opinions coming from experts, taking into account the effectiveness and safety aspects of the operation of the port oil terminal, we distinguish the following three safety states $(z = 2)$ of the system and its components:

- a safety state 2 – the components and the port oil terminal are fully safe,
- a safety state 1 – the components and the port oil terminal are less safe and more dangerous because of the possibility of environment pollution,
- a safety state 0 – the components and the port oil terminal are destroyed,

and we assume that:

- there are possible the transitions between the components safety states only from better to worse ones,
- the system and its components critical safety state is $r = 1$.
- the port oil terminal critical infrastructure risk function permitted level $\delta = 0.05$,
- the port oil terminal structure is series.

Moreover, the mean values of the assets A_i, $i = 1,2,\ldots,9$, of the port oil terminal critical infrastructure lifetimes in the safety state subsets {1, 2}, {2}, calculated on the basis of safety data of its components coming from experts, are as follows, (GMU, 2018; Kołowrocki & Magryta, 2020b; Kołowrocki & Soszyńska-Budny, 2011/2015):

- the mean values of the asset A_1, the port oil terminal critical infrastructure lifetimes in the safety state subsets {1, 2}, {2}:

– for safety state subsets {1, 2}

$$m_1^0(1) = 63 \text{ years}, \quad (30)$$

– for safety state subsets {2}

$$m_1^0(2) = 46 \text{ years}, \quad (31)$$

- the mean values of the asset $A_2 - A_9$ lifetime the safety state subsets {1, 2}, {2}, evaluated approximately by experts, are as follows, (Kołowrocki & Soszyńska-Budny, 2011/ 2015):
  – for safety state subsets {1, 2}

$$m_i^0(1) = 80 \text{ years}, \ i = 2,\ldots,9, \quad (32)$$

  – for safety state subsets {2}

$$m_i^0(2) = 50 \text{ years}, \ i = 2,\ldots,9, \quad (33)$$

We assume that assets A_i, $i = 1, 2,\ldots, 9$, have the piecewise exponential safety functions

$$\boldsymbol{S}^0(t,\cdot) = [\boldsymbol{S}^0(t,1), \boldsymbol{S}^0(t,2)], \ t \geq 0, \quad (34)$$

with coordinates given by

$$\boldsymbol{S}^0(t,u) = \exp[-l_i^0(u)t], \ t \geq 0, \quad (35)$$

where $u = 1,2$, and $l_i^0(u) > 0$, $i = 1,2,\ldots,9$, are intensities of ageing of the critical infrastructure assets A_i, $i = 1,2,\ldots,9$, / the intensities of the critical infrastructure assets A_i, $i = 1,2,\ldots,9$, departure from safety state subsets {1, 2} and {2} defined by the formula

$$l_i^0(u) = \frac{1}{m_i^0(u)}, \ u = 1,2, \ i = 1,2,\ldots,9. \quad (36)$$

Applying (36) to mean lifetimes of assets A_i, $i = 1,2,\ldots,9$, in the safety state subsets given by (30)–(33), we receive intensities of the assets departure from the safety state subsets {1, 2} and {2}:

- for asset A_1

$$l_1^0(1) = 0.015873, \ l_1^0(2) = 0.021739, \quad (37)$$

- for assets $A_2 - A_9$

$$l_i^0(1) = 0.0125, \ l_i^0(2) = 0.02, \ i = 2,3,\ldots,9. \quad (38)$$

4.4. Safety indicators

According to the formulae for the safety function of the series system and the assets' intensities of ageing, we receive the safety function of the port oil terminal critical infrastructure

$$\boldsymbol{S}^0(t,\cdot) = [\boldsymbol{S}^0(t,1), \boldsymbol{S}^0(t,2)], \ t \geq 0, \quad (39)$$

with coordinates given by

$$\begin{aligned} \boldsymbol{S}^0(t,1) &= \exp[-0.015873t]\exp[-0.0125t] \\ &\exp[-0.0125t]\exp[-0.0125t]\exp[-0.0125t] \\ &\exp[-0.0125t]\exp[-0.0125t]\exp[-0.0125t] \\ &\exp[-0.0125t] = \exp[-0.115873t] \end{aligned} \quad (40)$$

$$\begin{aligned} \boldsymbol{S}^0(t,2) &= \exp[-0.021739t]\exp[-0.02t] \\ &\exp[-0.02t]\exp[-0.02t]\exp[-0.02t] \\ &\exp[-0.02t]\exp[-0.02t]\exp[-0.02t] \\ &= \exp[-0.18739t] \end{aligned} \quad (41)$$

As the critical safety state is $r = 1$, then the port oil terminal critical infrastructure risk function, is given by

$$\boldsymbol{r}^0(t) = 1 - \boldsymbol{S}(t,1) = 1 - \exp[-0.115873t], \ t \geq 0. \quad (42)$$

Further, we get the following safety indicators of the port oil terminal critical infrastructure:

- the mean values of lifetimes in the safety state subsets {1, 2}, {2}:

$$\boldsymbol{\mu}^0(1) \cong 8.63, \ \boldsymbol{\mu}^0(2) \cong 5.50 \text{ years}, \quad (43)$$

- the standard deviations of lifetimes in the safety state subsets {1, 2}, {2}:

$$\boldsymbol{\mu}^0(1) \cong 8.63, \ \boldsymbol{\mu}^0(2) \cong 5.50 \text{ years}, \quad (44)$$

- the mean values of lifetimes in the particular safety states {1}, {2}:

$$\bar{\boldsymbol{\mu}}^0(1) \cong 3.13, \ \bar{\boldsymbol{\mu}}^0(2) \cong 5.50 \text{ years}, \quad (45)$$

- the moment when the risk function exceeds a permitted level $\delta = 0.05$ is

$$\tau \cong 0.440 \text{ year}, \quad (46)$$

- the intensities of degradation (ageing)

$$\lambda^0(t,1) \cong 0.116,\ \lambda^0(t,2) \cong 0.182. \quad (47)$$

4.5. Operation process impact

We assume that the port oil terminal critical infrastructure assets A_i, $i = 1,2,\ldots,9$, at the critical infrastructure operation process $Z(t)$ states z_b, $b = 1,2,\ldots,7$, conditional safety function

$$[S_i(t,\cdot)]^{(b)} = [[S_i(t,1)]^{(b)}, [S_i(t,2)]^{(b)}],\ t \geq 0,$$

$$b = 1,2,\ldots,7,\ i = 1,2,\ldots,9, \quad (48)$$

are piecewise exponential with the coordinates

$$[S_i(t,u)]^{(b)} = \exp[-[l_i(u)]^{(b)} t],\ t \geq 0,$$

$$u = 1,2,\ b = 1,2,\ldots,7,\ i = 1,2,\ldots,9, \quad (49)$$

where

$$[l_i(u)]^{(b)} = [r_i(u)]^{(b)} \cdot l_i^0(u),\ u = 1,2,$$

$$b = 1,2,\ldots,7,\ i = 1,2,\ldots,9, \quad (50)$$

and

$$[r_i(u)]^{(b)},\ u = 1,2,\ b = 1,2,\ldots,7,\ i = 1,2,\ldots,9,$$

are the coefficients of operation process impact on the intensities of degradation of the port oil critical infrastructure assets A_i, $i = 1,2,\ldots,9$, at the operation states z_b, $b = 1,2,\ldots,7$, and $l_i^0(u)$, $u = 1,2$, $i = 1,2,\ldots,9$, are the intensities of degradation of the port oil critical infrastructure assets without the operation process impact. From (Kołowrocki & Soszyńska-Budny, 2011/2015), it follows that the intensities of assets departure from the safety subset {1, 2} and {2} are given by (37)–(38).

