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Safety analysis of critical infrastructure impacted by operation and climate-weather changes – theoretical backgrounds

Keywords

critical infrastructure, ageing, safety, operation impact, climate-weather impact, resilience

Abstract

The methodology and general approach to critical infrastructure safety and resilience analysis is proposed. The principles of multistate approach to critical infrastructure safety analysis are introduced. There are introduced the notions of critical infrastructure basic safety indicators like, the critical infrastructure safety function, the critical infrastructure risk function and the critical infrastructure fragility curve. The critical infrastructure safety and resilience indicators are proposed to be obtained using probabilistic approach to modelling of operation threats and extreme weather hazard impacts on its assets safety. There are proposed safety and resilience indicators, crucial for operators and users of the critical infrastructure, defined as a complex system in its operating environment. The safety of a critical infrastructure free of any outside impacts is discussed and modelled. The safety indicators of this critical infrastructure are defined. The safety of critical infrastructure impacted by its operation process is considered. The critical infrastructure operation process and its parameters are defined and its characteristics are determined. The safety and resilience indicators of the critical infrastructure related to the operation process impact are proposed. The safety of critical infrastructure impacted by the climate-weather change process at its operating area is considered. The climate-weather change process at the critical infrastructure operating area and its parameters are defined and its characteristics are determined. The safety and resilience indicators of the critical infrastructure related to the climate-weather change process at its operating area impact are proposed. The safety of critical infrastructure impacted by its operation process and climate-weather change process at its operating area is considered. The critical infrastructure operation process related to climate-weather change process at its operating area and its parameters are defined and its characteristics are determined. The safety and resilience indicators of the critical infrastructure impacted by the operation process related to climate-weather change process are proposed. Real critical infrastructures and their assets impacted by their operation processes related to climate-weather change process at their operating area are suggested to be examined and their safety and resilience indicators are proposed to be determined by the proposed methods.

1. Introduction

The chapter is focused on the critical infrastructure safety analysis (Dąbrowska, 2020; Kołowrocki, 2019/2020, 2020a; Kołowrocki & Kuligowska, 2018; Kołowrocki & Magryta-Mut, 2020; Magryta-Mut, 2020; Torbicki, 2019a, 2019c; Torbicki & Drabiński, 2020) based on reliability modelling of multistate ageing system (Brunelle, 1999; Kołowrocki, 2014, 2020a; Kołowrocki & Magryta, 2020a; Lisnianski et al., 2010; Magryta, 2020; Natvig, 2007; Ramirez-Marqueza, 2017; Xue & Yang, 1995a, 1995b; Yingkui & Jing, 2012) with its development through the assumption of its operation process (Dąbrowska, 2020; Kołowrocki, 2014, 2019/2020, 2020a, 2020b, 2021; Kołowrocki &

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Magryta-Mut, 2020; Magryta, 2020) and the climate-weather process (Kołowrocki & Kuligowska, 2018; Torbicki, 2019a, 2019c; Torbicki & Drabiński, 2020) in its operating area changes influence on the system components' safety parameters and consequently on the system safety characteristics degradation. Considering that the paper is devoted to critical infrastructure safety mathematical modelling and prediction, the critical infrastructure is defined as a complex multistate ageing system composed of multistate ageing components/assets in its operating environment. In practical usage, the critical infrastructure significant features are its inside dependencies and its outside dependencies, that in the case of its degradation have a significant destructive influence on the health, safety and security, economics and social conditions of large human communities and territory areas (Rinaldi et al., 2001).

Many multistate methods used in complex system reliability and safety analysis are difficult to apply practically with enough good accuracy because using them causes that it is necessary to consider a large number of transitions between the reliability and safety states what results that receiving exact solutions is not possible and evaluation of the approximate solution is often not precise enough. The proposed approach allows to eliminate this problem and get the exact values of basic critical infrastructure safety indicators, independently of the number of assets and the number of their safety states. More exactly, it means that in the case the critical infrastructure free of outside impacts, the large number of its assets and their safety states do not restrict the possibility of the proposed approach use and receiving exact solutions. However, further main practically important idea of the approach to the multistate aging critical infrastructure safety analysis presented in this paper is to consider the critical infrastructure operation process and the climate-weather change process in its operating area impact on its assets safety states' degradation that causes decreasing the safety level of the whole critical infrastructure. And, in order to realize this significant idea, in joining the exact results coming from the safety model of the critical infrastructure free of outside impacts with the approximate results obtained from the operation process model and the climate-weather change process model we are forced to use those two processes approximate characteristics, what does the final results approximate (Kołowrocki & Kuligowska, 2018).

The common usage of the multistate complex critical infrastructure safety model (Kołowrocki, 2019/2020, 2021) and the semi-Markov models (Ferreira & Pacheco, 2007; Glynn & Haas, 2006; Grabski, 2014; Klabjan & Adelman, 2006; Limnios & Oprisan 2005; Tang et al., 2007) for its operation process and the climate-weather change process in its operating area in order to construct the joint general safety model of the critical infrastructure related to its operation process (Kołowrocki, 2021) and the climate-weather change process in its operating area (Kołowrocki, 2019/2020; Torbicki, 2019a, 2019c; Torbicki & Drabiński, 2020) is the paper main principle of the critical infrastructure safety modelling (De Porcellinis et al., 2009; Gouldby et al., 2010; Kołowrocki, 2019/2020; Kołowrocki & Kuligowska, 2018; Magryta, 2020; Svedsen & Wolthunsen, 2007). This principle allows to create useful practical tool in safety examination of real critical infrastructures (Gdynia Maritime University, 2018; Kołowrocki, 2020; Kołowrocki & Kuligowska, 2018; Magryta, 2020; Torbicki, 2019a, 2019c; Torbicki & Drabiński, 2020). The joint model linking the safety model of the multistate critical infrastructure and its varying in time operation process model and the climate-weather change process in its operating area model is constructed. The proposed in the paper model and methods of finding critical infrastructure safety, risk and resilience indicators can be practically applied to analysis, identification and prediction of various kinds of real complex systems related to varying in time their operation process and the climate-weather change process in their operating area influence on changing in time its safety structure and its assets safety parameters (Kołowrocki, 2019/2020; Kołowrocki & Kuligowska, 2018).

The paper delivers a complete and current elaboration on the newest mathematical methods of safety identification, evaluation and prediction for as wide as possible a range of critical infrastructures. Pointing out the possibility of these method's extensive practical application in the operating processes of these critical infrastructures and climate-weather change processes is also an important reason for this paper. The chapter contains complete current solutions of the formulated problems for the considered critical infrastructures under the assumption of the piecewise exponential safety functions of their assets. This assumption is necessary in the analytical approach to the considered subject, to get the significant results considering real ageing of critical infrastructures and their degradation caused by extreme outside operation and climate-weather conditions. While, the notation climate-weather change process corresponds to the weather change process in a short-term impact (Torbicki & Drabiński, 2020) analysis and to the climate change process in long-term impact (Torbicki, 2019a, 2019c) analysis.

The chapter consists of 7 parts, including this Introduction as Section 1, Sections 2–6 and Conclusion as Section 7. In Section 2, the methodology and general approach to critical infrastructure safety and resilience analysis is proposed. The principles of multistate approach to critical infrastructure safety analysis are introduced. There are introduced the notions of critical infrastructure basic safety indicators like, the critical infrastructure safety function, the critical infrastructure risk function and the critical infrastructure fragility curve. The critical infrastructure safety and resilience indicators are proposed to be obtained using probabilistic approach to modelling of operation threats and extreme weather hazard impacts on its assets safety.

There are proposed safety and resilience indicators (SafIs, ResIs): the critical infrastructure safety function (SafI1), the critical infrastructure risk function (SafI2), the critical infrastructure fragility curve (SafI3), the mean value of the critical infrastructure lifetime up to exceeding critical safety state (SafI4), the standard deviation of the critical infrastructure lifetime up to the exceeding the critical safety state (SafI5), the moment of exceeding acceptable value of critical infrastructure risk function level (SafI6), the mean values of the critical infrastructure lifetimes in the safety state subsets (SafI7), the standard deviations of the critical infrastructure lifetimes in the safety state subsets (SafI8), the mean values of the critical infrastructure lifetimes in particular safety states (SafI9), the intensities of degradation (ageing) of the critical infrastructure / the intensities of critical infrastructure departure from the safety state subsets (SafI10), the coefficients of operation process and climate-weather change process impact on the critical infrastructure intensities of degradation / the coefficients of operation process and climate-weather change process impact on critical infrastructure intensities of departure from the safety state subsets (ResI1), the indicator of critical infrastructure resilience to operation process and climate-weather change process impact (ResI2), crucial for operators and users of the critical infrastructure, defined as a complex system in its operating environment.

Section 3 is devoted to modelling critical infrastructure safety without considering its outside impacts. The critical infrastructure free of outside impacts assets' safety parameters are defined. The safety indicators SafI 1–10 of the critical infrastructure free of any outside impacts are introduced.

Section 4 is concerned with modelling safety of a critical infrastructure impacted by its operation process. The semi-Markov process is used to construct a probabilistic model of the critical infrastructure operation process. The critical infrastructure operation process and its parameters are defined. The operation characteristics of the critical infrastructure operation process are determined. Next, the integrated general safety model of a critical infrastructure impacted by its operation process, linking its multistate safety model constructed in Section 3 and its operation process model and considering variable at the different operation states its safety structure and its assets' safety parameters is created. The critical infrastructure impacted by its operation process assets' safety parameters are defined. The safety and resilience indicators SafI 1–10 and ResI 1–2 of the critical infrastructure impacted by its operation process are introduced.

Section 5 is devoted to modelling safety of a critical infrastructure impacted by the climate-weather change process at its operating area. The semi-Markov process is used to construct a probabilistic model of the climate-weather change process influencing critical infrastructure at its operation area. The climate-weather change process at the critical infrastructure operating area and its parameters are defined. The characteristics of this climate-weather change process are determined. Next, the integrated general safety model of a critical infrastructure impacted by the climate-weather change process at its operating area, linking its multistate safety model constructed in Section 3 and the model of the climate-weather change process at its operation area and considering variable at the different climate-weather states its safety structure and its assets' safety parameters is created. The critical infrastructure impacted by the climate-weather change process at its operating area assets' safety parameters are defined. The safety and resilience indicators SafI 1–10 and ResI 1–2 of the critical infrastructure impacted by the climate-weather change process at its operating area are introduced.

In Section 6, the integration of the safety model of the critical infrastructure impacted by its operation process introduced in Section 4 and the safety model of the critical infrastructure impacted by the climate-weather change process safety model at its operating area introduced in Section 5 is performed to construct the general joint safety model of critical infrastructure safety influenced simultaneously by its operation process and by the climate-weather change process at its operating area. This general joint safety model is proposed in two cases when the critical infrastructure operation process and climate-weather change process are independent and when they are dependent. The critical infrastructure operation process related to climate-weather change process and its parameters are defined and its characteristics are determined. The critical infrastructure impacted by its operation process related to the climate-weather change at its operating area assets' safety parameters are defined. The safety and resilience indicators SafI 1–10 and ResI 1–2 of the critical infrastructure impacted by its operation process related to the climate-weather change at its operating area are proposed.

In Section 7, the conclusions on the chapter context are done and the perspective for future research is formulated.

2. General approach to critical infrastructure safety modelling

2.1. Multistate approach to critical infrastructure safety

Many of the terms and notions needed for the critical infrastructure safety analysis are used in different and sometimes conflicting ways across various disciplines and approaches. Some of them are incorrect. Thus, a standard set of definitions should be fixed to support a shared understanding of the foci of this chapter and be applied by all its readers. Therefore, the definitions concerned with the methodology including the notions and the meanings of the critical infrastructure and its safety, the climate and weather change and the resilience should be convergent with those used in available literature. The spectrum of the terms concerned with those three main notions should be sufficiently wide and exhaustive in depth. The main fault in defining some of the terms is mixing the meaning of the defined notion with the values of its parameters it is characterized by. Having in mind this terminology state of the art and considering its imperfection and faults, the main principles in the proposed approach are:

- to differ between the notion and the values of the parameters it is defined by,
- to illustrate shortly the notion and its parameters together with short illustrations/interpretations of their meanings and their expected practical usage in order to provide a better understanding.

The first and most important term for the approach is the notion of the critical infrastructure. To follow the European Commission approach, the critical infrastructure is an asset or system which is essential for the maintenance of vital societal functions. The damage of a critical infrastructure, its destruction or disruption by natural disasters, terrorism, criminal activity or malicious behavior, may have a significant negative impact for the security of the European Union and the well-being of its citizens.

The critical infrastructure is a term used by governments to describe assets that are essential for the functioning of a society and economy. Most commonly associated with the term of critical infrastructure are facilities for:

- electricity generation, transmission and distribution,
- gas production, transport and distribution,
- oil and oil products production, transport and distribution,
- telecommunication,
- water supply,
- agriculture, food production and distribution,
- heating,
- public health,
- transportation systems,
- financial services,
- security services.

Critical infrastructures are usually interconnected and mutually dependent in various and complex ways, creating critical infrastructure networks.

They are interacting directly and indirectly at various levels of their complexity and operating activity (Nieuwenhuijs et al., 2008; Ouyang, 2014; Rinaldi, 2001). Identifying and modeling dependencies depend on the level of analysis. The selected level of analysis can vary from micro to macro level (De Porcellinis et al., 2009; Gdynia Maritime University, 2018; Holden et al., 2013). A holistic approach as in (Kossow & Preuss, 1995) can be considered or a reductionist approach in which elementary components are identified and their behavior is described. For example, Svedsen and Wolthunsen (Svedsen & Wolthunsen, 2007) focus on the components of critical infrastructure networks and they demonstrate several types of multi-dependency structures.

Considering that this chapter is devoted to critical infrastructure safety mathematical modelling and prediction the critical infrastructure is defined as a complex multistate ageing system in its operating environment that significant features are its inside dependencies and its outside impacts, that in the case of its degradation have a significant destructive influence on the health, safety and security, economics and social conditions of large human communities and territory areas.

The multistate system used in the proposed approach was introduced in (Xue, 1985; Xue & Yang, 1995a, 1995b; Yingkui & Jing, 2012). Different approaches to describe multistate systems and to estimate their reliability can be found in (Brunelle & Kapur, 1999; Lauge et al., 2015; Limnios & Oprisan, 2005; Magryta, 2020; Svedsen & Wolthunsen, 2007).

Multistate large systems are also widely discussed in the literature (Kołowrocki, 2000, 2003, 2005, 2008, 2014, 2020, 2021). Another practically important approach to multistate ageing system reliability analysis consider the assumption about component degradation (departures from the reliability state subsets) instead of component failures (Dąbrowska, 2020; Kołowrocki, 2019/2020, 2020; Kołowrocki & Kuligowska, 2018; Torbicki & Drabiński, 2020; Wang et al., 2011; Xue, 1985).

In real technical systems, components often degrade with time by going to states corresponding to different performance levels. Degradation of components and subsystems in case of complex systems, causes the decreasing of system reliability and its operation safety.

Considering the above performed analysis, similarly as in the case of multistate approach to critical infrastructure reliability, in the multistate safety analysis to define the critical infrastructure with degrading/ageing components/assets, we assume that (Kołowrocki, 2014, 2019/2020):

- n is the number of the critical infrastructure assets,
- A_i, $i = 1,2,...,n$, are the critical infrastructure assets,
- all assets and the critical infrastructure have the safety state set $\{0,1,...,z\}$, $z \geq 1$,
- the safety states are ordered, the safety state 0 is the worst and the safety state z is the best,
- r, $r \in \{1,2,\ldots,z\}$, is the critical safety state,
- $T_i(u)$, $u = 1,2,...,z$, $i = 1,2,...,n$, are independent random variables representing the lifetimes of assets A_i in the safety state subset $\{u,u+1,...,z\}$, $u = 1,2,...,z$, while they were in the safety state z at the moment $t = 0$,
- $T(u)$, $u = 1,2,...,z$, is a random variable representing the lifetime of the critical infrastructure in the safety state subset $\{u,u+1,...,z\}$, $u = 1,2,...,z$, while it was in the safety state z at the moment $t = 0$,
- the safety states degrade with time t,
- the assets and the critical infrastructure degrade with time t,
- $s_i(t)$, $i = 1,2,...,n$, is the asset A_i, safety state at the moment t, $t \geq 0$, given that it was in the safety state z at the moment $t = 0$,
- $s(t)$ is the critical infrastructure safety state at the moment t, $t \geq 0$, given that it was in the safety state z at the moment $t = 0$.

The critical safety state r means that the critical infrastructure and its assets staying in the safety states less than this safety state is highly dangerous for them and for their operating environment. The above assumptions mean that the safety states of the critical infrastructure with degrading assets may be changed in time only from better to worse. We denote by $T(u)$, $u = 1,2,...,z$, the critical infrastructure lifetime in the safety state subset $\{u, u+1,\ldots,z\}$, $u = 1,2,...,z$, and we define the critical infrastructure safety function by the vector (Kołowrocki et al., 2018; Kołowrocki, 2019/2020)

$$\boldsymbol{S}(t,\cdot) = [\boldsymbol{S}(t,1)\ \boldsymbol{S}(t,2)...,\ \boldsymbol{S}(t,z)],\ t \geq 0, \qquad (1)$$

where

$\boldsymbol{S}(t,u) = P(s(t) \geq u \mid s(0) = z) = P(T(u) > t),$

$$t \geq 0, u = 1,2,\ldots,z, \tag{2}$$

is the probability that the critical infrastructure is in the safety state subset $\{u, u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, at the moment t, $t \geq 0$, while it was in the safety state z at the moment $t = 0$.

The safety functions $\boldsymbol{S}(t,u)$, $t \geq 0$, $u = 1,2,\ldots, z$, defined by (2) are called the coordinates of the critical infrastructure safety function $\boldsymbol{S}(t,\cdot)$, $t \geq 0$, given by (1). Thus, the relationship between the distribution function $\boldsymbol{F}(t,u)$, $t \geq 0$, $u = 1,2,\ldots,z$, of the critical infrastructure lifetime $T(u)$, $u = 1,2,\ldots,z$, in the safety state subset $\{u, u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, and the coordinate $\boldsymbol{S}(t,u)$, $t \geq 0$, $u = 1,2,\ldots, z$, of its safety function (1) is given by

$$\boldsymbol{F}(t,u) = P(T(u) \leq t) = 1 - P(T(u) > t)$$

$$= 1 - \boldsymbol{S}(t,u), t \geq 0, u = 1, 2,\ldots,z.$$

If r is the critical safety state, then the critical infrastructure risk function

$$\boldsymbol{r}(t) = P(s(t) < r \mid s(0) = z)$$

$$= P(T(r) \leq t), t \geq 0, \tag{3}$$

is defined as a probability that the critical infrastructure is in the subset of safety states worse than the critical safety state r, $r \in \{1,\ldots,z\}$ while it was in the best safety state z at the moment $t = 0$ and given by (Szymkowiak, 2019; Torbicki, 2019a)

$$\boldsymbol{r}(t) = 1 - \boldsymbol{S}(t,r), t \geq 0, \tag{4}$$

where $\boldsymbol{S}(t,r)$, $t \in \langle 0,\infty)$, is the coordinate of the critical infrastructure unconditional safety function given by (2) for $u = r$.

Similarly, we define the asset A_i, $i = 1,2,\ldots,n$ safety function by the vector (Kołowrocki, 2019/2020, Kołowrocki et al., 2018)

$$S_i(t,\cdot) = [S_i(t,1), S_i(t,2),\ldots, S_i(t,z)], t \geq 0, \tag{5}$$

$i = 1,2,\ldots,n,$

where

$S_i(t,u) = P(s_i(t) \geq u \mid s_i(0) = z) = P(T_i(u) > t),$

$$t \geq 0, u = 1,2,\ldots,z, i = 1,2,\ldots,n, \tag{6}$$

is the probability that the asset A_i is in the safety state subset $\{u, u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, at the moment t, $t \geq 0$, while it was in the safety state z at the moment $t = 0$.

The safety functions $S_i(t,u)$, $t \geq 0$, $u = 1,\ldots,z$, $i = 1,2,\ldots,n$, defined by (6) are called the coordinates of the asset A_i, $i = 1,2,\ldots,n$, safety function $S_i(t,\cdot)$, $t \geq 0$, $i = 1,2,\ldots,n$, given by (5). Thus, the relationship between the distribution function $F_i(t,u)$, $t \geq 0$, $u = 1,2,\ldots,z$, $i = 1,2,\ldots,n$, of the asset A_i, $i = 1,2,\ldots,n$, lifetime $T_i(u)$, $u = 1,2,\ldots,z$, $i = 1,2,\ldots,n$, in the safety state subset $\{u, u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, and the coordinate $S_i(t,u)$, $t \geq 0$, $u = 1,2,\ldots,z$, $i = 1,2,\ldots,n$, of its safety function is given by

$$F_i(t,u) = P(T_i(u) \leq t) = 1 - P(T_i(u) > t)$$

$$= 1 - S_i(t,u),$$

$$t \geq 0, u = 1,2,\ldots,z, i = 1,2,\ldots,n. \tag{7}$$

2.2. General scheme of operation and climate-weather influence on critical infrastructure safety modelling

Most real complex technical systems are strongly influenced by their, changing in time, operation conditions and the climate-weather conditions at their operating areas. The time dependent interactions between the operation process, related to varying states of the climate-weather change process at the system operating area, and the system safety structure and its changing components/assets safety states, are evident features of most real technical systems including critical infrastructures (Kołowrocki, 2020a, 2020b, 2021; Kołowrocki & Kuligowska, 2018; Torbicki, 2019c). The critical infrastructure is defined as a complex system in its operating environment that significant features are inside-system dependencies (Kołowrocki, 2021) and outside-system impacts (Dąbrowska, 2020; Kołowrocki, 2020; Kołowrocki & Kuligowska, 2018; Torbicki, 2019b). In case of system degradation these dependencies have a significant destructive influence on the health, safety, security, economics and social conditions of large human communities and territories (Li & Pham, 2005).

The critical infrastructure may be affected by outside processes which have negative influence on its safety. By outside processes we understand critical infrastructure operation process and the process of weather change in the area of operation. Therefore critical infrastructure safety analysis related to its operation process and the climate-weather change process at its operating area has a great value in the industrial practice due to often negative impacts of these processes on the critical infrastructure safety and resilience (Kołowrocki, 2019/2020; Kołowrocki & Kuligowska, 2018; Torbicki, 2019a, 2019b, 2019c).

As a rule the safety analysis of the critical infrastructure impacted by those processes is very complex. This problem can be solved by the multistate critical infrastructures safety modelling performed similarly to reliability modelling of multistate complex systems (Torbicki, 2019b; Wang et al., 2011; Xue, 1985) commonly used with the semi-Markov modelling (Ferreira & Pacheco, 2007; Glynn & Haas, 2006; Grabski, 2014; Klabjan & Adelman, 2006; Kołowrocki, 2014, 2019/2020; Lisnianski et al., 2010; Torbicki, 2019a) of the joint operation process and climate-weather change process (Kołowrocki, 2019/2020; Kołowrocki & Kuligowska, 2018; Torbicki, 2019a, 2019b, 2019c). This approach leads to the construction of the joint general safety model of the critical infrastructure impacted by the operation process and changing weather at its operating area, considered partly in (Torbicki, 2019a, 2019b, 2019c), originally and significantly developed earlier in the report (Kołowrocki et al., 2018) and later in (Kołowrocki, 2019/2020).

This original and innovative general approach to common influence of operation process and climate-weather change process on critical infrastructure safety and resilience modelling and analysis consists in combining of the critical infrastructure operation process model and the climate-weather change process model and constructing one general joint safety model of the critical infrastructure impacted by its operation process and climate-weather process at its operating area. This approach can be basis for the formulation and development of the new solutions, which consists of the improvement and optimization of the safety of the critical infrastructure related to their operation processes and outside climate-weather change processes, the mitigation the consequences of accidents caused by their degradation. A novel concept, which would induce further complexity to the approach, is an introduction of impacts of climate pressures in the interconnection between critical infrastructures. New and innovative part of this approach lies in inclusion of linkages and dependencies, both internal and external, to critical infrastructures that are impacted by climate-weather hazards.

The content in the approaches of the above scheme is concerned in:

- *Critical Infrastructure Operation Process (CIOP) and Climate-Weather Change Process (C-WCOP)*, with modelling of critical infrastructure operation process and climate-weather change process in its operation area through defining parameters of these processes and giving the ways of their characteristics determination,
- *Integration of CIOP and C-WCP*, with creating of a joint model of critical infrastructure operation process and climate-weather change process at its operating area through defining the critical infrastructure operation process impacted by climate-weather change at its operating area, defining its parameters and giving the procedures of its characteristics determinations,
- *Critical Infrastructure (free of outside impact) Safety (CIS) Modelling*, with constructing of critical infrastructure multistate safety model independent of outside operation and climate-weather impacts,
- *Critical Infrastructure Safety Indicators (SafIs)*, with introducing of practically useful critical infrastructure safety characteristics, called safety indicators,
- *Integration of CIOP, C-WCP and CIOP&C-WCP Models 1–3 with CIS Model 0*, with constructing critical infrastructure safety models, separately and jointly dependent on outside operation and climate-weather impacts,
- *Critical Infrastructure Safety and Resilience Indicators (SafIs, ResIs)*, with introducing of practically useful safety characteristics of critical infrastructure impacted separately and jointly by outside operation and climate-weather conditions, called safety indicators and resilience indicators,

- *Inventory of Critical Infrastructure Safety and Resilience Indicators*, with creating of detailed list of safety indicators of critical infrastructure free of outside impacts and safety indicators and resilience indicators of critical infrastructure outside impacts,
- *Models Application and Validation*, with all proposed models and creating safety and resilience indicators application and validation in practice to port oil terminal critical infrastructure safety examination,
- *Suggestion for Further Research on Critical Infrastructure Safety Examination*, with possible future research and development of proposed models in creating practically important tools for critical infrastructure safety strengthening and optimization, examination and mitigation of critical infrastructure accident consequences and critical infrastructure business continuity analysis.

Thus, starting from the simplest, pure safety *CIS Model 0*, defined as a multistate ageing system without considering outside impacts, several functions and indicators are defined. Namely, the critical infrastructure and its assets' safety functions, the critical infrastructure mean values and variances of lifetimes in the safety state subsets and in the particular safety states, the critical infrastructure risk function, its fragility curve, the moment of exceeding by the critical infrastructure the critical safety state and its intensities of ageing/degrading, are introduced.

Next, the *CIS Model 0* is combined with the critical infrastructure operation process *CIOP Model 1* to create the integrated *CIS Model 1*, which is intended to safety modelling and prediction of critical infrastructure impacted by its operation process. In *CIS Model 1* we define the critical infrastructure as a complex system in its operating environment that significant features are its operation impacts (Dąbrowska, 2020; Kołowrocki, 2014; Kołowrocki & Magryta-Mut, 2020). That safety model of a critical infrastructure related to its operation process links its multistate safety model and its operation process model, to create the critical infrastructure operation impact safety model. Moreover, *CIS Model 1* considers also variable safety structure and its components' safety parameters at different operation states (Dąbrowska, 2020; Kołowrocki, 2014; Kołowrocki & Magryta-Mut, 2020). In this model, we introduce additional safety indicators, which are typical for the critical infrastructure and are related to its varying in time safety structures and its components' safety parameters. Namely, *CIS Model 1* extends the set of safety indicators of *CIS Model 0* by the components and critical infrastructure conditional intensities of ageing at particular operation states and conditional and unconditional coefficients of the operation process impact on the critical infrastructure intensities of ageing and the critical infrastructure coefficient of resilience to its operation process.

Further, an integrated safety *CIS Model 2* of critical infrastructure safety is proposed. This critical infrastructure safety model is related to influence of the climate-weather change process in the critical infrastructure operating area on its safety. It is the integrated model of critical infrastructure safety, linking its multistate safety *CIS Model 0* and the *CIOP Model 1* of the climate-weather change process at its operating area, to create the critical infrastructure climate-weather impact safety model. The *CIS Model 2* considers variable system components safety parameters impacted by different climate-weather states. The conditional safety functions at the particular climate-weather states, the unconditional safety function and the risk function of the critical infrastructure at changing in time climate-weather conditions are defined. Other, practically significant, critical infrastructure safety indicators introduced in the *CIS Model 2* are, its mean lifetime to the moment of exceeding a critical safety state, the moment when its risk function value exceeds the acceptable safety level, the intensities of ageing of the critical infrastructure related to the climate-weather change process at its operating area and coefficients of the impact of the climate-weather change process on the critical infrastructure and its components intensities of ageing and the critical infrastructure coefficient of resilience to climate-weather change process at its operating area.

Finally, the general critical infrastructure safety *CIS Model 3* is proposed that simultaneously considers the operation process and the climate-weather change process influence on the safety of a critical infrastructure (Brunelle & Kapur, 1999; Kołowrocki, 2019/2020; Torbicki, 2019a, 2019b, 2019c). It is a safety model of a critical infrastructure under the influence of the operation process related to climate-weather change at its operating area. It is an integrated model of a critical infrastructure safety, linking its multistate safety *CIS*

Model 0 and the joint *CIOP&C-WCP Model 3* of its operation process related to climate-weather change process at its operating area, to create the critical infrastructure joint operation and climate-weather impact safety model. Thus, *CIS Model 3* considers variable system safety structures and its components safety parameters, impacted by climate-weather states, at different operation states. The conditional safety functions at the operation and climate-weather states of the operation process related the climate-weather change, the unconditional safety function and the risk function of a critical infrastructure at changing in time operation and climate-weather conditions are defined. Other useful critical infrastructure safety indicators introduced in *CIS Model 3* are, its mean lifetime up to the moment of exceeding a critical safety state and the moment when its risk function value exceeds the acceptable safety level, the intensities of ageing of the critical infrastructure and its components impacted by the operation process related to the climate-weather change process, coefficients of the operation process related to climate-weather change impact on the critical infrastructure and its components intensities of ageing and the critical infrastructure coefficient of resilience to operation process related to climate-weather change process at its operating area.