The coefficients of the operation process impact on the port oil terminal critical infrastructure intensities of ageing at the operation states z_b, $b = 1,2,\ldots,7$, are as follows (GMU, 2018):

- for assets A_1

$$[\rho_i(1)]^{(b)} = 1.1,\ [\rho_i(2)]^{(b)} = 1.1,\ b = 1,2,7,\ i = 1,$$
$$[\rho_i(1)]^{(b)} = 1.2,\ [\rho_i(2)]^{(b)} = 1.2,\ b = 3,5,\ i = 1,$$
$$[\rho_i(1)]^{(b)} = 1.3,\ [\rho_i(2)]^{(b)} = 1.3,\ b = 4,6,\ i = 1, \quad (51)$$

- for assets A_2

$$[\rho_i(1)]^{(b)} = 1.1,\ [\rho_i(2)]^{(b)} = 1.1,$$
$$b = 1, 2, 7,\ i = 2,$$
$$[\rho_i(1)]^{(b)} = 1.2,\ [\rho_i(2)]^{(b)} = 1.2,\ b = 3, 5,\ i = 2,$$
$$[\rho_i(1)]^{(b)} = 1.3,\ [\rho_i(2)]^{(b)} = 1.3,\ b = 4, 6,\ i = 2, \quad (52)$$

- for assets A_3

$$[\rho_i(1)]^{(b)} = 1,\ [\rho_i(2)]^{(b)} = 1,\ b = 1,2,3,7,\ i = 3,$$
$$[\rho_i(1)]^{(b)} = 1.2,\ [\rho_i(2)]^{(b)} = 1.2,\ b = 5,\ i = 3,$$
$$[\rho_i(1)]^{(b)} = 1.3,\ [\rho_i(2)]^{(b)} = 1.3,\ b = 4,6,\ i = 3, \quad (53)$$

- for assets A_4

$$[\rho_i(1)]^{(b)} = 1.1,\ [\rho_i(2)]^{(b)} = 1.1,\ b = 1,2,7,\ i = 4,$$
$$[\rho_i(1)]^{(b)} = 1.2,\ [\rho_i(2)]^{(b)} = 1.2,\ b = 3,5,\ i = 4,$$
$$[\rho_i(1)]^{(b)} = 1.3,\ [\rho_i(2)]^{(b)} = 1.3,\ b = 4,6,\ i = 4, \quad (54)$$

- for assets A_5

$$[\rho_i(1)]^{(b)} = 1,\ [\rho_i(2)]^{(b)} = 1,\ b = 1,2,7,\ i = 5,$$
$$[\rho_i(1)]^{(b)} = 1.2,\ [\rho_i(2)]^{(b)} = 1.2,\ b = 3,5,\ i = 5,$$
$$[\rho_i(1)]^{(b)} = 1.3,\ [\rho_i(2)]^{(b)} = 1.3,\ b = 4,6,\ i = 5, \quad (55)$$

- for assets A_6

$$[\rho_i(1)]^{(b)} = 1,\ [\rho_i(2)]^{(b)} = 1,\ b = 2,5,\ i = 6,$$
$$[\rho_i(1)]^{(b)} = 1.1,\ [\rho_i(2)]^{(b)} = 1.1,\ b = 1,7,\ i = 6,$$
$$[\rho_i(1)]^{(b)} = 1.2,\ [\rho_i(2)]^{(b)} = 1.2,\ b = 3,\ i = 6,$$
$$[\rho_i(1)]^{(b)} = 1.3,\ [\rho_i(2)]^{(b)} = 1.3,\ b = 4,6,\ i = 6, \quad (56)$$

- for assets A_7

$$[\rho_i(1)]^{(b)} = 1,\ [\rho_i(2)]^{(b)} = 1,\ b = 2,3,5,\ i = 7,$$
$$[\rho_i(1)]^{(b)} = 1.1,\ [\rho_i(2)]^{(b)} = 1.1,\ b = 1,7,\ i = 7,$$
$$[\rho_i(1)]^{(b)} = 1.3,\ [\rho_i(2)]^{(b)} = 1.3,\ b = 4,6,\ i = 7, \quad (57)$$

- for assets A_8

$[\rho_i(1)]^{(b)} = 1$, $[\rho_i(2)]^{(b)} = 1$, $b = 1,3,4,5,6$, $i = 8$,
$[\rho_i(1)]^{(b)} = 1.1$, $[\rho_i(2)]^{(b)} = 1.1$, $b = 2,7$, $i = 8$, (58)

- for assets A_9

$[\rho_i(1)]^{(b)} = 1.1$, $[\rho_i(2)]^{(b)} = 1.1$, $b = 1,2,7$, $i = 9$,
$[\rho_i(1)]^{(b)} = 1.2$, $[\rho_i(2)]^{(b)} = 1.2$, $b = 3,5$, $i = 9$,
$[\rho_i(1)]^{(b)} = 1.3$, $[\rho_i(2)]^{(b)} = 1.3$, $b = 4,6$, $i = 9$, (59)

4.6. Safety parameters impacted by operation process

Under the assumption (50), considering (51)–(59) and (37)–(38), it follows that the intensities of assets departure from the safety states subset {1, 2}, {2}, with operation impact on their safety are:

- for assets A_1

$[\lambda_i(1)]^{(b)} = 0.01746$, $[\lambda_i(2)]^{(b)} = 0.023913$,
$b = 1,2,7$, $i = 1$,
$[\lambda_i(1)]^{(b)} = 0.019048$, $[\lambda_i(2)]^{(b)} = 0.026087$,
$b = 3,5$, $i = 1$,
$[\lambda_i(1)]^{(b)} = 0.020635$, $[\lambda_i(2)]^{(b)} = 0.028261$,
$b = 4,6$, $i = 1$, (60)

- for assets A_2

$[\lambda_i(1)]^{(b)} = 0.01375$, $[\lambda_i(2)]^{(b)} = 0.022$,
$b = 1,2,7$, $i = 2$,
$[\lambda_i(1)]^{(b)} = 0.015$, $[\lambda_i(2)]^{(b)} = 0.024$,
$b = 3,5$, $i = 2$,
$[\lambda_i(1)]^{(b)} = 0.01625$, $[\lambda_i(2)]^{(b)} = 0.026$,
$b = 4,6$, $i = 2$, (61)

- for assets A_3

$[\lambda_i(1)]^{(b)} = 0.0125$, $[\lambda_i(2)]^{(b)} = 0.02$,
$b = 1,2,3,7$, $i = 3$,
$[\lambda_i(1)]^{(b)} = 0.015$, $[\lambda_i(2)]^{(b)} = 0.024$,
$b = 5$, $i = 3$,
$[\lambda_i(1)]^{(b)} = 0.01625$, $[\lambda_i(2)]^{(b)} = 0.026$,
$b = 4,6$, $i = 3$, (62)