These all, above mentioned, safety indicators, proposed in *CIS Models 0–3*, are defined in general for any critical infrastructures with varying in time their safety structures and components/assets safety parameters, which are influenced by, changing in time, operation and climate-weather conditions at their operating areas.

The next step that can be done to perform the tasks formulated in scheme items is these models application and validation, what is practically realized in (Kołowrocki et al., 2018) through the port oil terminal critical infrastructure examination.

The path we should follow in our future research activity is to investigate and solve problems of safety and resilience strengthening of critical infrastructure impacted by operation and climate-weather change. This activity will lead to establishing of elaborate models of business continuity for critical infrastructure under operation and climate pressures, as well as to solving the critical infrastructure safety optimization (Kołowrocki & Magryta, 2020a; Magryta-Mut, 2020) and its degradation and accident consequences identification and mitigation (Bogalecka, 2020).

All presented models are the basis for preparation of procedures, which are very easy to use by the practitioners and operators of the critical infrastructures in their operation and safety analysis. The use of these procedures for real critical infrastructure is presented in details in all sections of this chapter. All created models, and procedures based on them, can be modified and developed for other problems of safety features of critical infrastructure analysis. In this context, modelling and prediction of critical infrastructure safety presented in this paper developed by considering inside dependences between the critical infrastructure assets (Kołowrocki, 2020a, 2021) would be very important broadening to real practice in critical infrastructure safety examination to build the model considering commonly the critical infrastructure ageing its inside dependences and outside impacts as an innovative general approach significant and breakthrough applications this new theoretical results.

2.3. Critical infrastructure safety and resilience indicators

In the first step of the approach proposed in Section 3 we start with the simplest pure safety model *CIS Model 0*, without considering outside impacts. For the critical infrastructure (and its assets) following useful safety indicators are defined:

- the critical infrastructure safety function (SafI1),
- the critical infrastructure risk function (SafI2),
- the critical infrastructure fragility curve (SafI3),
- the mean value of the critical infrastructure lifetime up to the exceeding the critical safety state (SafI4),
- the standard deviation of the critical infrastructure lifetime up to the exceeding the critical safety state (SafI5),
- the moment of exceeding acceptable value of critical infrastructure risk function level (SafI6),
- the mean values of the critical infrastructure lifetimes in the safety state subsets (SafI7),
- the standard deviations of the critical infrastructure lifetimes in the safety state subsets (SafI8),
- the mean value of the critical infrastructure lifetimes in particular safety states (SafI9),

- the intensities of degradation (ageing) of the critical infrastructure / the intensities of critical infrastructure departure from the safety state subsets (SafI10).

In Section 4, in the second step of the proposed approach, the simplest safety model *CIS Model 0* is combined with the critical infrastructure operation process model *CIOP Model 1*, in order to create a safety model *CIS Model 1* of critical infrastructure related to its operating environment. Next, in Section 5, an impact model on critical infrastructure safety *CIS Model 2* related to the climate-weather change process in its operating area is proposed. The most general safety impact model *CIS Model 3* which consider jointly the operation process and climate-weather change process influence on the safety of a critical infrastructure is presented in Section 6. It is the integrated model of a critical infrastructure safety, linking its multistate safety model *CIS Model 0* and the joint model *CIOP&C-WCP* of its operation process and the climate-weather change process in its operating area. These model consider variable safety structures of the critical infrastructure at different operation and climate-weather states, as well as safety parameters of critical infrastructure assets. For those models, the following safety indicators are respectively defined:

- the critical infrastructure safety function (SafI1),
- the critical infrastructure risk function (SafI2),
- the critical infrastructure fragility curve (SafI3),
- the mean value of the critical infrastructure lifetime up to the exceeding the critical safety state (SafI4),
- the standard deviation of the critical infrastructure lifetime up to the exceeding the critical safety state (SafI5),
- the moment of exceeding acceptable value of critical infrastructure risk function level (SafI6),
- the mean values of the critical infrastructure lifetimes in the safety state subsets (SafI7),
- the standard deviations of the critical infrastructure lifetimes in the safety state subsets (SafI8),
- the mean value of the critical infrastructure lifetimes in particular safety states (SafI9),
- the intensities of degradation (ageing) of the critical infrastructure / the intensities of critical infrastructure departure from the safety state subsets (SafI10).

These all safety indicators are defined, in general, for any critical infrastructures with varying in time their safety structures and their assets safety parameters influenced by changing in time operation conditions and climate-weather conditions at their operating areas.

We can make a next step in order to terminate methodological framework, for critical infrastructures with outside impacts to define the following critical infrastructure resilience indicators:

- the coefficients of operation process impact on the critical infrastructure intensities of degradation / the coefficients of operation process impact on critical infrastructure intensities of departure from the safety state subset (ResI1),
- the indicator of critical infrastructure resilience to operation process impact (ResI2),
- the coefficients of climate-weather change process impact on the critical infrastructure intensities of degradation / the coefficients of climate-weather change process impact on critical infrastructure intensities of departure from the safety state subset (ResI1),
- the indicator of critical infrastructure resilience to climate-weather change process impact (ResI2),
- the coefficients of operation process and climate-weather change process impact on the critical infrastructure intensities of degradation / the coefficients of operation process and climate-weather change process impact on critical infrastructure intensities of departure from the safety state subset (ResI1),
- the indicator of critical infrastructure resilience to operation process and climate-weather change process impact (ResI2).

All the proposed indicators and other safety and resilience tools can be validated through their practical application to the real critical infrastructures.

Further research activities could concentrate on investigating and solving of optimization problems for critical infrastructure safety. These research should include finding of optimal values of safety and resilience indictors, as well as analysis of resilience and strengthening of critical infrastructure to climate-weather change. This activity will result in elaboration of business continuity models for critical infrastructure under the operation and climate-weather pressures, cost-effectiveness analysis and modelling and critical infrastructure degradation and accident consequences analysis and mitigation.

3. Modelling safety of critical infrastructure without outside impacts

3.1. Safety indicators of critical infrastructure without outside impacts

We denote the critical infrastructure free of outside impacts lifetime in the safety state subset $\{u, u+1, \ldots, z\}$, $u = 1,2,\ldots,z$, by $T^0(u)$ and define the first safety indicator, the critical infrastructure safety function (SafI1) by the vector (Kołowrocki et al., 2018; Kołowrocki, 2019/2020)

$$\boldsymbol{S}^0(t,\cdot) = [1, \boldsymbol{S}^0(t,1)\ldots, \boldsymbol{S}^0(t,z)],\ t \geq 0, \tag{8}$$

with the coordinates

$$\boldsymbol{S}^0(t,u) = P(s(t) \geq u \mid s(0) = z) = P(T^0(u) > t)$$

$$\text{for } t \geq 0,\ u = 1,2,\ldots,z, \tag{9}$$

defined as the probability that the critical infrastructure is in the safety state subset $\{u, u+1, \ldots, z\}$, $u = 1,2,\ldots,z$, at the moment t, $t \geq 0$, while it was in the safety state z at the moment $t = 0$.

Moreover, if r is the critical safety state, then the second safety indicator, the critical infrastructure risk function (SafI2)

$$\boldsymbol{r}0(t) = P(s(t) < r \mid s(0) = z)$$

$$= P(T0(r) \leq t),\ t \geq 0, \tag{10}$$

is defined as a probability that the critical infrastructure is in the subset of safety states worse than the critical safety state r, $r \in \{1,\ldots,z\}$ while it was in the best safety state z at the moment $t = 0$ and given by (Kołowrocki et al., 2018)

$$\boldsymbol{r}0(t) = 1 - \boldsymbol{S}^0(t,r),\ t \geq 0, \tag{11}$$

where $\boldsymbol{S}^0(t,r)$ is the coordinate of the critical infrastructure safety function given by (9) for $u = r$.

The graph of the critical infrastructure risk function is the third safety indicator called the critical infrastructure fragility curve (Gouldby, 2010) (SafI3).

The critical infrastructure safety function (SafI1), the critical infrastructure risk function (SafI2) and the critical infrastructure fragility curve (SafI3) are proposed as main basic critical infrastructure safety indicators.

Other practically useful critical infrastructure safety indicators are:

- the mean value of the critical infrastructure lifetime $T^0(r)$ up to exceeding critical safety state r (SafI4) given by

$$\boldsymbol{\mu}^0(r) = \int_0^\infty \boldsymbol{S}^0(t,r)dt, \tag{12}$$

  where $\boldsymbol{S}^0(t,r)$, $t \geq 0$, is defined by (9) for $u = r$,

- the standard deviation of the critical infrastructure lifetime $T^0(r)$ up to the exceeding the critical safety state r (SafI5) given by

$$\boldsymbol{\sigma}^0(r) = \sqrt{\boldsymbol{n}^0(r) - [\boldsymbol{\mu}^0(r)]^2}, \tag{13}$$

  where

$$\boldsymbol{n}^0(r) = 2\int_0^\infty t \cdot \boldsymbol{S}^0(t,r)dt, \tag{14}$$

  and $\boldsymbol{S}^0$, $t \geq 0$, is given by (9) for $u = r$ and $\boldsymbol{\mu}^0(r)$ is given by (12),

- the moment t of exceeding acceptable value of critical infrastructure risk function level d (SafI6) given by

$$\boldsymbol{\tau}^0 = (\boldsymbol{r}^0)^{-1}(\delta), \tag{15}$$

  where $(\boldsymbol{r}^0)^{-1}(t)$, $t \geq 0$, is the inverse function of the risk function $\boldsymbol{r}^0(t)$ given by (11),

- the mean values of the critical infrastructure lifetimes in the safety state subsets $\{u, u+1, \ldots, z\}$, $u = 1,2,\ldots,z$, (SafI7) given by

$$\boldsymbol{\mu}^0(u) = \int_0^\infty \boldsymbol{S}^0(t,u)dt,\ u = 1,2,\ldots,z, \tag{16}$$

  $i = 1,2,\ldots,n$,

- the standard deviations of the critical infrastructure lifetimes in the safety state subsets $\{u, u+1, \ldots, z\}$, $u = 1,2,\ldots,z$, (SafI8) given by

$\boldsymbol{\sigma}^0(u) = \sqrt{\boldsymbol{n}^0(u) - [\boldsymbol{\mu}^0(u)]^2},$

$u = 1,2,...,z,$ (17)

where

$\boldsymbol{n}^0(r) = 2\int_0^\infty t \cdot \boldsymbol{S}^0(t,r)dt, u = 1,2,...,z,$

$i = 1,2,...,n,$ (18)

- the mean lifetimes $\bar{\boldsymbol{\mu}}^0(u)$, $u = 1,2,...,z$, of the critical infrastructure in the particular safety states (SafI9)

  $\bar{\boldsymbol{\mu}}^0(u) = \boldsymbol{\mu}^0(u) - \boldsymbol{\mu}^0(u+1),$

  $u = 0,1,\ldots,z-1, \bar{\boldsymbol{\mu}}^0(z) = \boldsymbol{\mu}^0(z),$ (19)

- the intensities of degradation (ageing) of the critical infrastructure / the intensities of critical infrastructure departure from the safety state subset $\{u, u+1, \ldots, z\}$, $u = 1,2,...,z$, (SafI10), i.e. the coordinates of the vector

  $\boldsymbol{\lambda}^0(t,\cdot) = [\boldsymbol{\lambda}^0(t,1), \ldots, \boldsymbol{\lambda}^0(t,z)], t \geq 0,$ (20)

  where

  $\boldsymbol{\lambda}^0(t,u) = -\frac{d\boldsymbol{S}^0(t,u)}{dt} \cdot \frac{1}{\boldsymbol{S}^0(t,u)}, t \geq 0,$

  $u = 1,2,\ldots,z,$ (21)

In the particular case, when the critical infrastructure has the piecewise exponential safety function (SafI1), i.e.

$\boldsymbol{S}^0(t,\cdot) = [\boldsymbol{S}^0(t,1),\ldots, \boldsymbol{S}^0(t,z)], t \geq 0,$ (22)

where

$\boldsymbol{S}^0(t,u) = \exp[-\boldsymbol{\lambda}^0(u)t], t \geq 0,$

$\boldsymbol{\lambda}^0(u) \geq 0, u = 1,2,\ldots,z,$ (23)

the intensities of degradation of the critical infrastructure / the intensities of critical infrastructure departure from the safety state subset $\{u, u+1, \ldots, z\}$, $u = 1,2,...,z$, (SafI7), i.e. the coordinates of the vector

$\boldsymbol{\lambda}^0(\cdot) = [\boldsymbol{\lambda}^0(1),\ldots, \boldsymbol{\lambda}^0(z)],$ (24)

are constant and

$\boldsymbol{\lambda}^0(u) = \frac{1}{\boldsymbol{\mu}^0(u)}, u = 1,2,\ldots,z,$ (25)

where $\boldsymbol{\mu}^0(u)$ is the mean value of the critical infrastructure lifetime $T^0(u)$ in the safety state subset $\{u, u+1, \ldots, z\}$, $u = 1,2,\ldots,z$, defined by

$\boldsymbol{\mu}^0(u) = \int_0^\infty \boldsymbol{S}^0(t,u)dt,$ (26)

and $\boldsymbol{S}^0(t,u)$, $t \geq 0$, is defined by (9) for $u = 1,2,\ldots,z$, and given in this case by (23).
The assets safety parameters of the critical infrastructure free of outside impacts can by introduced in an analogous way (Kołowrocki, 2019/2020; Kołowrocki et al., 2018).