- for assets A_4

$[\lambda_i(1)]^{(b)} = 0.01375$, $[\lambda_i(2)]^{(b)} = 0.022$,
$b = 1,2,7$, $i = 4$,
$[\lambda_i(1)]^{(b)} = 0.015$, $[\lambda_i(2)]^{(b)} = 0.024$,
$b = 3,5$, $i = 4$,
$[\lambda_i(1)]^{(b)} = 0.01625$, $[\lambda_i(2)]^{(b)} = 0.026$,
$b = 4,6$, $i = 4$, (63)

- for assets A_5

$[\lambda_i(1)]^{(b)} = 0.0125$, $[\lambda_i(2)]^{(b)} = 0.02$,
$b = 1,2,7$, $i = 5$,
$[\lambda_i(1)]^{(b)} = 0.015$, $[\lambda_i(2)]^{(b)} = 0.024$,
$b = 3,5$, $i = 5$,
$[\lambda_i(1)]^{(b)} = 0.01625$, $[\lambda_i(2)]^{(b)} = 0.026$,
$b = 4,6$, $i = 5$, (64)

- for assets A_6

$[\lambda_i(1)]^{(b)} = 0.0125$, $[\lambda_i(2)]^{(b)} = 0.02$,
$b = 2,5$, $i = 6$,
$[\lambda_i(1)]^{(b)} = 0.01375$, $[\lambda_i(2)]^{(b)} = 0.022$,
$b = 1,7$, $i = 6$,
$[\lambda_i(1)]^{(b)} = 0.015$, $[\lambda_i(2)]^{(b)} = 0.024$,
$b = 3$, $i = 6$,
$[\lambda_i(1)]^{(b)} = 0.01625$, $[\lambda_i(2)]^{(b)} = 0.026$,
$b = 4,6$, $i = 6$, (65)

- for assets A_7

$[\lambda_i(1)]^{(b)} = 0.0125$, $[\lambda_i(2)]^{(b)} = 0.02$,
$b = 2,3,5$, $i = 7$,
$[\lambda_i(1)]^{(b)} = 0.01375$, $[\lambda_i(2)]^{(b)} = 0.022$,
$b = 1,7$, $i = 7$,
$[\lambda_i(1)]^{(b)} = 0.01625$, $[\lambda_i(2)]^{(b)} = 0.026$,
$b = 4,6$, $i = 7$, (66)

- for assets A_8

$[\lambda_i(1)]^{(b)} = 0.0125$, $[\lambda_i(2)]^{(b)} = 0.02$,
$b = 1,3,4,5,6$, $i = 8$,
$[\lambda_i(1)]^{(b)} = 0.01375$, $[\lambda_i(2)]^{(b)} = 0.022$,
$b = 2,7$, $i = 8$, (67)

- for assets A_9

$[\lambda_i(1)]^{(b)} = 0.01375$, $[\lambda_i(2)]^{(b)} = 0.022$,
$b = 1,2,7$, $i = 9$,
$[\lambda_i(1)]^{(b)} = 0.015$, $[\lambda_i(2)]^{(b)} = 0.024$,
$b = 3,5$, $i = 9$,
$[\lambda_i(1)]^{(b)} = 0.01625$, $[\lambda_i(2)]^{(b)} = 0.026$,
$b = 4,6$, $i = 9$, (68)

4.7. Safety indicators impacted by operation process

Considering that the coordinates of the conditional safety function (48) for the port oil terminal critical infrastructure assets A_1, i = 1, 2,…,9, with the intensities of ageing at the operation states z_b, b = 1,2,…,7, given respectively by (60)–(68), as the port oil terminal critical infrastructure is a three-state (z = 2) series system. The influence of the system operation states changing on the changes of the system safety structure and its components safety functions is as follows.

At the system operation state z_1, the conditional safety function is given by

$$[\boldsymbol{S}(t,\cdot)]^{(1)} = [[\boldsymbol{S}(t,1)]^{(1)}, [\boldsymbol{S}(t,2)]^{(1)}],\ t \geq 0,$$

where

$$[\boldsymbol{S}(t,1)]^{(1)} = \exp[-0.12371t],$$

$$[\boldsymbol{S}(t,2)]^{(1)} = \exp[-0.193913t], \tag{69}$$

The expected values and standard deviations of the port oil terminal conditional lifetimes in the safety state subsets {1, 2}, {2} at the operation state z_1, calculated from the results given by (69), respectively are:

$$m_1(1) \cong 8.083,\ m_1(2) \cong 5.157 \text{ year}, \tag{70}$$

and further, the mean values of the conditional lifetimes in the particular safety states 1, 2 at the operation state z_1, respectively are:

$$\bar{m}_1(1) \cong 2.926,\ \bar{m}_1(2) \cong 5.157 \text{ year}. \tag{71}$$

At the system operation state z_2, the conditional safety function is given by

$$[\boldsymbol{S}(t,\cdot)]^{(2)} = [[\boldsymbol{S}(t,1)]^{(2)}, [\boldsymbol{S}(t,2)]^{(2)}],\ t \geq 0,$$

where

$$[\boldsymbol{S}(t,1)]^{(2)} = \exp[-0.12246t],$$

$$[\boldsymbol{S}(t,2)]^{(2)} = \exp[-0.191913t], \tag{72}$$

The expected values and standard deviations of the port oil terminal conditional lifetimes in the safety state subsets {1, 2}, {2} at the operation state z_2, calculated from the results given by (72), respectively are:

$$m_2(1) \cong 8.1666,\ m_2(2) \cong 5.211 \text{ year}, \tag{73}$$

and further, the mean values of the conditional lifetimes in the particular safety states 1, 2 at the operation state z_2, respectively are:

$$\bar{m}_2(1) \cong 2.955,\ \bar{m}_2(2) \cong 5.211 \text{ year}. \tag{74}$$

At the system operation state z_3, the conditional safety function is given by

$$[\boldsymbol{S}(t,\cdot)]^{(3)} = [[\boldsymbol{S}(t,1)]^{(3)}, [\boldsymbol{S}(t,2)]^{(3)}],\ t \geq 0,$$

where

$$[\boldsymbol{S}(t,1)]^{(3)} = \exp[-0.131548t],$$

$$[\boldsymbol{S}(t,2)]^{(3)} = \exp[-0.206087t], \tag{75}$$

The expected values and standard deviations of the port oil terminal conditional lifetimes in the safety state subsets {1, 2}, {2} at the operation state z_3, calculated from the results given by (75), respectively are:

$$m_3(1) \cong 7.602,\ m_3(2) \cong 4.852 \text{ year}, \tag{76}$$

and further, the mean values of the conditional lifetimes in the particular safety states 1, 2 at the operation state z_3, respectively are:

$$\bar{m}_3(1) \cong 2.749,\ \bar{m}_3(2) \cong 4.852 \text{ year}. \tag{77}$$

At the system operation state z_4, the conditional safety function is given by

$$[\boldsymbol{S}(t,\cdot)]^{(4)} = [[\boldsymbol{S}(t,1)]^{(4)}, [\boldsymbol{S}(t,2)]^{(4)}],\ t \geq 0,$$

where

$$[\boldsymbol{S}(t,1)]^{(4)} = \exp[-0.146885t],$$

$$[\boldsymbol{S}(t,2)]^{(4)} = \exp[-0.230261t], \tag{78}$$

The expected values and standard deviations of the port oil terminal conditional lifetimes in the safety state subsets {1, 2}, {2} at the operation state z_4, calculated from the results given by (78), respectively are:

$$m_4(1) \cong 6.808, \ m_4(2) \cong 4.343 \text{ year}, \tag{79}$$

and further, the mean values of the conditional lifetimes in the particular safety states 1, 2 at the operation state z_3, respectively are:

$$\bar{m}_4(1) \cong 2.465, \ \bar{m}_4(2) \cong 4.343 \text{ year}. \tag{80}$$

At the system operation state z_5, the conditional safety function is given by

$$[\boldsymbol{S}(t,\cdot)]^{(5)} = [[\boldsymbol{S}(t,1)]^{(5)}, [\boldsymbol{S}(t,2)]^{(5)}], \ t \geq 0,$$

where

$$[\boldsymbol{S}(t,1)]^{(5)} = \exp[-0.131548t],$$

$$[\boldsymbol{S}(t,2)]^{(5)} = \exp[-0.206087t], \tag{81}$$

The expected values and standard deviations of the port oil terminal conditional lifetimes in the safety state subsets {1, 2}, {2} at the operation state z_5, calculated from the results given by (81), respectively are:

$$m_5(1) \cong 7.602, \ m_5(2) \cong 4.852 \text{ year}, \tag{82}$$

and further, the mean values of the conditional lifetimes in the particular safety states 1, 2 at the operation state z_5, respectively are:

$$\bar{m}_5(1) \cong 2.750, \ \bar{m}_5(2) \cong 4.852 \text{ year}. \tag{83}$$

At the system operation state z_6, the conditional safety function is given by

$$[\boldsymbol{S}(t,\cdot)]^{(6)} = [[\boldsymbol{S}(t,1)]^{(6)}, [\boldsymbol{S}(t,2)]^{(6)}], \ t \geq 0,$$

where

$$[\boldsymbol{S}(t,1)]^{(6)} = \exp[-0.146885t],$$

$$[\boldsymbol{S}(t,2)]^{(6)} = \exp[-0.230261t], \tag{84}$$

The expected values and standard deviations of the port oil terminal conditional lifetimes in the safety state subsets {1, 2}, {2} at the operation state z_6, calculated from the results given by (84), respectively are:

$$m_6(1) \cong 6.808, \ m_6(2) \cong 4.343 \text{ year}, \tag{85}$$

and further, the mean values of the conditional lifetimes in the particular safety states 1, 2 at the operation state z_6, respectively are:

$$\bar{m}_6(1) \cong 2.465, \ \bar{m}_6(2) \cong 4.343 \text{ year}. \tag{86}$$

At the system operation state z_7, the conditional safety function is given by

$$[\boldsymbol{S}(t,\cdot)]^{(7)} = [[\boldsymbol{S}(t,1)]^{(7)}, [\boldsymbol{S}(t,2)]^{(7)}], \ t \geq 0,$$

where

$$[\boldsymbol{S}(t,1)]^{(7)} = \exp[-0.12496t],$$

$$[\boldsymbol{S}(t,2)]^{(7)} = \exp[-0.195913t], \tag{87}$$

The expected values and standard deviations of the port oil terminal conditional lifetimes in the safety state subsets {1, 2}, {2} at the operation state z_7, calculated from the results given by (87), respectively are:

$$m_7(1) \cong 8.003, \ m_7(2) \cong 5.104 \text{ year}, \tag{88}$$

and further, the mean values of the conditional lifetimes in the particular safety states 1, 2 at the operation state z_7, respectively are:

$$\bar{m}_7(1) \cong 2.899, \ \bar{m}_7(2) \cong 5.104 \text{ year}. \tag{89}$$

In the case when the operation time is large enough, the port oil terminal system unconditional safety function is given by the vector

$$\boldsymbol{S}(t,\cdot) = [\boldsymbol{S}(t,1), \boldsymbol{S}(t,2)], \ t \geq 0, \tag{90}$$

where according to (5) and considering the pipeline system operation process transient probabilities at the operation states determined by (28), the vector coordinates are given respectively by

$$\begin{aligned}\boldsymbol{S}(t,1) = {} & 0.395 \cdot [\boldsymbol{S}(t,1)]^{(1)} + 0.060 \cdot [\boldsymbol{S}(t,1)]^{(2)} \\ & + 0.003 \cdot [\boldsymbol{S}(t,1)]^{(3)} + 0.002 \cdot [\boldsymbol{S}(t,1)]^{(4)} \\ & + 0.2 \cdot [\boldsymbol{S}(t,1)]^{(5)} + 0.058 \cdot [\boldsymbol{S}(t,1)]^{(6)} \\ & + 0.282 \cdot [\boldsymbol{S}(t,1)]^{(7)}, \ t \geq 0,\end{aligned} \tag{91}$$

$$\boldsymbol{S}(t,2) = 0.395\cdot[\boldsymbol{S}(t,2)]^{(1)} + 0.060\cdot[\boldsymbol{S}(t,2)]^{(2)} + 0.003\cdot[\boldsymbol{S}(t,2)]^{(3)} + 0.002\cdot[\boldsymbol{S}(t,2)]^{(4)} + 0.2\cdot[\boldsymbol{S}(t,2)]^{(5)} + 0.058\cdot[\boldsymbol{S}(t,2)]^{(6)} + 0.282\cdot[\boldsymbol{S}(t,2)]^{(7)},\ t \ge 0. \quad (92)$$

The graph of the three-state port oil terminal system safety function is presented in Figure 2.

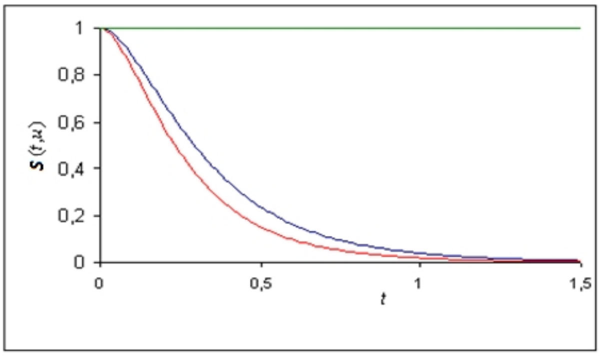

Figure 2. The graph of the port oil terminal system safety function coordinates.

The expected values and standard deviations of the system unconditional lifetimes in the safety state subsets {1, 2}, {2}, calculated from the above results given by (70), (73), (76), (79), (82), (85), (88), and according to (22)–(23), respectively are:

$$\boldsymbol{\mu}(1) = 0.395 \cdot 8.083 + 0.060 \cdot 8.166 + 0.003 \cdot 7.602 + 0.002 \cdot 6.808 + 0.2 \cdot 7.602 + 0.058 \cdot 6.808 + 0.282 \cdot 8.003 \cong 7.891 \text{ year}, \quad (93)$$

$$\boldsymbol{\sigma}(1) \cong 7.91 \text{ year}, \quad (94)$$

$$\boldsymbol{\mu}(2) = 0.395 \cdot 5.157 + 0.060 \cdot 5.211 + 0.003 \cdot 4.852 + 0.002 \cdot 4.343 + 0.2 \cdot 4.852 + 0.058 \cdot 4.343 + 0.282 \cdot 5.104 \cong 5.034 \text{ year}, \quad (95)$$

$$\boldsymbol{\sigma}(2) \cong 5.05 \text{ year}, \quad (96)$$

and further, considering (8) and (93) and (95), the mean values of the unconditional lifetimes in the particular safety states 1, 2, respectively are:

$$\bar{\boldsymbol{\mu}}(1) = \boldsymbol{\mu}(1) - \boldsymbol{\mu}(2) = 2.857,$$
$$\bar{\boldsymbol{\mu}}(2) = \boldsymbol{\mu}(2) = 5.034 \text{ year}. \quad (97)$$

Since the critical safety state is $r = 1$, then according to (9), the system risk function is given by

$$\boldsymbol{r}(t) = 1 - \boldsymbol{S}(t,1) \text{ for } t \ge 0, \quad (98)$$

where $\boldsymbol{S}(t,1)$ is given by (91).
Hence, according to (10), the moment when the system risk function exceeds a permitted level $d = 0.05$ is

$$t = \boldsymbol{r}^{-1}(d) \cong 0.404 \text{ year}. \quad (99)$$

The graph of the port oil terminal system risk function $\boldsymbol{r}(t)$ is presented in Figure 3.