4. Modelling safety of critical infrastructure impacted by operation process

4.1. Critical infrastructure operation process

We denote by $Z(t)$, $t \geq 0$, the critical infrastructure operation process, and we assume that it is impacted in a various way at this process operation states z_b, $b = 1,2,\ldots,v$. We assume that the changes of the operation states of the critical infrastructure operation process $Z(t)$ have an influence on the critical infrastructure safety structure and on the safety of the critical infrastructure assets A_i, $i = 1,2,\ldots,n$, (Kołowrocki et al., 2018).
The critical infrastructure operation process is defined by the following parameters (OPP) that can be identified either statistically using the methods given in (Kołowrocki et al., 2018) or evaluated approximately by experts:

- the number of operation states (OPP1)

  v,

- the operation states (OPP2)

  $z_1, z_2, \ldots, z_v,$

- the vector

  $[p_b(0)]_{1xv} = [p_1(0), p_2(0),\ldots, p_v(0)],$ (27)

  of the initial probabilities (OPP3)

$p_b(0) = P(Z(0) = z_b), b = 1,2,\dots,\nu,$

of the critical infrastructure operation process $Z(t)$ staying at particular operation states z_b at the moment $t = 0$,

- the matrix

$$[p_{bl}]_{\nu\times\nu} = \begin{bmatrix} p_{11} & p_{12} & \dots & p_{1\nu} \\ p_{21} & p_{22} & \dots & p_{2\nu} \\ \dots \\ p_{\nu 1} & p_{\nu 2} & \dots & p_{\nu\nu} \end{bmatrix} \quad (28)$$

of probabilities of transition (OPP4)

$p_{bl}, b, l = 1,2,\dots,\nu, p_{bb} = 0, b = 1,2,\dots,\nu,$

of the critical infrastructure operation process $Z(t)$ between the operation states z_b and z_l,

- the matrix

$$[M_{bl}]_{\nu\times\nu} = \begin{bmatrix} M_{11} & M_{12} & \dots & M_{1\nu} \\ M_{21} & M_{22} & \dots & M_{2\nu} \\ \dots \\ M_{\nu 1} & M_{\nu 2} & \dots & M_{\nu\nu} \end{bmatrix}, \quad (29)$$

of the mean values of conditional sojourn times (OPP5)

$$M_{bl} = E[\theta_{bl}]$$

$$= \int_0^\infty t dH_{bl}(t) = \int_0^\infty t h_{bl}(t) dt,$$

$b, l = 1,2,\dots,\nu, b \neq l,$

$$M_{bb} = 0, b = 1,2,\dots,\nu, \quad (30)$$

of the critical infrastructure operation process $Z(t)$ conditional sojourn times θ_{bl} at the operation state z_b when the next state is z_l, where

$H_{bl}(t) = P(\theta_{bl} < t), t \geq 0,$

$b, l = 1,2,\dots,\nu, b \neq l,$

are conditional distribution functions of the critical infrastructure operation process $Z(t)$ conditional sojourn times θ_{bl} at the operation states corresponding to conditional density functions $b \neq l$

$h_{bl}(t) = \frac{dH_{bl}(t)}{dt}, t \geq 0,$

$b, l = 1,2,\dots,\nu, b \neq l.$

The following critical infrastructure operation process characteristics (OPC) can be either calculated analytically using the above parameters of the operation process or evaluated approximately by experts (Kołowrocki et al., 2018):

- the vector

$$[M_b]_{1\times\nu} = [M_1, M_2, \dots, M_\nu], \quad (31)$$

of mean values of the critical infrastructure operation process $Z(t)$ unconditional sojourn times θ_b, $b = 1,2,\dots,\nu$, at the operation states (OPC1)

$$M_b = E[\theta_b] = \sum_{l=1}^{\nu} p_{bl} M_{bl}, \ b = 1,2,\dots,\nu, \quad (32)$$

where M_{bl} are defined by the formula (30),

- the vector

$$[p_b]_{1\times\nu} = [p_1, p_2, \dots, p_\nu], \quad (33)$$

of limit values of transient probabilities (OPC2)

$p_b(t) = P(Z(t) = z_b), t \geq 0, b = 1,2,\dots,\nu,$

of the critical infrastructure operation process $Z(t)$ at the particular operation states z_b, $b = 1,2,\dots,\nu$, given by

$$p_b = \lim_{t\to\infty} p_b(t) = \frac{\pi_b M_b}{\sum_{l=1}^{\nu} \pi_l M_l},$$

$$b = 1,2,\dots,\nu, \quad (34)$$

where $M_b, b = 1,2,\dots,\nu$, are given by (32) and the steady probabilities π_b of the vector $[\pi_b]_{1\times\nu}$ satisfy the system of equations (Kołowrocki, 2019/2020; Kołowrocki et al., 2018)

$$\begin{cases} [\pi_b] = [\pi_b][p_{bl}] \\ \sum_{l=1}^{\nu} \pi_l = 1, \end{cases} \quad (35)$$

- the vector

$$[\widehat{M}_b]_{1\times\nu} = [\widehat{M}_1, \widehat{M}_2, \dots, \widehat{M}_\nu], \quad (36)$$

of the mean values of the total sojourn times $\hat{\theta}_b$ (OPC3)

$$\hat{M}_b = E[\hat{\theta}_b] \cong p_b\theta, b = 1,2,\ldots,v, \quad (37)$$

of the critical infrastructure operation process *Z*(*t*) at the particular operation states z_b, b = 1,2,…,v, during the fixed critical infrastructure opetation time θ, where p_b are given by (34).

4.2. Safety and resilience indicators of critical infrastructure impacted by its operation process

We denote by $[T^1(u)]^{(b)}$, u = 1,2,…,z, b = 1,2,…,v, the critical infrastructure conditional lifetime in the safety state subset $\{u, u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, while its operation process *Z*(*t*), $t \geq 0$, is at the operation state z_b, $b = 1,2,\ldots,v$, and the conditional safety function of the critical infrastructure at this operation state by the vector (Kołowrocki et al., 2018; Kołowrocki, 2019/2020)

$$[\boldsymbol{S}^1(t,\cdot)]^{(b)} = [[\boldsymbol{S}^1(t,1)]^{(b)},\ldots, [\boldsymbol{S}^1(t,z)]^{(b)}], \quad (38)$$

with the coordinates

$$[\boldsymbol{S}^1(t,u)]^{(b)} = P([T^1(u)]^{(b)} > t|Z(t) = z_b)$$

for $t \geq 0, u = 1,2,\ldots,z, b = 1,2,\ldots,v.$ (39)

The safety function $[\boldsymbol{S}^1(t,u)]^{(b)}$, $t \geq 0$, $u = 1,2,\ldots,z$, is the conditional probability that the critical infrastructure impacted by its operation process *Z*(*t*), $t \geq 0$, lifetime $[T^1(u)]^{(b)}$, $u = 1,2,\ldots,z$, in the safety state subset $\{u, u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, is greater than *t*, while the critical infrastructure operation process *Z*(*t*), $t \geq 0$, is at the operation state z_b.

Next, we denote by $T^1(u), u = 1,2,\ldots,z$, the critical infrastructure impacted by its operation process *Z*(*t*), $t \in \langle 0,\infty)$, unconditional lifetime in the safety state subset $\{u, u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, and the firs safety indicator, the unconditional safety function (SafI1) of the critical infrastructure impacted by its operation process *Z*(*t*), $t \geq 0$, by the vector

$$\boldsymbol{S}^1(t,\cdot) = [\boldsymbol{S}^1(t,1),\ldots, \boldsymbol{S}^1(t,z)], \quad (40)$$

with the coordinates

$$\boldsymbol{S}^1(t,u) = P(T^1(u) > t) \quad (41)$$

for $t \geq 0, u = 1,2,\ldots,z.$

In the case when the system operation time θ is large enough, the coordinates of the unconditional safety function of the critical infrastructure related to the operation process *Z*(*t*), $t \geq 0$, defined by (41), are given by

$$\boldsymbol{S}^1(t,u) \cong \sum_{b=1}^{v} p_b [\boldsymbol{S}^1(t,u)]^{(b)}$$

for $t \geq 0, u = 1,2,\ldots,z,$ (42)

where $[\boldsymbol{S}^1(t,u)]^{(b)}, u = 1,2,\ldots,z, b = 1,2,\ldots,v$, are the coordinates of the critical infrastructure impacted by its operation process *Z*(*t*), $t \geq 0$, conditional safety functions defined by (38)–(39) and p_b, $b = 1,2,\ldots,v$, are the critical infrastructure operation process *Z*(*t*), $t \in \langle 0,\infty)$, limit transient probabilities at the operation states z_b, $b = 1,2,\ldots,v$, given by (34).

If r is the critical safety state, then the second safety indicator of the critical infrastructure impacted by its operation process *Z*(*t*), $t \in \langle 0,\infty)$, the risk function (SafI2)

$$\boldsymbol{r}1(t) = P(s(t) < r \mid s(0) = z)$$

$$= P(T1(r) \leq t), t \geq 0, \quad (43)$$

is defined as a probability that the critical infrastructure impacted by its operation process *Z*(*t*), $t \geq 0$, is in the subset of safety states worse than the critical safety state *r*, $r \in \{1,\ldots,z\}$ while it was in the best safety state *z* at the moment $t = 0$ and given by (Kołowrocki et al., 2018)

$$\boldsymbol{r}1(t) = 1 - \boldsymbol{S}^1(t,r), t \geq 0, \quad (44)$$

where $\boldsymbol{S}^1(t,r)$ is the coordinate of the critical infrastructure impacted by its operation process *Z*(*t*), $t \geq 0$, unconditional safety function given by (42) for $u = r$.

The graph of the critical infrastructure risk function $\boldsymbol{r}^1(t)$, $t \geq 0$, defined by (44), is the safety indicator called the fragility curve (SafI3) of the critical infrastructure impacted by its operation process *Z*(*t*), $t \geq 0$.

Other practically useful safety and resilience indicators of the critical infrastructure impacted by its operation process $Z(t)$, $t \geq 0$, are:

- the mean value of the critical infrastructure unconditional lifetime $T^1(r)$ up to exceeding critical safety state r (SafI4) given by

$$\boldsymbol{\mu}^1(r) = \int_0^\infty [\boldsymbol{S}^1(t,r)]dt \cong \sum_{b=1}^{v} p_b [\boldsymbol{\mu}^1(r)]^{(b)}, \quad (45)$$

  where $[\boldsymbol{\mu}^1(r)]^{(b)}$ are the mean values of the critical infrastructure conditional lifetimes $[T^1(r)]^{(b)}$ in the safety state subset $\{r, r+1, \ldots, z\}$ at the operation state z_b, $b = 1,2,\ldots,v$, given by

$$[\boldsymbol{\mu}^1(r)]^{(b)} = \int_0^\infty [\boldsymbol{S}^1(t,r)]^{(b)} dt,$$

$$b = 1,2,\ldots,v, \quad (46)$$

  and $[\boldsymbol{S}^1(t,r)]^{(b)}$, $t \geq 0$, $b = 1,2,\ldots,v$, are defined by (38)–(39) and p_b are given by (34),
- the standard deviation of the critical infrastructure lifetime $T^1(r)$ up to the exceeding the critical safety state r (SafI5) given by

$$\boldsymbol{\sigma}^1(r) = \sqrt{\boldsymbol{n}^1(r) - [\boldsymbol{\mu}^1(r)]^2}, \quad (47)$$

  where

$$\boldsymbol{n}^1(r) = 2\int_0^\infty t \cdot \boldsymbol{S}^1(t,r)dt, \quad (48)$$

  and $\boldsymbol{S}^1(t,r)$, $t \geq 0$, is defined by (39) for $u = r$ and $\boldsymbol{\mu}^1(r)$ is given by (45),
- the moment $\boldsymbol{\tau}^1$ of exceeding acceptable value of critical infrastructure risk function level d (SafI6) given by

$$\boldsymbol{\tau}^1 = (\boldsymbol{r}^1)^{-1}(\delta), \quad (49)$$

  where $(\boldsymbol{r}^1)^{-1}(t)$, $t \geq 0$, is the inverse function of the risk function $\boldsymbol{r}^1(t)$ given by (43),
- the mean values of unconditional lifetimes of the critical infrastructure in the safety state subsets $\{u, u+1, \ldots, z\}$, $u = 1,2,\ldots,z$, (SafI7) given by

$$\boldsymbol{\mu}^1(u) = \int_0^\infty [\boldsymbol{S}^1(t,u)]dt$$

$$\cong \sum_{b=1}^{v} p_b [\boldsymbol{\mu}^1(u)]^{(b)}, \; u = 1,2,\ldots,z, \quad (50)$$

  where $[\boldsymbol{\mu}^1(u)]^{(b)}$ are the mean values of the critical infrastructure conditional lifetimes $[T^1(u)]^{(b)}$ in the safety state subsets $\{u, u+1, \ldots, z\}$ at the operation state z_b, $b = 1,2,\ldots,v$, given by

$$[\boldsymbol{\mu}^1(u)]^{(b)} = \int_0^\infty [\boldsymbol{S}^1(t,u)]^{(b)} dt,$$

$$u = 1,2,\ldots,z, \; b = 1,2,\ldots,v, \quad (51)$$

  and $[\boldsymbol{S}(t,u)]^{(b)}$, $t \geq 0$, $u = 1,2,\ldots,z$, $b = 1,2,\ldots,v$, are defined by (38)–(39) and p_b are given by (34),
- the standard deviations of the critical infrastructure lifetimes in the safety state subsets $\{u, u+1, \ldots, z\}$, $u = 1,2,\ldots,z$, (SafI8) given by

$$\boldsymbol{\sigma}^1(u) = \sqrt{\boldsymbol{n}^1(u) - [\boldsymbol{\mu}^1(u)]^2}, \; u = 1,2,\ldots,z, \quad (52)$$

  where

$$\boldsymbol{n}^1(u) = 2\int_0^\infty t\boldsymbol{S}^1(t,u)dt, \; u = 1,2,\ldots,z, \quad (53)$$

- the mean lifetimes $\bar{\boldsymbol{\mu}}^1(u)$, $u = 1,2,\ldots,z$, of the critical infrastructure in the particular safety states (SafI9)

$$\bar{\boldsymbol{\mu}}^1(u) = \boldsymbol{\mu}^1(u) - \boldsymbol{\mu}^1(u+1),$$

$$u = 0,1,\ldots,z-1,$$

$$\bar{\boldsymbol{\mu}}^1(z) = \boldsymbol{\mu}^1(z), \quad (54)$$

- the intensities of degradation of the critical infrastructure / the intensities of critical infrastructure departure from the safety state subset $\{u, u+1, \ldots, z\}$, $u = 1,2,\ldots,z$, (SafI10), i.e. the coordinates of the vector
-

$$\boldsymbol{\lambda}^1(t,\cdot) = [\boldsymbol{\lambda}^1(t,1), \ldots, \boldsymbol{\lambda}^1(t,z)], \; t \geq 0, \quad (55)$$

  where

$$\boldsymbol{\lambda}^1(t,u) = -\frac{d\boldsymbol{S}^1(t,u)}{dt} \cdot \frac{1}{\boldsymbol{S}^1(t,u)}, \; t \geq 0,$$

$$u = 1,2,\ldots,z, \quad (56)$$

- the coefficients of operation process impact on the critical infrastructure intensities of degradation / the coefficients of operation process impact on critical infrastructure intensities of departure from the safety state subset $\{u, u+1, \ldots, z\}$ (ResI1), i.e. the coordinates of the vector

$$\boldsymbol{\rho}^1(t,\cdot) = [\boldsymbol{\rho}^1(t,1), \ldots, \boldsymbol{\rho}^1(t,z)],\ t \geq 0, \quad (57)$$

where

$$\boldsymbol{\lambda}^1(t,u) = \boldsymbol{\rho}^1(t,u) \cdot \boldsymbol{\lambda}^0(t,u),\ t \geq 0,$$

$$u = 1,2,\ldots,z, \quad (58)$$

i.e.

$$\boldsymbol{\rho}^1(t,u) = \frac{\boldsymbol{\lambda}^1(t,u)}{\boldsymbol{\lambda}^0(t,u)},\ t \geq 0,\ u = 1,2,\ldots,z, \quad (59)$$

and $\boldsymbol{\lambda}^0(t,u)$, $t \geq 0$, $u = 1,2,\ldots,z$, are the intensities of degradation of the critical infrastructure without of operation process impact, defined by (20), i.e. the coordinate of the vector

$$\boldsymbol{\lambda}^0(t,\cdot) = [\boldsymbol{\lambda}^0(t,1), \ldots, \boldsymbol{\lambda}^0(t,z)],\ t \geq 0, \quad (60)$$

and $\boldsymbol{\lambda}^1(t,u)$, $t \geq 0$, $u = 1,2,\ldots,z$, are the intensities of degradation of the critical infrastructure with of operation process impact, defined by (56), i.e. the coordinate of the vector

$$\boldsymbol{\lambda}^1(t,\cdot) = [\boldsymbol{\lambda}^1(t,1), \ldots, \boldsymbol{\lambda}^1(t,z)],\ t \geq 0, \quad (61)$$

- the indicator of critical infrastructure resilience to operation process impact (ResI2) defined by

$$\boldsymbol{RI}^1(t,r) = \frac{1}{\boldsymbol{\rho}^1(t,r)},\ t \geq 0, \quad (62)$$

where $\boldsymbol{\rho}^1(t,r)$, $t \geq 0$, is the coefficients of operation process impact on the critical infrastructure intensities of degradation given by (58)–(59) for $\boldsymbol{u} = \boldsymbol{r}$.