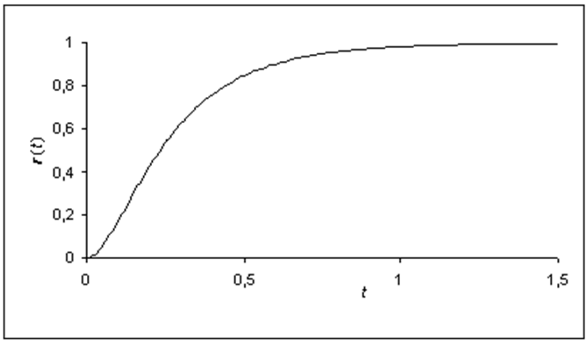

Figure 3. The graph of the port oil terminal system risk function.

The port oil terminal critical infrastructure mean intensities of ageing are:

$$\lambda(1) = 0.127,\ \lambda(2) = 0.199. \quad (100)$$

Considering (47) and (100) and applying (12), the coefficients of the operation process impact on the port oil terminal critical infrastructure intensities of ageing are:

$$\boldsymbol{\rho}(1) \cong 1.094,\ \boldsymbol{\rho}(2) \cong 1.094. \quad (101)$$

Finally, by (13) and (101), the port oil terminal critical infrastructure resilience indicator, i.e. the coefficient of the port oil terminal critical infrastructure resilience to the operation process impact is

$$\boldsymbol{RI}(1) = 1/\boldsymbol{\rho}(1) \cong 0.914 = 91.4\%. \quad (102)$$

4.8. Optimal safety indicators

After applying the procedure of the system safety maximization, the maximal value of the port oil terminal lifetime $\boldsymbol{\mu}(1)$ is (Magryta-Mut, 2021)

$$\dot{\mu}(1) = \dot{p}_1 \cdot 8.083 + \dot{p}_2 \cdot 8.166 + \dot{p}_3 \cdot 7.602 + \dot{p}_4 \cdot 6.808 + \dot{p}_5 \cdot 7.602 + \dot{p}_6 \cdot 6.808 + \dot{p}_7 \cdot 8.003 \cong 7.943 \text{ years}, \quad (103)$$

where the optimal limit transient probabilities of the port oil terminal at the particular operation states are:

$$\dot{p}_1 = 0.46,\ \dot{p}_2 = 0.08,\ \dot{p}_3 = 0.002,\ \dot{p}_4 = 0.001,\ \dot{p}_5 = 0.15,\ \dot{p}_6 = 0.004,\ \dot{p}_7 = 0.267.$$

Moreover, the corresponding optimal unconditional safety function coordinate of the port oil terminal system takes the form

$$\dot{\boldsymbol{S}}(t,1) = 0.46\exp[-0.12371t] + 0.08\exp[-0.12246t] + 0.002\exp[-0.131548t] + 0.001\exp[-0.146885t] + 0.15\exp[-0.131548t] + 0.04\exp[-0.146885t] + 0.267\exp[-0.12496t],\ t \geq 0. \quad (104)$$

Moreover, considering (103) and (104), the corresponding optimal standard deviation of the port oil terminal system unconditional lifetime in the state subset is (Magryta-Mut, 2021).

$$\dot{\boldsymbol{\sigma}}(1) \cong 7.943 \text{ years}. \quad (105)$$

As the port oil terminal system critical safety state is $\boldsymbol{r} = 1$, then considering (104), its optimal system risk function, is given by

$$\dot{\boldsymbol{r}}(t) \cong 1 - \dot{\boldsymbol{S}}(t,1),\ t \geq 0. \quad (106)$$

Hence, and considering (106) the moment when the optimal system risk function exceeds a permitted level $d = 0.05$ is

$$\dot{\tau} = \dot{\boldsymbol{r}}^{-1}(d) \cong 0.407 \text{ year}. \quad (107)$$

By (103) the port oil terminal system mean value of the optimal intensity of ageing is

$$\dot{\lambda}(1) = \frac{1}{\dot{\mu}(1)} \cong 0.126. \quad (108)$$

Considering (108) and the values of the analyzed system without operation impact intensity of ageing $\lambda^0(1)$, determined in (Kołowrocki & Soszyńska-Budny, 2018b; Magryta-Mut, 2021) and given by (47), the optimal coefficient of the operation process impact on the port oil terminal system intensity of ageing is

$$\dot{\rho}(1) = \frac{\dot{\lambda}(1)}{\lambda^0(1)} = \frac{0.126}{0.116} \cong 1.086. \quad (109)$$

Finally, the port oil terminal system optimal resilience indicator, i.e. the optimal coefficient of the port oil terminal system resilience to operation process impact, is

$$\dot{\boldsymbol{RI}}(1) = \frac{1}{\dot{\rho}(1)} = \frac{1}{1.086} \cong 0.921 = 92.1\%. \quad (110)$$

5. Port oil terminal critical infrastructure operation cost

5.1. Operation cost

The port oil terminal critical infrastructure operation process $Z(t)$ main characteristics are the limit values of transient probabilities of the operation process $Z(t)$ at the particular operation states z_b, $b = 1,2,\ldots,7$, (Kołowrocki & Soszyńska-Budny, 2011/2015; Magryta, 2020).

The asset A_1, the port oil terminal system is composed of 2880 components and the number of the system components operating at the various operation states, are different. Namely, there are operating 1086 system components at the operation states z_1, z_2 and z_7, 1794 system components at the operation states z_3 and z_5, 2880 system components at the operation states z_4 and z_6.

According to the information coming from experts, the approximate value of the instantaneous operation cost of the single basic component of the asset A_1 used during the operation time interval of $q = 1$ year at the operation state z_b, $b = 1,2,\ldots,7$, is constant and amounts 9.6 PLN, $t \in \langle 0,1 \rangle$, $b = 1,2,\ldots,7$, whereas, the cost of each its singular basic component that is not used is equal to 0 PLN.