In the case, when the critical infrastructure have the piecewise exponential safety functions, i.e.

$$\boldsymbol{S}^1(t,\cdot) = [\boldsymbol{S}^1(t,1), \ldots, \boldsymbol{S}^1(t,z)],\ t \geq 0, \quad (63)$$

where

$$\boldsymbol{S}^1(t,u) = \exp[-\boldsymbol{\lambda}^1(u)t],\ t \geq 0,$$

$$\boldsymbol{\lambda}^1(u) \geq 0,\ u = 1,2,\ldots,z, \quad (64)$$

the critical infrastructure safety and resilience indicators defined by (55)–(62) take forms:

- the intensities of degradation of the critical infrastructure related to the operation process impact, (SafII10), i.e. the coordinates of the vector

$$\boldsymbol{\lambda}^1(\cdot) = [\boldsymbol{\lambda}^1(1), \ldots, \boldsymbol{\lambda}^1(z)], \quad (65)$$

are constant and

$$\boldsymbol{\lambda}^1(u) = \frac{1}{\boldsymbol{\mu}^1(u)}, \quad (66)$$

- the coefficients of the operation process impact on the critical infrastructure intensities of degradation / the coefficients of the operation process impact on critical infrastructure intensities of departure from the safety state subset $\{u, u+1, \ldots, z\}$, (ResI1) i.e. the coordinates of the vector

$$\boldsymbol{\rho}^1(\cdot) = [\boldsymbol{\rho}^1(1), \ldots, \boldsymbol{\rho}^1(z)], \quad (67)$$

where

$$\boldsymbol{\rho}^1(u) = \frac{\boldsymbol{\lambda}^1(u)}{\boldsymbol{\lambda}^0(u)} = \frac{\boldsymbol{\mu}^0(u)}{\boldsymbol{\mu}^1(u)},\ u = 1,2,\ldots,z, \quad (68)$$

and $\boldsymbol{\lambda}^0(u)$, $u = 1,2,\ldots,z$, are the intensities of degradation of the critical infrastructure without of operation process impact, given by (25), i.e. the coordinate of the vector

$$\boldsymbol{\lambda}^0(u) = [\boldsymbol{\lambda}^0(1), \ldots, \boldsymbol{\lambda}^0(z)] \quad (69)$$

and $\boldsymbol{\lambda}^1(u)$, $u = 1,2,\ldots,z$, are the intensities of degradation of the critical infrastructure related to the operation impact, given by (66), i.e. the coordinates of the vector

$$\boldsymbol{\lambda}^1(\cdot) = [\boldsymbol{\lambda}^1(1), \ldots, \boldsymbol{\lambda}^1(z)], \quad (70)$$

- the indicator of critical infrastructure resilience to operation process impact (ResI2) defined by

$$\boldsymbol{RI}^1(r) = \frac{1}{\boldsymbol{\rho}^1(r)},\ t \geq 0, \quad (71)$$

where $\boldsymbol{\rho}^1(r)$ is the coefficient of operation process impact on the critical infrastructure intensities of degradation given by (68) for $u = r$.

The assets safety parameters of the critical infrastructure impacted by the operation process can by introduced in an analogous way (Kołowrocki, 2019/2020; Kołowrocki et al., 2018).

5. Modelling critical infrastructure safety impacted by climate-weather change process

5.1. Climate-weather change process at critical infrastructure operating area

5.1.1. States of climate-weather change process

To define the climate-weather states at the fixed area, we distinguish a, $a \in$ N, parameters that define (describe) the climate-weather states in this area and mark the values they can take by $w_1, w_2, ..., w_a$. Further, we assume that the possible values of the i-th parameter w_i, $i = 1,2,\ldots,a$, can belong to the interval $t \in \langle b_i, d_i)$, $i = 1,2,\ldots,a$. We divide each of the intervals $\langle b_i, d_i)$, $i = 1,2,\ldots,a$, into n_i, $n_i \in N$, disjoint subintervals

$$\langle b_{i1}, d_{i1}), \langle b_{i2}, d_{i2}), \ldots, \langle b_{in_i}, d_{in_i}),\ i = 1,2,\ldots,a,$$

such that

$$\langle b_{i1}, d_{i1}) \cup \langle b_{i2}, d_{i2}) \cup \ldots \cup \langle b_{in_i}, d_{in_i}) = \langle b_i, d_i),$$

$$d_{ij_i} = b_{ij_i+1},\ j_i = 1,2,\ldots,n_i - 1,\ i = 1,2,\ldots,a.$$

Thus, the points $(w_1, w_2, ..., w_a)$ describing the values of the climate-weather parameters are the points from the set of the a dimensional space of the Cartesian product

$$\langle b_1, d_1) \times \langle b_2, d_2) \times \ldots \times \langle b_a, d_a)$$

that is composed of the a dimensional space domains of the form

$$\langle b_{1j_1}, d_{1j_1}) \times \langle b_{2j_2}, d_{2j_2}) \times \ldots \times \langle b_{aj_a}, d_{aj_a}),$$

where $j_i = 1,2,\ldots,n_i$, $i = 1,2,\ldots,a$, called the climate-weather states and

$$w_i,\ i = 1,2,\ldots,a,$$

can takes values from one of the interwals

$$\langle b_{i1}, d_{i1}), \langle b_{i2}, d_{i2}), \ldots, \langle b_{in_i}, d_{in_i}),\ i = 1,2,\ldots,a.$$

The domains of the above form called the climate-weather states of the climate-weather change process are numerated from 1 up to the value $w = n_1 \cdot n_2 \cdot \ldots \cdot n_a$ that is the number of all possible climate-weather states and marked by $c_1, c_2, \ldots, c_w$.

The interpretation of the states of the climate-weather change process in the case they are defined by $a = 2$ parameters is given in Figure 1. In this case, we have $w = n_1 \cdot n_2$ climate-weather states of the climate-weather change process represented in Figure 1 by the squares marked by $c_1, c_2, \ldots, c_w$.

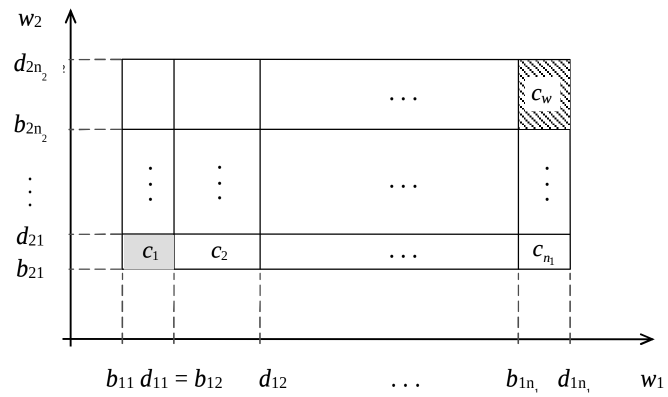

Figure 1. Interpretation of the climate-weather change process two dimensional climate-weather states.

In this particular case, the climate-weather change process can take values (w_1, w_2) from the climate-weather states defined by the domains

$$\langle b_{1j_1}, d_{1j_1}) \times \langle b_{2j_2}, d_{2j_2}),$$

where $j_i = 1,2,\ldots n_i$, $i = 1,2$, in the way such that w_1 can take value from one of the intervals

$$\langle b_{11}, d_{11}), \langle b_{12}, d_{12}), \ldots, \langle b_{1n_1}, d_{1n_1})$$

and w_2 can take value from one of the intervals

$$\langle b_{21}, d_{21}), \langle b_{22}, d_{22}), \ldots, \langle b_{2n_2}, d_{2n_2})$$

and marked by

$$c_1, c_2, \ldots, c_w,$$

where

$w = n_1 \cdot n_2$

is the number of all possible climate-weather states.

5.1.2. Semi-Markov model of climate-weather change process

To model the climate-weather change process for the critical infrastructure operating area, we assume that the climate-weather process in this area is taking w, $w \in N$, different climate-weather states $c_1, c_2, \ldots, c_w$. Further, we define the climate-weather change process $C(t)$, $t \in \langle 0, +\infty)$, with discrete operation states from the set $\{c_1, c_2, \ldots, c_w\}$.

Assuming that the climate-weather change process $C(t)$ is a semi-Markov process (Grabski, 2014; Kołowrocki, 2014, 2019/2020) it can be described by the following climate-weather change process parameters (C-WCPP), that can be identified either statistically using the methods given in (Kołowrocki et al., 2018) or evaluated approximately by experts:

- the number of climate-weather states (C-WCPP1)

  w,

- the climate-weather states (C-WCPP2)

  $\{c_1, c_2, \ldots, c_w\}$,

- the vector

  $[q_l(0)]_{1xw} = [q_1(0), q_2(0), \ldots, q_w(0)]$,
  of the initial probabilities (C-WCPP3) of the climate-weather change process $C(t)$ staying at particular climate-weather states c_l at the moment $t = 0$

  $q_l(0) = P(C(0) = c_l), l = 1,2, \ldots, w$,

- the matrix

$$[q_{lk}]_{v \times w} = \begin{bmatrix} q_{11} \ q_{12} \ . \ . \ . \ q_{1w} \\ q_{21} \ q_{22} \ . \ . \ . \ q_{2w} \\ . \ . \ . \\ q_{w1} \ q_{w2} \ . \ . \ . \ q_{ww} \end{bmatrix},$$

  of probabilities of transition (C-WCPP4) of the climate-weather change process $C(t)$ between the climate-weather states c_l and c_k

  $q_{lk}, l, k = 1,2, \ldots, w, l \neq k,$

  $q_{ll} = 0, l = 1,2, \ldots, w,$

- the matrix

$$[N_{lk}]_{w \times w} = \begin{bmatrix} N_{11} \ N_{12} \ . \ . \ . \ N_{1w} \\ N_{21} \ N_{22} \ . \ . \ . \ N_{2w} \\ . \ . \ . \\ N_{w1} \ N_{w2} \ . \ . \ . \ N_{ww} \end{bmatrix},$$

  of mean values of the climate-weather change process $C(t)$ conditional sojourn times (C-WCPP5) C_{lk} at the climate-weather state c_l when the next state is c_k

  $N_{lk} = E[C_{lk}] = \int_0^\infty t dC_{lk}(t) = \int_0^\infty t c_{lk}(t) dt,$

  $l, k = 1,2, \ldots, w, l \neq k,$

  $N_{ll} = 0, l = 1,2, \ldots, w,$ (72)

  where

  $C_{lk}(t) = P(\theta_{lk} < t), t \geq 0,$

  $l, k = 1,2, \ldots, w, l \neq k,$

  are conditional distribution functions of the critical infrastructure climate-weather change process $C(t)$, $t \geq 0$, conditional sojourn times θ_{lk} at the climate-weather states corresponding to conditional density functions

  $c_{lk}(t) = \frac{dC_{lk}(t)}{dt}, t \geq 0,$

  $l, k = 1,2, \ldots, w, l \neq k.$

Assuming that we have identified the above parameters (C-WCPP1-5) of the climate-weather change process semi-Markov model, we can predict this process basic characteristics (C-WCPC), that can be either calculated analytically or evaluated approximately by experts (Kołowrocki, 2019/2020; Kołowrocki et al., 2018):

- the vector

  $[N_l]_{1 \times w} = [N_1, N_2, \ldots, N_w],$ (73)

of mean values of the climate-weather change process $C(t)$ critical infrastructure at the operating area unconditional sojourn times C_l, $l = 1,2,\dots,w$, at the climate-weather states c_l, $l = 1,2,\dots,w$, (C-WCPC1)

$$N_l = E[C_l] = \sum_{k=1}^{v} q_{lk} N_{lk},\ l = 1,2,\dots,w, \tag{74}$$

where N_{lk} are defined by the formula (72),

- the vector

$$[q_l]_{1\times w} = [q_1, q_2, \dots, q_w],$$

of limit values of the climate-weather change process $C(t)$, $t \ge 0$, transient probabilities

$$q_l(t) = P(C(t) = c_l),\ t \ge 0,\ l = 1,2,\dots,w,$$

at the particular climate-weather states (C-WCPC2)

$$q_l = \lim_{t\to\infty} q_l(t) = \frac{\pi_l N_l}{\sum_{k=1}^{w} \pi_k N_k},\ l = 1,2,\dots,w,, \tag{75}$$

where N_l, $l = 1,2,\dots,w$, are given by (74), while the steady probabilities π_l of the vector $[\pi_l]_{1\times w}$ satisfy the system of equations

$$\begin{cases} [\pi_l] = [\pi_l][q_{lk}] \\ \sum_{k=1}^{v} \pi_k = 1. \end{cases} \tag{75}$$

In the case of a periodic climate-weather change process, the limit transient probabilities q_l, $l = 1,2,\dots,w$, at the climate-weather states determined by (75), are the long term proportions of the climate-weather change process $C(t)$, $t \ge 0$, sojourn times at the particular climate-weather states c_l, $l = 1,2,\dots,w$.
Another interesting characteristic of the system climate-weather change process $C(t)$ possible to obtain is

- the vector of the mean values (C-WCPC3)

$$[\hat{N}_l]_{1\times w} = [\hat{N}_1, \hat{N}_2, \dots, \hat{N}_w],$$

of the total sojourn times $\hat{C}_l$, $l = 1,2,\dots,w$, of the climate-weather change process $C(t)$ at the critical infrastructure operating area at the particular climate-weather states c_l, $l = 1,2,\dots,w$, during the fixed time C. It is well known, (Kołowrocki, 2014, 2019/2020, 2020a, 2020b), that the climate-weather change process total sojourn times $\hat{C}_l$ at the particular climate-weather states c_l for sufficiently large time C have approximately normal distributions with the mean values given by

$$\hat{N}_l = E[\hat{C}_l] = q_l C,\ l = 1,2,\dots,w, \tag{77}$$

where q_l are given by (75).

5.2. Safety and resilience indicators of critical infrastructure impacted by climate-weather change process

We denote by $[T^2(u)]^{(l)}$, $u = 1,2,\dots,z$, $l = 1,2,\dots,w$, the critical infrastructure conditional lifetime in the safety state subset $\{u, u+1,\dots,z\}$, $u = 1,2,\dots,z$, while the climate-weather change process $C(t)$, $t \ge 0$, at the critical infrastructure operating area is at the climate-weather state c_l, $l = 1,2,\dots,w$, and the conditional safety function of the critical infrastructure related to the climate-weather change process at its operating area $C(t)$, $t \ge 0$, by the vector (Kołowrocki, 2019/2020; Kołowrocki et al., 2018)

$$[\boldsymbol{S}^2(t,\cdot)]^{(l)} = [[\boldsymbol{S}^2(t,1)]^{(l)},\dots,[\boldsymbol{S}^2(t,z)]^{(l)}],$$

$$t \ge 0,\ l = 1,2,\dots,w, \tag{78}$$

with the coordinates defined by

$$[\boldsymbol{S}^2(t,u)]^{(l)} = P([T^2(u)]^{(l)} > t \,|\, Z(t) = z_l) \tag{79}$$

for $t \ge 0$, $u = 1,2,\dots,z$, $l = 1,2,\dots,w$.

The safety function $[\boldsymbol{S}^2(t,u)]^{(l)}$, $t \ge 0$, $u = 1,2,\dots,z$, $l = 1,2,\dots,w$, is the conditional probability that the critical infrastructure impacted by the climate-weather change process $C(t)$, $t \ge 0$, lifetime $[T^2(u)]^{(l)}$, $u = 1,2,\dots,z$, $l = 1,2,\dots,w$, in the safety state subset $\{u, u+1,\dots,z\}$, $u = 1,2,\dots,z$, is greater than t, while the climate-weather change process $C(t)$, $t \ge 0$, is at the climate-weather state c_l, $l = 1,2,\dots,w$.
Next, we denote by $T^2(u)$, $u = 1,2,\dots,z$, the critical infrastructure impacted by the climate-

weather change process $C(t)$, $t \geq 0$,unconditional lifetime in the safety state subset $\{u, u+1, \ldots, z\}$, $u = 1,2,\ldots,z$, and the unconditional safety function (SafI1) of the critical infrastructure impacted by the climate-weather change process $C(t)$, $t \geq 0$, by the vector

$$\boldsymbol{S}^2(t,\cdot) = [\boldsymbol{S}^2(t,1),\ldots,\boldsymbol{S}^2(t,z)],\ t \geq 0, \tag{80}$$

with the coordinates defined by

$$\boldsymbol{S}^2(t,u) = P(T^2(u) > t),\ t \geq 0,\ u = 1,2,\ldots,z. \tag{81}$$

In the case when the critical infrastructure operation time θ is large enough, the coordinates of the unconditional safety function of the critical infrastructure related to the climate-weather change process $C(t)$, $t \geq 0$, defined by (81), are given by

$$\boldsymbol{S}^2(t,u) \cong \sum_{l=1}^{w} q_l [\boldsymbol{S}^2(t,u)]^{(l)},$$

$$\text{for } t \geq 0,\ u = 1,2,\ldots,z, \tag{82}$$

where $[\boldsymbol{S}^2(t,u)]^{(l)}$, $t \geq 0$, $u = 1,2,\ldots,z$, $l = 1,2,\ldots,w$, are the coordinates of the critical infrastructure related to the climate-weather change process $C(t)$, $t \geq 0$, conditional safety functions defined by (78)–(79) and q_l, $l = 1,2,\ldots,w$, are the climate-weather change process $C(t)$, $t \geq 0$, at the critical infrastructure operating area limit transient probabilities at the climate-weather states c_l, $l = 1,2,\ldots,w$, given by (75).

If r is the critical safety state, then the second safety indicator of the critical infrastructure impacted by the climate-weather change process $C(t)$, $t \geq 0$, the risk function (SafI2)

$$\boldsymbol{r}2(t) = P(s(t) < r \mid s(0) = z) = P(T2(r) \leq t),$$

$$t \geq 0, \tag{83}$$

is defined as a probability that the critical infrastructure impacted by the climate-weather change process $C(t)$, $t \geq 0$, is in the subset of safety states worse than the critical safety state r, $r \in \{1,\ldots,z\}$, while it was in the best safety state z at the moment $t = 0$ and given by (Kołowrocki et al., 2018)

$$\boldsymbol{r}^2(t) = 1 - \boldsymbol{S}^2(t,r),\ t \geq 0, \tag{84}$$

where $\boldsymbol{S}^2(t,r)$, $t \geq 0$, is the coordinate of the critical infrastructure impacted by the climate-weather change process $C(t)$, $t \geq 0$, unconditional safety function given by (82) for $u = r$.

The graph of the critical infrastructure risk function $\boldsymbol{r}^2(t)$, $t \geq 0$, defined by (84), is the safety indicator called the fragility curve (SafI3) of the critical infrastructure impacted by the climate-weather change process $C(t)$, $t \geq 0$.

Other practically useful safety and resilience indicators of the critical infrastructure impacted by climate-weather change process $C(t)$, $t \geq 0$, at its operating area are:

- the mean value of the critical infrastructure unconditional lifetime $T^2(r)$ up to exceeding critical safety state r (SafI4) given by

$$\boldsymbol{\mu}^2(r) = \int_0^\infty [\boldsymbol{S}^2(t,r)]dt$$

$$\cong \sum_{l=1}^{w} q_l [\boldsymbol{\mu}^2(r)]^{(l)}, \tag{85}$$

  where $[\boldsymbol{\mu}^2(r)]^{(l)}$ are the mean values of the critical infrastructure conditional lifetimes $[T^2(r)]^{(l)}$ in the safety state subset $\{r, r+1, \ldots, z\}$ at the climate-weather state c_l, $l = 1,2,\ldots,w$, given by

$$[\boldsymbol{\mu}^2(r)]^{(l)} = \int_0^\infty [\boldsymbol{S}^2(t,r)]^{(l)} dt,\ l = 1,2,\ldots,w, \tag{86}$$

  and $[\boldsymbol{S}^2(t,r)]^{(l)}$, $t \geq 0$, $l = 1,2,\ldots,w$, are defined by (78)–(79) and q_l, $l = 1,2,\ldots,w$, are given by (75),
- the standard deviation of the critical infrastructure lifetime $T^2(r)$ up to the exceeding the critical safety state r (SafI5) given by

$$\boldsymbol{\sigma}^2(r) = \sqrt{\boldsymbol{n}^2(r) - [\boldsymbol{\mu}^2(r)]^2}, \tag{87}$$

  where

$$\boldsymbol{n}^2(r) = 2\int_0^\infty t \cdot \boldsymbol{S}^2(t,r)dt, \tag{88}$$

  and $\boldsymbol{S}^2(t,r)$, $t \geq 0$, is defined by (82) for $u = r$ and $\boldsymbol{\mu}^2(r)$ is given by (85),
- the moment $\boldsymbol{\tau}^2$ of exceeding acceptable value of critical infrastructure risk function level d (SafI6) given by

$$\boldsymbol{\tau}^2 = (\boldsymbol{r}^2)^{-1}(\delta), \tag{89}$$

where $(\boldsymbol{r}^2)^{-1}(t)$ is the inverse function of the risk function $\boldsymbol{r}^2(t)$, $t \geq 0$, given by (84),

- the mean lifetimes of the critical infrastructure in the safety state subsets $\{u, u+1, \ldots, z\}$, $u = 1,2,...,z$, (SafI7), given by

$$\boldsymbol{\mu}^2(u) = \int_0^\infty [\boldsymbol{S}^2(t,u)]dt \cong \sum_{l=1}^{w} p_l [\boldsymbol{\mu}^2(u)]^{(l)},$$

$$u = 1,2,\ldots,z, \quad (90)$$

where $[\boldsymbol{\mu}^2(u)]^{(l)}$, $u = 1,2,\ldots,z$, $l = 1,2,\ldots,w$, are the mean values of the critical infrastructure conditional lifetimes $[T^2(u)]^{(l)}$, $u = 1,2,\ldots,z$, $l = 1,2,\ldots,w$, in the safety state subsets $\{u, u+1, \ldots, z\}$ at the climate-weather state c_l, $l = 1,2,\ldots,w$, given by

$$[\boldsymbol{\mu}^2(u)]^{(l)} = \int_0^\infty [\boldsymbol{S}^2(t,u)]^{(l)} dt,$$

$$u = 1,2,\ldots,z, \; l = 1,2,\ldots,w, \quad (91)$$

and $[\boldsymbol{S}^2(t,u)]^{(l)}$, $t \geq 0$, $u = 1,2,\ldots,z$, $l = 1,2,\ldots,w$, are defined by (78)–(79) and q_l, $l = 1,2,\ldots,w$, are given by (75),

- the standard deviations of the critical infrastructure lifetimes in the safety state subsets $\{u, u+1, \ldots, z\}$, $u = 1,2,...,z$, (SafI8), given by

$$\boldsymbol{\sigma}^2(u) = \sqrt{\boldsymbol{n}^2(u) - [\boldsymbol{\mu}^2(u)]^2}, \; u = 1,2,...,z, \quad (92)$$

where

$$\boldsymbol{n}^2(u) = 2\int_0^\infty t \cdot \boldsymbol{S}^2(t,u)dt, \; u = 1,2,...,z, \quad (93)$$

and $\boldsymbol{S}^2(t,u)$, $t \geq 0$, $u = 1,2,\ldots,z$, are given by (82);

- the mean lifetimes $\bar{\boldsymbol{\mu}}^2(u)$, $u = 1,2,...,z$, of the critical infrastructure in the particular safety states (SafI9)

$$\bar{\boldsymbol{\mu}}^2(u) = \boldsymbol{\mu}^2(u) - \boldsymbol{\mu}^2(u+1),$$

$$u = 0,1,\ldots,z-1, \; \bar{\boldsymbol{\mu}}^2(z) = \boldsymbol{\mu}^2(z), \quad (94)$$

where $\boldsymbol{\mu}^2(u)$, $u = 1,2,\ldots,z$, are given by (91),

- the intensities of degradation of the critical infrastructure / the intensities of critical infrastructure departure from the safety state subset $\{u, u+1, \ldots, z\}$, $u = 1,2,...,z$, (SafI10), i.e. the coordinates of the vector

$$\boldsymbol{\lambda}^2(t,\cdot) = [\boldsymbol{\lambda}^2(t,1), \ldots, \boldsymbol{\lambda}^2(t,z)], \; t \geq 0, \quad (95)$$

where

$$\boldsymbol{\lambda}^2(t,u) = -\frac{d\boldsymbol{S}^2(t,u)}{dt} \cdot \frac{1}{\boldsymbol{S}^2(t,u)},$$

$$t \geq 0, \; u = 1,2,\ldots,z, \quad (96)$$

- the coefficients of climate-weather change process impact on the critical infrastructure intensities of degradation / the coefficients of climate-weather change process impact on critical infrastructure intensities of departure from the safety state subset $\{u, u+1, \ldots, z\}$ (ResI1), i.e. the coordinates of the vector

$$\boldsymbol{\rho}^2(t,\cdot) = [\boldsymbol{\rho}^2(t,1), \ldots, \boldsymbol{\rho}^2(t,z)], \; t \geq 0, \quad (97)$$

where

$$\boldsymbol{\lambda}^2(t,u) = \boldsymbol{\rho}^2(t,u) \cdot \boldsymbol{\lambda}^0(t,u), \; t \geq 0,$$

$$u = 1,2,\ldots,z, \quad (98)$$

i.e.

$$\boldsymbol{\rho}^2(t,u) = \frac{\boldsymbol{\lambda}^2(t,u)}{\boldsymbol{\lambda}^0(t,u)}, \; t \geq 0, \; u = 1,2,\ldots,z, \quad (99)$$

and $\boldsymbol{\lambda}^0(t,u)$, $t \geq 0$, $u = 1,2,\ldots,z$, are the intensities of degradation of the critical infrastructure without of climate-weather change process impact, defined by (21), i.e. the coordinates of the vector

$$\boldsymbol{\lambda}^0(t,\cdot) = [\boldsymbol{\lambda}^0(t,1), , \ldots, \boldsymbol{\lambda}^0(t,z)], \; t \geq 0, \quad (100)$$

and $\boldsymbol{\lambda}^2(t,u)$, $t \geq 0$, $u = 1,2,\ldots,z$, are the intensities of degradation of the critical infrastructure impacted by the climate-weather change process, defined by (96), i.e. the coordinates of the vector

$$\boldsymbol{\lambda}^2(t,\cdot) = [\boldsymbol{\lambda}^2(t,1), \ldots, \boldsymbol{\lambda}^2(t,z)], \; t \geq 0, \quad (101)$$

- the indicator of critical infrastructure resilience to climate-weather change process impact (ResI2) defined by

$$\boldsymbol{RI}^2(t,r) = \frac{1}{\boldsymbol{\rho}^2(t,r)},\ t \geq 0, \tag{102}$$

where $\boldsymbol{\rho}^2(t,r)$, $t \geq 0$, is the coefficients of climate-weather change process impact on the critical infrastructure intensities of degradation given by (99) for $u = r$.