Hence, the number of components in a subsystems S_1, S_2, S_3 and their use at particularly operation states imply that the asset A_1 conditional instantaneous operation costs $[C_1(t)]^{(b)}$, $t \in \langle 0,q \rangle$, $b = 1,2,\ldots,7$, introduced by (21), are:

$[\boldsymbol{C}_1(t)]^{(1)} = 1086 \cdot 9.6 = 10425.6$ PLN,
$[\boldsymbol{C}_1(t)]^{(2)} = 1086 \cdot 9.6 = 10425.6$ PLN,
$[\boldsymbol{C}_1(t)]^{(3)} = 1794 \cdot 9.6 = 17222.4$ PLN,
$[\boldsymbol{C}_1(t)]^{(4)} = 2880 \cdot 9.6 = 27648$ PLN,
$[\boldsymbol{C}_1(t)]^{(5)} = 1794 \cdot 9.6 = 17222.4$ PLN,
$[\boldsymbol{C}_1(t)]^{(6)} = 2880 \cdot 9.6 = 27648$ PLN,
$[\boldsymbol{C}_1(t)]^{(7)} = 1086 \cdot 9.6 = 10425.6$ PLN. (111)

Through (3) and (28), the approximate mean values $\hat{M}_b$, of total sojourn times of the port oil terminal at the particular operation states are:

$$\hat{M}_1 = 144.175,\ \hat{M}_2 = 21.9,\ \hat{M}_3 = 1.095,$$
$$\hat{M}_4 = 0.73,\ \hat{M}_5 = 73,\ \hat{M}_6 = 21.17,$$
$$\hat{M}_7 = 102.93. \quad (112)$$

Applying the formula (7) to (29) and (111), we get the approximate mean values $[C_1(\theta)]^{(b)}$, $b = 1,2,\ldots,7$, of the asset A_1 conditional operation total costs at the operation state z_b, $b = 1,2,\ldots,7$, during the operation time $q = 1$ year:

$[\hat{\boldsymbol{C}}_1(q)]^{(1)} = 144.175 \cdot 10425.6 = 1503110.88$ PLN,
$[\hat{\boldsymbol{C}}_1(q)]^{(2)} = 21.9 \cdot 10425.6 = 228320.64$ PLN,
$[\hat{\boldsymbol{C}}_1(q)]^{(3)} = 1.095 \cdot 17222.4 = 18858.528$ PLN,
$[\hat{\boldsymbol{C}}_1(q)]^{(4)} = 0.73 \cdot 27648 = 20183.04$ PLN,
$[\hat{\boldsymbol{C}}_1(q)]^{(5)} = 73 \cdot 17222.4 = 1257235.2$ PLN,
$[\hat{\boldsymbol{C}}_1(q)]^{(6)} = 21.17 \cdot 27648 = 585308.16$ PLN,
$[\hat{\boldsymbol{C}}_1(q)]^{(7)} = 102.93 \cdot 10425.6 = 1073107.008$ PLN. (113)

The corresponding mean values of the total conditional operation costs for the remaining assets $A_2 - A_9$, during the operation time $\theta = 1$ year, assumed arbitrarily (we do not have data at the moment) equal to 10000 PLN, in all operation states if they are used and equal to 0 PLN if they are not used. Under this assumption, considering the procedure of using assets $A_2 - A_9$ at particular operation states and the total operation costs of asset A_1 given in (28), we fix the total costs of the entire port oil terminal at the particular operation states z_b, $b = 1,2,\ldots,7$, given by:

$[\hat{\boldsymbol{C}}(q)]^{(1)} = 1503110.88 + 50000$
$= 1553110.88$ PLN,
$[\hat{\boldsymbol{C}}(q)]^{(2)} = 228320.64 + 40000$
$= 26832.64$ PLN,
$[\hat{\boldsymbol{C}}(q)]^{(3)} = 18858.528 + 40000$
$= 58858.528$ PLN,
$[\hat{\boldsymbol{C}}(q)]^{(4)} = 20183.04 + 70000$
$= 90183.04$ PLN,
$[\hat{\boldsymbol{C}}(q)]^{(5)} = 1257235.2 + 50000$
$= 130735.2$ PLN,
$[\hat{\boldsymbol{C}}(q)]^{(6)} = 585308.16 + 70000$
$= 655308.16$ PLN,
$[\hat{\boldsymbol{C}}(q)]^{(7)} = 1073107.008 + 60000$
$= 1133107.008$ PLN. (114)

Considering the values of the total costs $[\hat{C}_1(q)]^{(b)}$, $b = 1,2,\ldots,7$, from (113) and the values of transient probabilities p_b, $b = 1,2,\ldots,7$, given by (28), the port oil terminal total operation mean cost during the operation time $\theta = 1$ year, according to (22), is given by

$$\hat{\boldsymbol{C}}(q) \cong p_1[\hat{\boldsymbol{C}}(q)]^{(1)} + p_2[\hat{\boldsymbol{C}}(q)]^{(2)} + p_3[\hat{\boldsymbol{C}}(q)]^{(3)} + p_4[\hat{\boldsymbol{C}}(q)]^{(4)} + p_5[\hat{\boldsymbol{C}}(q)]^{(5)} + p_6[\hat{\boldsymbol{C}}(q)]^{(6)} + p_7[\hat{\boldsymbol{C}}(q)]^{(7)}$$
$$\cong 0.395 \cdot 1503110.88 + 0.06 \cdot 268320.64 + 0.003 \cdot 58858.528 + 0.002 \cdot 90183.04 + 0.2 \cdot 130735.2 + 0.058 \cdot 655308.16 + 0.282 \cdot 1133107.008 \cong 1013630. \quad (115)$$

5.2. Operation cost optimization

Considering (28) to find the minimum value of the port oil terminal mean cost, we define the objective function given by (22), in the following form

$$\boldsymbol{C}(q) = p_1 \cdot 1553110.88 + p_2 \cdot 268320.64 + p_3 \cdot 58858.528 + p_4 \cdot 90183.04 + p_5 \cdot 130735.2 + p_6 \cdot 655308.16 + p_7 \cdot 1133107.008. \quad (116)$$

The lower $\breve{p}_b$, and upper $\hat{p}_b$ bounds of the unknown optimal values of transient probabilities p_b, $b = 1,2,\ldots,7$, respectively are (Magryta, 2020):

$$\breve{p}_1 = 0.31,\ \breve{p}_2 = 0.04,\ \breve{p}_3 = 0.002,\ \breve{p}_4 = 0.001,$$
$$\breve{p}_5 = 0.15,\ \breve{p}_6 = 0.04,\ \breve{p}_7 = 0.25,$$

$$\hat{p}_1 = 0.46,\ \hat{p}_2 = 0.08,\ \hat{p}_3 = 0.006,\ \hat{p}_4 = 0.004,$$
$$\hat{p}_5 = 0.26,\ \hat{p}_6 = 0.08,\ \hat{p}_7 = 0.40. \quad (117)$$

Therefore, according to (24)–(25), we assume the following bound constraints

$$0.31 \le p_1 \le 0.46,\ 0.04 \le p_2 \le 0.08,$$
$$0.002 \le p_3 \le 0.006,\ 0.001 \le p_4 \le 0.004,$$
$$0.15 \le p_5 \le 0.26,\ 0.04 \le p_6 \le 0.08,$$
$$0.25 \le p_7 \le 0.40,\ \sum_{i=1}^{7} p_b = 1. \quad (118)$$

Now, before we find optimal values $\dot{p}_b$ of the transient probabilities p_b, $b = 1,2,\ldots,7$, that minimize the objective function (116), we arrange the mean values of the port oil terminal conditional operation costs $[\boldsymbol{C}(q)]^{(b)}$, $b = 1,2,\ldots,7$, in non-decreasing order

$$58858.528 \le 90183.04 \le 130735.2$$
$$\le 268320.64 \le 655308.16 \le 1133107.008$$
$$\le 1553110.88,$$

i.e.