In the case, the critical infrastructure have the piecewise exponential safety functions, i.e.

$$\boldsymbol{S}^2(t,\cdot) = [\boldsymbol{S}^2(t,1),\ldots,\boldsymbol{S}^2(t,z)\ ],\ t \geq 0, \tag{103}$$

where

$$\boldsymbol{S}^2(t,u) = \exp[-\boldsymbol{\lambda}^2(u)t],\ t \geq 0,$$

$$\boldsymbol{\lambda}^2(u) \geq 0,\ u = 1,2,\ldots,z, \tag{104}$$

the critical infrastructure safety and resilience indicators defined by (95)–(101) take following forms:

- the intensities of degradation of the critical infrastructure impacted by the climate-weather change process impact, (SafI10), i.e. the coordinates of the vector

$$\boldsymbol{\lambda}^2(\cdot) = [\boldsymbol{\lambda}^2(1),\ldots,\ \boldsymbol{\lambda}^2(z)], \tag{105}$$

are constant and

$$\boldsymbol{\lambda}^2(u) = \frac{1}{\boldsymbol{\mu}^2(u)},\ u = 1,2,\ldots,z, \tag{106}$$

where $\boldsymbol{\mu}^2(u)$, $u = 1,2,\ldots,z$, is the mean value of the critical infrastructure lifetime in the safety state subset $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$,

- the coefficients of the climate-weather change process impact on the critical infrastructure intensities of degradation / the coefficients of the climate-weather change process impact on critical infrastructure intensities of departure from the safety state subset $\{u,u+1,\ldots,z\}$, (ResI1), i.e. the coordinates of the vector

$$\boldsymbol{\rho}^2(\cdot) = [\boldsymbol{\rho}^2(1),\ldots,\ \boldsymbol{\rho}^2(z)], \tag{107}$$

where

$$\boldsymbol{\rho}^2(u) = \frac{\boldsymbol{\lambda}^2(u)}{\boldsymbol{\lambda}^0(u)} = \frac{\boldsymbol{\mu}^0(u)}{\boldsymbol{\mu}^2(u)},\ u = 1,2,\ldots,z, \tag{108}$$

and $\boldsymbol{\lambda}^0(u)$, $u = 1,2,\ldots,z$, are the intensities of degradation of the critical infrastructure without of climate-weather change process impact, defined by (25), i.e. the coordinates of the vector

$$\boldsymbol{\lambda}^0(u) = [\boldsymbol{\lambda}^0(1),\ldots,\boldsymbol{\lambda}^0(z)] \tag{109}$$

and $\boldsymbol{\lambda}^2(u)$, $u = 1,2,\ldots,z$, are the intensities of degradation of the critical infrastructure related to the climate-weather process impact, defined by (106), i.e. the coordinates of the vector

$$\boldsymbol{\lambda}^2(\cdot) = [\boldsymbol{\lambda}^2(1),\ldots,\boldsymbol{\lambda}^2(z)], \tag{110}$$

- the indicator of critical infrastructure resilience to climate-weather change process impact (ResI2) defined by

$$\boldsymbol{RI}^2(r) = \frac{1}{\boldsymbol{\rho}^2(r)},\ t \geq 0, \tag{111}$$

where $\boldsymbol{\rho}^2(r)$ is the coefficient of climate-weather change process impact on the critical infrastructure intensities of degradation given by (108) for $u = r$.

The assets safety parameters of the critical infrastructure impacted by the climate-weather change process can by introduced in an analogous way (Kołowrocki, 2019/2020; Kołowrocki et al., 2018).

6. Modelling critical infrastructure safety impacted by operation process and climate-weather change process

6.1. Critical infrastructure operation process related to climate-weather change process at its operating area

We consider the critical infrastructure impacted by its operation process $ZC(t)$, $t \geq 0$, related to the climate-weather change process at its operating area in a various way at this process states zc_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$ We assume that the changes of the states of operation process related to the climate-weather change process $ZC(t)$, $t \geq 0$, at the critical infrastructure operating area have an influence on the critical infrastructure safety structure and on the safety parameters of the critical infrastructure assets A_i, $i = 1,2,\ldots,n$, as well (Kołowrocki et al., 2018).

We assume, as in Section 4, that the critical infrastructure during its operation process is taking v, $v \in N$, different operation states $z_1, z_2, ..., z_v$. We define the critical infrastructure operation process $Z(t)$, $t \geq 0$, with discrete operation states from the set $\{z_1, z_2, ..., z_v\}$. Moreover, we assume that the critical infrastructure operation process $Z(t)$, $t \geq 0$, is a semi-Markov process that can be described by the following parameters:

- the number of operation states v,
- the operation states $\{z_1, z_2, ..., z_v\}$,
- the vector $[p_b(0)]_{1\times v}$ of the initial probabilities $p_b(0)$, $b = 1,2,\ldots,v$, of the critical infrastructure operation process $Z(t)$ staying at particular operation state z_b, $b = 1,2,\ldots,v$, at the moment $t = 0$,
- the matrix $[p_{bl}]_{v\times v}$ of probabilities p_{bl}, $b,l = 1,2,\ldots,v$, of the critical infrastructure operation process $Z(t)$ transitions between the operation states z_b and z_l, $b, l = 1,2,\ldots,v$,
- the matrix $[H_{bl}(t)]_{v\times v}$ of conditional distribution functions $H_{bl}(t)$, $t \geq 0$, $b, l = 1,2,\ldots,v$, of the critical infrastructure operation process $Z(t)$ conditional sojourn times q_{bl} at the operation states z_b under the condition that the next operation state will be z_l, $b, l = 1,2,\ldots,v$.

Further, we assume that we have either calculated analytically using the above parameters of the operation process or evaluated approximately by experts the vector of limit values (OPC1)

$$[p_b]_{1\times v} = [p_1, p_2, \ldots, p_v],$$

of transient probabilities

$$p_b(t) = P(Z(t) = z_b),\ t \geq 0,\ b = 1,2,\ldots,v,$$

of the critical infrastructure operation process $Z(t)$ at the particular operation states z_b, $b = 1,2,\ldots,v$.

Moreover, as in Section 5, we assume that the climate-weather change process $C(t)$, $t \geq 0$, at the critical infrastructure operating area is taking w, $w \in N$, different climate-weather states $c_1, c_2, \ldots, c_w$. We assume that the climate-weather change process $C(t)$, $t \geq 0$, is a semi-Markov process and it can be described by:

- the number of climate-weather states w,
- the climate-weather states $\{c_1, c_2, \ldots, c_w\}$,
- the vector $[q_b(0)]_{1\times w}$ of the initial probabilities $q_b(0)$, $b = 1,2,\ldots,w$, of the climate-weather change process $C(t)$ staying at particular climate-weather states c_b, $b = 1,2,\ldots,w$, at the moment $t = 0$,
- the matrix $[q_{bl}]_{w\times w}$ of the probabilities q_{bl}, $b,l = 1,2,\ldots,w$, of transitions of the climate-weather change process $C(t)$ from the climate-weather states c_b to the climate-weather state c_l, $b, l = 1,2,\ldots,w$,
- the matrix $[C_{bl}(t)]_{wxw}$ of the conditional distribution functions $C_{bl}(t)$, $t \geq 0$, $b, l = 1,2,\ldots,w$, of the conditional sojourn times C_{bl} at the climate-weather states c_b when its next climate-weather state is c_l, $b, l = 1,2,\ldots,w$.

Further, we assume that we have either calculated analytically using the above parameters of the climate-weather change process or evaluated approximately by experts the vector of limit values (C-WCPC1)

$$[q_l]_{1\times w} = [q_1, q_2, \ldots, q_v],$$

of transient probabilities

$$q_l(t) = P(C(t) = c_l),\ t \geq 0,\ l = 1,2,\ldots,w,$$

of the climate-weather change process $C(t)$ at the particular climate-weather states c_l, $l = 1,2,\ldots,w$.

6.1.1. Joint model of critical infrastructure independent operation process and climate-weather change process

Under the assumption that the critical infrastructure operation process $Z(t)$, $t \geq 0$, and the climate-weather change process $C(t)$, $t \geq 0$, at its operating area are independent, we introduce the joint process of critical infrastructure operation process and climate-weather change process called the critical infrastructure operation process related to climate-weather change marked by

$$ZC(t),\ t \geq 0, \tag{111}$$

and we assume that it can take vw, $v, w \in N$, different operation and climate-weather states

$$zc_{11}, zc_{12}, \ldots, zc_{vw},$$

$$b = 1,2,\ldots,v,\ l = 1,2,\ldots,w. \tag{112}$$

We assume that the critical infrastructure operation process related to climate-weather change process $ZC(t)$, at the moment t, $t \geq 0$, is at the operation and climate-weather state zc_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, if and only if at that moment t, the operation process $Z(t)$, $t \geq 0$, is at the operation states z_b, $b = 1,2,\ldots,v$, and the climate-weather change process $C(t)$ is at the climate-weather state c_l, $l = 1,2,\ldots,w$, at the moment t, $t \geq 0$, what we express as follows:

$$(ZC(t) = zc_{bl}) \Leftrightarrow (Z(t) = z_b \cap C(t) = c_l),$$

$$t \geq 0, b = 1,2,\ldots,v, l = 1,2,\ldots,w. \quad (113)$$

Further, we define the initial probabilities

$$pq_{bl}(0) = P(ZC(0) = zc_{bl}),$$

$$b = 1,2,\ldots,v, l = 1,2,\ldots,w, \quad (114)$$

of the critical infrastructure operation process related to climate-weather change process $ZC(t)$, at the initial moment $t = 0$ at the operation and climate-weather state zc_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, and this way we have the vector

$$[pq_{bl}(0)]_{1\times vw} = [pq_{11}(0), pq_{12}(0), \ldots, pq_{1w}(0),$$

$$pq_{21}(0), pq_{22}(0), \ldots, pq_{2w}(0), \ldots,$$

$$pq_{v1}(0), pq_{v2}(0), \ldots, pq_{vw}(0)] \quad (115)$$

of the initial probabilities the critical infrastructure operation process related to climate-weather change process $ZC(t)$, $t \geq 0$, staying at the particular operation and climate-weather states at the initial moment $t = 0$

From the assumption that the critical infrastructure operation process $Z(t)$, $t \geq 0$, and climate-weather change process $C(t)$, $t \geq 0$, are independent, it follows that (Kołowrocki, 2019/2020)

$$pq_{bl}(0) = P(ZC(0) = zc_{bl})$$

$$= P(Z(0) = z_b \cap C(0) = c_l)$$

$$= P(Z(0) = z_b) \cdot P(C(0) = c_l)$$

$$= p_b(0) \cdot q_l(0), b = 1,2,\ldots,v, l = 1,2,\ldots,w, \quad (116)$$

where $p_b(0)$, $b = 1,2,\ldots,v$, and $q_l(0)$, $l = 1,2,\ldots,w$, are the critical infrastructure initial probabilities of the operation process and the critical infrastructure initial probabilities of the climate-weather change process at its operating area, respectively introduced in Section 4 and Section 5.

Hence, the vector of the initial probabilities of the critical infrastructure operation process related to climate-weather change $ZC(t)$ defined by (111)–(112) take the following form (Kołowrocki, 2019/2020)

$$[pq_{bl}(0)]_{1\times vw} = [p_b(0)q_l(0)]_{1\times vw}$$

$$= [p_1(0)q_1(0), p_1(0)q_2(0), \ldots, p_1(0)q_w(0),$$

$$p_2(0)q_1(0), p_2(0)q_2(0), \ldots, p_2(0)q_w(0), \ldots,$$

$$p_v(0)q_1(0), p_v(0)q_2(0), \ldots, p_v(0)q_w(0)]. \quad (117)$$

Further, we introduce the probabilities (Kołowrocki, 2019/2020)

$$pq_{bl\,mn}, b = 1,2,\ldots,v, l = 1,2,\ldots,w,$$

$$m = 1,2,\ldots,v, n = 1,2,\ldots,w, \quad (118)$$

of the transitions of the critical infrastructure operation process related to climate-weather change process $ZC(t), t \geq 0$, between the operation and climate-weather states

$$zc_{bl} \text{ and } zc_{mn}, b = 1,2,\ldots,v, l = 1,2,\ldots,w,$$

$$m = 1,2,\ldots,v, n = 1,2,\ldots,w, \quad (119)$$

and get their following matrix form

$$[pq_{bl\,mn}]_{vw\mathrm{x}vw}$$

$$= \begin{bmatrix} pq_{11\,11}\; pq_{11\,12} \;\cdots\; pq_{11\,1w}; pq_{11\,21}\; pq_{11\,22} \cdots pq_{11\,2w}; \ldots; pq_{11\,v1}\; pq_{11\,v2} \cdots pq_{11\,vw} \\ pq_{12\,11}\; pq_{12\,12} \;\cdots\; pq_{12\,1w}; pq_{12\,21}\; pq_{12\,22} \cdots pq_{12\,2w}; \ldots; pq_{12\,v1}\; pq_{12\,v2} \cdots pq_{12\,vw} \\ \cdots \\ pq_{vw\,11}\; pq_{vw\,12} \;\cdots\; pq_{vw\,1w}; pq_{vw\,21}\; pq_{vw\,22} \cdots pq_{vw\,2w}; \ldots; pq_{vw\,v1}\; pq_{vw\,v2} \cdots pq_{vw\,vw} \end{bmatrix} \quad (120)$$

From the assumption that the critical infrastructure operation process $Z(t), t \geq 0$, and climate-weather change process $C(t)$, $t \geq 0$, are independent, it follows that (Kołowrocki, 2019/2020)

$$pq_{bl\,mn} = p_{bm} \cdot q_{ln},\ b = 1,2,\ldots,v,\ l = 1,2,\ldots,w,$$

$$m = 1,2,\ldots,v,\ n = 1,2,\ldots,w, \qquad (121)$$

where

p_{bm}, $b = 1,2,\ldots,v$, $m = 1,2,\ldots,v$ and

$$q_{ln},\ l = 1,2,\ldots,w,\ n = 1,2,\ldots w, \qquad (122)$$

are the probabilities of critical infrastructure operation process transitions between the operation states and the probabilities of critical infrastructure climate-weather change process transitions between climate-weather states, respectively defined in Section 4 and in Section 5.