$$[\boldsymbol{C}(q)]^{(3)} \le [\boldsymbol{C}(q)]^{(4)} \le [\boldsymbol{C}(q)]^{(5)} \le [\boldsymbol{C}(q)]^{(2)}$$
$$\le [\boldsymbol{C}(q)]^{(6)} \le [\boldsymbol{C}(q)]^{(7)} \le [\boldsymbol{C}(q)]^{(1)}. \quad (119)$$

Further, we substitute

$$x_1 = p_3,\ x_2 = p_4,\ x_3 = p_5,\ x_4 = p_2,\ x_5 = p_6,$$
$$x_6 = p_7,\ x_7 = p_1, \quad (120)$$

and

$$\breve{x}_1 = \breve{p}_3 = 0.002,\ \breve{x}_2 = \breve{p}_4 = 0.001,\ \breve{x}_3 = \breve{p}_5 = 0.15,$$
$$\breve{x}_4 = \breve{p}_2 = 0.04,\ \breve{x}_5 = \breve{p}_6 = 0.04,\ \breve{x}_6 = \breve{p}_7 = 0.25,$$
$$\breve{x}_7 = \breve{p}_1 = 0.31,$$
$$\hat{x}_1 = \hat{p}_3 = 0.006,\ \hat{x}_2 = \hat{p}_4 = 0.004,\ \hat{x}_3 = \hat{p}_5 = 0.26,$$
$$\hat{x}_4 = \hat{p}_2 = 0.08,\ \hat{x}_5 = \hat{p}_6 = 0.08,\ \hat{x}_6 = \hat{p}_7 = 0.40,$$
$$\hat{x}_7 = \hat{p}_1 = 0.46, \quad (121)$$

and we minimize with respect to x_i, $i = 1,2,\ldots,7$, the linear form (116) which takes the form

$$\boldsymbol{C}(q) = x_1 \cdot 58858.528 + x_2 \cdot 90183.04$$
$$+ x_3 \cdot 130735.2 + x_4 \cdot 268320.64$$
$$+ x_5 \cdot 655308.16 + x_6 \cdot 1133107.008$$
$$+ x_7 \cdot 1553110.88, \quad (122)$$

with the following bound constraints

$$0.002 \le x_1 \le 0.006,\ 0.001 \le x_2 \le 0.004,$$
$$0.15 \le x_3 \le 0.26,\ 0.04 \le x_4 \le 0.08,$$
$$0.04 \le x_5 \le 0.08,\ 0.25 \le x_6 \le 0.40,$$
$$0.31 \le x_7 \le 0.46,\ \sum_{i=1}^{7} x_i = 1. \quad (123)$$

We calculate

$$\breve{x} = \sum_{i=1}^{7} \breve{x}_i = 0.793,$$

$$\hat{y} = 1 - \breve{x} = 1 - 0.793 = 0.207 \quad (124)$$

and we find

$$\breve{x}^0 = 0,\ \hat{x}^0 = 0,\ \hat{x}^0 - \breve{x}^0 = 0,$$
$$\breve{x}^1 = 0.002,\ \hat{x}^1 = 0.006,\ \hat{x}^1 - \breve{x}^1 = 0.004,$$
$$\breve{x}^2 = 0.003,\ \hat{x}^2 = 0.01,\ \hat{x}^2 - \breve{x}^2 = 0.007,$$
$$\breve{x}^3 = 0.153,\ \hat{x}^3 = 0.27,\ \hat{x}^3 - \breve{x}^3 = 0.117,$$
$$\breve{x}^4 = 0.193,\ \hat{x}^4 = 0.35,\ \hat{x}^4 - \breve{x}^4 = 0.157,$$
$$\breve{x}^5 = 0.233,\ \hat{x}^5 = 0.43,\ \hat{x}^5 - \breve{x}^5 = 0.197,$$
$$\breve{x}^6 = 0.483,\ \hat{x}^6 = 0.83,\ \hat{x}^6 - \breve{x}^6 = 0.347,$$
$$\breve{x}^7 = 0.793,\ \hat{x}^7 = 1.29,\ \hat{x}^7 - \breve{x}^7 = 0.497. \quad (125)$$

From the above, since the expression takes the form

$$\hat{x}^I - \breve{x}^I < 0.207, \quad (126)$$

then it follows that the largest value $I \in \{0,1,\ldots,7\}$ such that this inequality holds is $I = 5$. Therefore, we fix the optimal solution that minimize linear function (116). Namely, we get

$$\dot{x}_1 = \hat{x}_1 = 0.006,\ \dot{x}_2 = \hat{x}_2 = 0.004,\ \dot{x}_3 = \hat{x}_3 = 0.26,$$
$$\dot{x}_4 = \hat{x}_4 = 0.08,\ \dot{x}_5 = \hat{x}_5 = 0.08,$$
$$\dot{x}_6 = \hat{y} - \hat{x}^5 + \breve{x}^5 + \breve{x}_6$$
$$= 0.207 - 0.43 + 0.233 + 0.25 = 0.26,$$
$$\dot{x}_7 = \breve{x}_7 = 0.31. \quad (127)$$

Finally, after making the substitution inverse to (120), we get the optimal transient probabilities

$$\hat{p}_1 = \hat{x}_7 = 0.31,\ \hat{p}_2 = \hat{x}_4 = 0.08,\ \hat{p}_3 = \hat{x}_1 = 0.006,$$
$$\hat{p}_4 = \hat{x}_2 = 0.004,\ \hat{p}_5 = \hat{x}_3 = 0.26,\ \hat{p}_6 = \hat{x}_5 = 0.08,$$
$$\hat{p}_7 = \hat{x}_6 = 0.26, \qquad (128)$$

that minimize the mean value of the port oil terminal operation total cost $\boldsymbol{C}(\theta)$ during the operation time $q = 1$ year, expressed by the linear form (115) and considering (128), its minimal value is

$$\hat{C}(q) \cong 0.31 \cdot 1553110.88 + 0.08 \cdot 268320.64$$
$$+ 0.006 \cdot 58858.528 + 0.004 \cdot 90183.04$$
$$+ 0.26 \cdot 130735.2 + 0.08 \cdot 655308.16$$
$$+ 0.26 \cdot 1133107.008 \cong 884667. \qquad (129)$$

6. Joint system safety optimization and operation cost analysis

6.1. System operation cost corresponding to its maximal safety

To analyze jointly the system safety and its operation cost, it is possible to propose the procedure of determining the optimal values of limit transient probabilities of the system operation process at the particular operation states that allows to find the maximal values of the system safety indicators, through applying the proposed system safety general model and linear programing. Next, to find the system conditional operation total cost during the fixed operation time, corresponding to this system maximal safety indicators, we replace the limit transient probabilities at the system particular operation states, existing in the formula for the system operation total cost during the fixed operation time by their optimal values existing in the formula for the system maximal safety function coordinates.

Thus, in Section 2.2, there is presented the procedure of determining the optimal values $\hat{p}_b$, $b = 1,2,\ldots,v$, of the limit transient probabilities of the system operation process $Z(t)$ at the particular operation states z_b, $b = 1,2,\ldots,v$, that allows to find the system maximal safety indicators, through applying the general system safety model and linear programing and determining their values. To find the system conditional operation total cost during the fixed operation time θ, corresponding to the system maximal safety indicators, we replace p_b, $b = 1,2,\ldots,v$, existing in the formula (22) for the the system operation total cost by $\hat{p}_b$, $b = 1,2,\ldots,v$, existing in the formula (19) for its maximal safety indicator, the system maximal mean lifetime in the system safety state subset not worse than the system critical safety state.

6.2. Port oil terminal operation cost corresponding to its maximal safety

In Section 4.8, there are given the optimal limit transient probabilities of the port oil terminal operation process $Z(t)$ at the particular operation states z_b, $b = 1,2,\ldots,7$:

$$\hat{p}_1 = 0.46,\ \hat{p}_2 = 0.08,\ \hat{p}_3 = 0.002,\ \hat{p}_4 = 0.001,$$
$$\hat{p}_5 = 0.15,\ \hat{p}_6 = 0.04,\ \hat{p}_7 = 0.267, \qquad (130)$$

determining the maximal safety indicators through applying the system general safety model impacted by its operation process and determining the system maximal mean life time in the system safety state subset not worse than the system critical safety state.

To find, the system conditional operation total cost during the fixed operation time of one year, corresponding to this system maximal safety indicators, we replace p_b, $b = 1,2,\ldots,7$, existing in the formula (115) for the the system total operation cost, by $\hat{p}_b$, $b = 1,2,\ldots,7$, given by (130). This way, we get the port oil terminal system conditional operation total cost during the fixed operation time of one year, corresponding to this system maximal safety indicators

$$\hat{C}(q) \cong 0.46 \cdot 1503110.88 + 0.08 \cdot 268320.64$$
$$+ 0.002 \cdot 58858.528 + 0.001 \cdot 90183.04$$
$$+ 0.15 \cdot 130735.2 + 0.04 \cdot 655308.16$$
$$+ 0.267 \cdot 1133107.008 \cong 1084467. \qquad (131)$$

6.3. Discussion of results

The obtained in Section 4.8 maximal port oil safety indicators are slightly better that those determined in Section 4.7 before safety maximization. Whereas, the conditional operation total cost of the port oil terminal given in Section 6.2 by (131), corresponding to its maximal safety indicators is slightly higher than this before the safety optimization given by (115) in Section 5.1, and much higher than this determined by (129) after the safety direct unconditional optimization performed in Section 5.2.

Thus, if we prefer the high safety of the port oil terminal safety more than ensuring the system lower operation total cost, we can modify this system operation process through replacing approximately the limit transient probabilities at the operation states p_b, $b = 1,2,\ldots,7$, in the particular operation states before the system safety maximization given by (28) by the values convergent to their optimal values $\dot{p}_b$, $b = 1,2,\ldots,7$, after the system safety maximization by (130).

In practice, it is easier to modify the considered system operation process through replacing approximately the system total operation time mean values at the particular operation states during the fixed operation time of $\theta = 1$ year = 365 days, determined by the approximate formula (Kołowrocki & Magryta-Mut, 2020c)

$$\hat{M}_b = p_b \cdot q, \; b = 1,2,...,7, \tag{132}$$

and given in days by (29) by the system total operation time mean values at the particular operation states during the fixed operation time of $\theta = 1$ year = 365 days, after the system operation total cost minimization determined according to the approximate formula (Kołowrocki & Magryta-Mut, 2020c)

$$\dot{M}_b = \dot{p}_b \cdot q, \; b = 1,2,...,7, \tag{133}$$

and after considering (130) given by:

$$\dot{M}_1 = 167.9, \; \dot{M}_2 = 29.2, \; \dot{M}_3 = 0.73, \; \dot{M}_4 = 0.365,$$
$$\dot{M}_5 = 54.75, \dot{M}_6 = 14.6, \; \dot{M}_7 = 97.455. \tag{134}$$

Equivalently, we can to modify the considered system operation process through replacing approximately the system total operation time mean values in the particular operation states during the fixed operation time of $\theta = 1$ month = 30 days = 720 hours, determined by the approximate formula (132) and after considering (28) given in hours by:

$$\hat{M}_1 = 284.4, \; \hat{M}_2 = 43.2, \; \hat{M}_3 = 2.16, \; \hat{M}_4 = 1.44,$$
$$\hat{M}_5 = 144, \; \hat{M}_6 = 41.76, \; \hat{M}_7 = 203.04. \tag{135}$$

by the system total operation time mean values in the particular operation states during the fixed operation time of $\theta = 1$ month = 30 days = 720 hours, after the system operation total cost optimization determined according to the approximate formula (133) and after considering (130) given by:

$$\dot{M}_1 = 331.2, \; \dot{M}_2 = 57.6, \; \dot{M}_3 = 1.44, \; \dot{M}_4 = 0.72,$$
$$\dot{M}_5 = 108, \; \dot{M}_6 = 28.8, \; \dot{M}_7 = 192.24. \tag{136}$$

The procedure of the port oil terminal operation process can be performed for other than the above fixed operation times of 1 month and 1 year, dependently to the system operator comfort in the achievement of the best results of the system operation total times in the particular operation states convergence to their optimal values resulting from the performed system safety maximization.

7. Conclusion

The procedures of using the general safety analytical model and the operation total cost model of complex multistate technical system related to its operation process (Kołowrocki, 2014) and the linear programing (Klabjan & Adelman, 2006) are presented and proposed to separate and joint analysis of the system safety maximization, its operation total cost minimization and determining its conditional operation total cost corresponding to this system maximal safety. The mean value of the complex multistate system in the system safety state subset not worse than the system critical safety state is maximize through the system operation process modification. This operation process modification allows to find the complex system conditional operation total cost during the fixed operation time corresponding to the system maximal safety indicators. The proposed system safety optimization procedure and corresponding system operation total cost finding gives practically important possibility of the system indicators maximization and keeping fixed corresponding the system operation total cost during the operation through the system new operation strategy. The proposed system safety and system operation cost optimization models and procedures are applied to the port oil terminal examination. These procedures can be used in safety and operation cost optimization of members of various real complex systems and critical infrastructures (Gouldby et al., 2010; Habibullah et al., 2009; Kołowrocki et al., 2016; Kołowrocki & Magryta, 2020b; Kołowrocki & Magryta-Mut, 2020c; Lauge et al., 2015; Magryta-Mut, 2020).

Further research can be related to considering other impacts on the system safety and its operation cost, for instance a very important impact related to climate-weather factors (Kołowrocki & Kuligowska, 2018; Kołowrocki & Soszyńska-Budny, 2017; Torbicki, 2019a-b; Torbicki & Drabiński, 2020) and resolving the issues of critical infrastructure (Lauge et al., 2015) safety and operation cost optimization and discovering optimal values of safety, operation cost and resilience indicators of system impacted by the operation and climate-weather conditions (Kołowrocki, 2021). These developments can also benefit the mitigation of critical infrastructure accident consequences (Bogalecka, 2020; Dąbrowska & Kołowrocki, 2019a-b; 2020a-c, Kołowrocki, 2021) and to minimize the system operation cost and to improve critical infrastructure resilience to operation and climate-weather conditions (Kołowrocki, 2021; Kołowrocki & Kuligowska, 2018; Kołowrocki & Soszyńska-Budny, 2017; Torbicki, 2019a-b; Torbicki & Drabiński; 2020).

The optimization procedures applied to the system operation cost and to safety and resilience optimization of complex systems and critical infrastructures can give practically important possibility of these systems' effectiveness improvement through their new operation strategy application.

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