Hence, the matrix of the probabilities of transitions between the critical infrastructure operation process related to climate-weather change process $ZC(t), t \geq 0$, defined by (111)–(112) takes the following form

$$[pq_{bl\,mn}]_{vw\times vw} = [p_{bm}q_{ln}]_{vw\times vw}$$

$$= \begin{bmatrix} p_{11}q_{11}\ p_{11}q_{12}\ \ldots\ p_{11}q_{1w};\ p_{12}q_{11}\ p_{12}q_{12}\ldots p_{12}q_{1w};\ \ldots;\ p_{1v}q_{11}\ p_{1v}q_{11}\ldots p_{1v}q_{1w} \\ p_{11}q_{21}\ p_{11}q_{22}\ \ldots\ p_{11}q_{2w};\ p_{12}q_{21}\ p_{12}q_{22}\ldots p_{12}q_{2w};\ \ldots;\ p_{1v}q_{21}\ p_{1v}q_{21}\ldots p_{1v}q_{2w} \\ \ldots \\ p_{v1}q_{w1}\ p_{v1}q_{w2}\ \ldots\ p_{v1}q_{ww};\ p_{v2}q_{w1}\ p_{v2}q_{w2}\ldots p_{v2}q_{ww};\ \ldots;\ p_{vv}q_{w1}\ p_{vv}q_{w2}\ldots p_{vv}q_{ww} \end{bmatrix}. \qquad (123)$$

The matrix of conditional distribution functions (Kołowrocki, 2019/2020)

$$HC_{bl\,mn}(t) = P(\theta C_{bl\,mn} < t),\ t \geq 0,$$

$$b = 1,2,\ldots,v,\ l = 1,2,\ldots,w,$$

$$m = 1,2,\ldots,v,\ n = 1,2,\ldots,w, \qquad (124)$$

of the critical infrastructure operation process related to climate-weather change process $ZC(t)$, $t \geq 0$, conditional sojourn times $\theta C_{bl\,mn}$, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, $m = 1,2,\ldots,v$, $n = 1,2,\ldots,w$, at the operation and climate-weather state zc_{bm}, $b = 1,2,\ldots,v$, $m = 1,2,\ldots,v$, when the next operation and climate-weather state is zc_{ln}, $l = 1,2,\ldots,w$, $n = 1,2,\ldots,w$, takes the following form

$$[HC_{bl\,mn}(t)]_{vw\times vw}$$

$$= \begin{bmatrix} HC_{11\ 11}(t)\ HC_{11\ 12}(t)\ \ldots\ HC_{11\ 1w}(t);\ HC_{11\ 21}(t)HC_{11\ 22}(t)\ldots HC_{11\ 2w}(t);\ldots;HC_{11\ v1}(t)HC_{11\ v2}(t)\ldots HC_{11\ vw}(t) \\ HC_{12\ 11}(t)HC_{12\ 12}(t)\ \ldots\ HC_{12\ 1w}(t);\ HC_{12\ 21}(t)HC_{12\ 22}(t)\ldots HC_{12\ 2w}(t);\ldots;HC_{12\ v1}(t)HC_{12\ v2}(t)\ldots HC_{12\ vw}(t) \\ \ldots \\ HC_{vw\ 11}(t)\ HC_{vw\ 12}(t)\ \ldots\ HC_{vw\ 1w}(t);\ HC_{vw\ 21}(t)HC_{vw\ 22}(t)\ldots HC_{vw\ 2w}(t);\ldots;HC_{vw\ v1}(t)HC_{vw\ v2}(t)\ldots HC_{vw\ vw}(t) \end{bmatrix} \qquad (125)$$

and the matrix of their corresponding conditional density functions

$$hc_{bl\,mn}(t) = \frac{d}{dt}[HC_{bl\,mn}(t)],\ t \geq 0, \qquad (126)$$

where $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, $m = 1,2,\ldots,v$, $n = 1,2,\ldots,w$, has the form

$$[hc_{bl\,mn}(t)]_{vw\times vw} =$$

$$\begin{bmatrix} hc_{11\ 11}(t)\ hc_{11\ 12}(t)\ \ldots\ hc_{11\ 1w}(t);\ hc_{11\ 21}(t)hc_{11\ 22}(t)\ldots hc_{11\ 2w}(t);\ldots;hc_{11\ v1}(t)hc_{11\ v2}(t)\ldots hc_{11\ vw}(t) \\ hc_{12\ 11}(t)hc_{12\ 12}(t)\ \ldots\ hc_{12\ 1w}(t);\ hc_{12\ 21}(t)hc_{12\ 22}(t)\ldots hc_{12\ 2w}(t);\ldots;hc_{12\ v1}(t)hc_{12\ v2}(t)\ldots hc_{12\ vw}(t) \\ \ldots \\ hc_{vw\ 11}(t)\ hc_{vw\ 12}(t)\ \ldots\ hc_{vw\ 1w}(t);\ hc_{vw\ 21}(t)hc_{vw\ 22}(t)\ldots hc_{vw\ 2w}(t);\ldots;hc_{vw\ v1}(t)hc_{vw\ v2}(t)\ldots hc_{vw\ vw}(t) \end{bmatrix} \qquad (127)$$

From the assumption that the critical infrastructure operation process $Z(t), t \geq 0$, and climate-weather change process $C(t)$, $t \geq 0$, are independent, it follows that (Kołowrocki, 2019/2020)

$$HC_{bl\,mn}(t) = P(\theta C_{bl\,mn} < t)$$

$$= P\big((\theta_{bm} < t) \cap (C_{ln} < t)\big) = H_{bm}(t)C_{ln}(t),$$

$$t \geq 0,\ b = 1,2,\dots,v,\ l = 1,2,\dots,w,$$

$$m = 1,2,\dots,v,\ n = 1,2,\dots,w, \qquad (128)$$

and

$$hc_{bl\,mn}(t) = \frac{d}{dt}[HC_{bl\,mn}(t)]$$

$$= \frac{d}{dt}[H_{bm}(t)C_{ln}(t)]$$

$$= h_{bm}(t)C_{ln}(t) + H_{bm}(t)c_{ln}(t),$$

$$t \geq 0,\ b = 1,2,\dots,v,\ m = 1,2,\dots,v,$$

$$l = 1,2,\dots,w,\ n = 1,2,\dots,w, \qquad (129)$$

where

$$H_{bm}(t),\ t \geq 0,\ b = 1,2,\dots,v,\ m = 1,2,\dots,v, \text{ and}$$

$$C_{ln}(t), t \geq 0,\ l = 1,2,\dots,w,\ n = 1,2,\dots,w, \qquad (130)$$

are conditional distribution functions of the critical infrastructure operation process lifetimes at the operation states and conditional distribution functions of the climate-weather change process lifetimes at climate-weather states and

$$h_{bm}(t),\ t \geq 0,\ b = 1,2,\dots,v,\ m = 1,2,\dots,v, \text{ and}$$

$$c_{ln}(t), t \geq 0,\ l = 1,2,\dots,w,\ n = 1,2,\dots,w, \qquad (131)$$

are conditional density functions correspond to them, respectively defined in (Holden et al., 2013).

Hence, the matrix of the conditional distribution functions and the matrix of the conditional density functions of the critical infrastructure operation process related to climate-weather change process $ZC(t)$ conditional sojourn times defined by (111) and (113) respectively take the following forms (Kołowrocki, 2019/2020)

$$[HC_{bl\,mn}(t)]_{vw\times vw} = [H_{bm}(t)C_{ln}(t)]_{vw\times vw}$$

$$= \begin{bmatrix} H_{11}(t)C_{11}(t)\ H_{11}(t)C_{12}(t)\ ...\ H_{11}(t)C_{1w}(t);\dots;H_{1v}(t)C_{11}(t)H_{1v}(t)C_{12}(t)\dots H_{1v}(t)C_{1w}(t) \\ H_{11}(t)C_{21}\ (t)H_{11}(t)C_{22}(t)\ ...\ H_{11}(t)C_{2w}(t);\dots;H_{1v}(t)C_{21}(t)H_{1v}(t)C_{22}(t)\dots H_{1v}(t)C_{1w}(t) \\ ... \\ H_{v1}(t)C_{w1}(t)\ H_{v1}(t)C_{w2}(t)\ ...\ H_{v1}(t)C_{ww}(t);\dots;H_{vv}(t)C_{w1}(t)H_{vv}(t)C_{w2}(t)\dots H_{vv}(t)C_{ww}(t) \end{bmatrix} \qquad (132)$$

and

$$[hc_{bl\,mn}(t)]_{vw\times vw} = [h_{bm}(t)C_{ln}(t) + H_{bm}(t)c_{ln}(t)]_{vw\times vw} =$$

$$\begin{bmatrix} h_{11}(t)C_{11}(t) + H_{11}(t)c_{11}(t)\ ...\ h_{11}(t)C_{1w}(t) + H_{11}(t)c_{1w}(t);\dots;h_{1v}(t)C_{11}(t) + H_{1v}(t)c_{11}(t)\dots h_{1v}(t)C_{1w}(t) + H_{1v}(t)c_{1w}(t) \\ h_{11}(t)C_{21}\ (t) + H_{11}(t)c_{21}(t)\ ...\ h_{11}(t)C_{2w}(t) + H_{11}(t)c_{2w}(t);\dots;h_{1v}(t)C_{21}(t) + H_{1v}(t)c_{21}(t)\dots h_{1v}(t)C_{1w}(t) + H_{1v}(t)c_{1w}(t) \\ ... \\ h_{v1}(t)C_{w1}(t) + H_{v1}(t)c_{w1}(t)\ ...\ h_{v1}(t)C_{ww}(t) + H_{v1}(t)c_{ww}(t);\dots;h_{vv}(t)C_{w1}(t) + H_{vv}(t)c_{w1}(t)\dots h_{vv}(t)C_{ww}(t) + H_{vv}(t)c_{ww}(t) \end{bmatrix}. \qquad (133)$$

We assume that the suitable and typical distributions to describe the critical infrastructure operation process related to climate-weather change process $ZC(t)$, $t \geq 0$, conditional sojourn times $\theta C_{bl\,mn}$, $b = 1,2,\dots,v$, $l = 1,2,\dots,w$, $m = 1,2,\dots,v$, $n = 1,2,\dots,w$, at the particular states are of the same kind as those distinguished in (Holden et al., 2013) for the critical infrastructure operation process $Z(t)$, $t \geq 0$, conditional sojourn times θ_{bl}, $b = 1,2,\dots,v$, $l = 1,2,\dots,w$.

6.1.2. Joint model of critical infrastructure dependent operation process and climate-weather change process

Under the assumption that the critical infrastructure operation process $Z(t)$, $t \geq 0$, and the climate-weather change process $C(t)$, $t \geq 0$, at its operating area are dependent, we introduce the joint process of critical infrastructure operation process and climate-weather change process called the critical infrastructure operation process related to climate-weather change process marked by (Kołowrocki, 2019/2020)

$$ZC(t),\ t \geq 0, \tag{134}$$

and we assume that it can take vw, $v, w \in N$, different operation states

$$zc_{11},\ zc_{12}, \ldots,\ zc_{vw},\ b = 1,2,\ldots,v,\ l = 1,2,\ldots,w. \tag{135}$$

We assume that the critical infrastructure operation process related to climate-weather change process $ZC(t)$, $t \geq 0$, at the moment t, $t \geq 0$, is at the operation and climate-weather state zc_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, if and only if at that moment t, the operation process $Z(t)$, $t \geq 0$, is at the operation states z_b, $b = 1,2,\ldots,v$, and the climate-weather change process $C(t)$, $t \geq 0$, is at the climate-weather state c_l, $l = 1,2,\ldots,w$, what we express as follows (Kołowrocki, 2019/2020):

$$(ZC(t) = zc_{bl}) \Leftrightarrow (Z(t) = z_b \cap C(t) = c_l),$$

$$t \geq 0,\ b = 1,2,\ldots,v,\ l = 1,2,\ldots,w. \tag{136}$$

Further, we define the initial probabilities

$$pq_{bl}(0) = P(ZC(0) = zc_{bl}), \tag{137}$$

$$b = 1,2,\ldots,v,\ l = 1,2,\ldots,w,$$

of the critical infrastructure operation process related to climate-weather change process $ZC(t)$, $t \geq 0$, at the initial moment $t = 0$ at the operation and climate-weather state zc_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, and this way we have the vector

$$[pq_{bl}(0)]_{1\times vw}$$

$$= [pq_{11}(0), pq_{12}(0), \ldots, pq_{1w}(0),$$

$$pq_{21}(0), pq_{22}(0), \ldots, pq_{2w}(0), \ldots,$$

$$pq_{v1}(0), pq_{v2}(0), \ldots, pq_{vw}(0)], \tag{138}$$

of the initial probabilities of the critical infrastructure operation process related to climate-weather change process $ZC(t)$, $t \geq 0$, staying at the particular operation and climate-weather state at the initial moment $t = 0$.

In the case when the processess $Z(t)$ and $C(t)$ are dependent the initial probabilities existing in (138) can be express either by (Kołowrocki, 2019/2020)

$$pq_{bl}(0) = P(ZC(0) = zc_{bl})$$

$$= P(Z(0) = z_b \cap C(0) = c_l)$$

$$= P(Z(0) = z_b) \cdot P(C(0) = c_l | Z(0) = z_b)$$

$$= p_b(0) \cdot q_{l|b}(0),\ b = 1,2,\ldots,v,\ l = 1,2,\ldots,w, \tag{139}$$

where

$$p_b(0),\ b = 1,2,\ldots,v, \tag{140}$$

are the initial probabilities of the operation process $Z(t)$ at the operation state defined in Section 4 and

$$q_{l|b}(0) = P(C(0) = c_l | Z(0) = z_b),$$

$$b = 1,2,\ldots,v,\ l = 1,2,\ldots,w, \tag{141}$$

are conditional initial probabilities of the climate-weather change process $C(t)$ at the climate-weather state corresponding to those defined in Section 5 in case they are not conditional or by

$$pq_{bl}(0) = P(ZC(0) = zc_{bl})$$

$$= P(Z(0) = z_b \cap C(0) = c_l)$$

$$= P(Z(0) = z_b | C(0) = c_l) \cdot P(C(0) = c_l)$$

$$= p_{b|l}(0) \cdot q_l(0),\ b = 1,2,\ldots,v,\ l = 1,2,\ldots,w, \tag{142}$$

where

$$q_l(0) = P(C(0) = c_l),\ l = 1,2,\ldots,w, \tag{143}$$

are initial probabilities of the climate-weather change process $C(t)$ at the climate-weather state defined in Section 5 and

$$p_{b|l}(0) = P(Z(0) = z_b | C(0) = c_l),$$

$$b = 1,2,\ldots,v,\ l = 1,2,\ldots,w, \tag{144}$$

are conditional initial probabilities of the operation process $Z(t)$ at the operation state corresponding to those defined in Section 4 in case they are not conditional.

Further, we introduce the probabilities (Kołowrocki, 2019/2020)

$pq_{bl\,mn}$, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$,

$m = 1,2,\ldots,v$, $n = 1,2,\ldots,w$, (145)

of the transitions of the critical infrastructure operation process related to climate-weather change process $ZC(t)$, $t \geq 0$, between the operation and climate-weather states

zc_{bl} and zc_{mn}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$,

$m = 1,2,\ldots,v$, $n = 1,2,\ldots,w$, (146)

and get their following matrix form (Kołowrocki, 2019/2020)

$$[pq_{bl\,mn}]_{vw\times vw}$$
$$= \begin{bmatrix} pq_{11\,11}\ pq_{11\,12}\ \cdot\cdot\cdot pq_{11\,1w}; pq_{11\,21}\ pq_{11\,22} \cdots pq_{11\,2w}; \ldots; pq_{11\,v1}\ pq_{11\,v2} \cdots pq_{11\,vw} \\ pq_{12\,11}\ pq_{12\,12}\ \cdot\cdot\cdot pq_{12\,1w}; pq_{12\,21}\ pq_{12\,22} \cdots pq_{12\,2w}; \ldots; pq_{12\,v1}\ pq_{12\,v2} \cdots pq_{12\,vw} \\ \cdot\cdot\cdot \\ pq_{vw\,11}\ pq_{vw\,12}\ \cdot\cdot\cdot pq_{vw\,1w}; pq_{vw\,21}\ pq_{vw\,22} \cdots pq_{vw\,2w}; \ldots; pq_{vw\,v1}\ pq_{vw\,v2} \cdots pq_{vw\,vw} \end{bmatrix} \quad (147)$$

In the case when the processess $Z(t)$ and $C(t)$ are dependent the probabilities of transitions between the operation and climate-weather states existing in (147) can be express either by (Kołowrocki, 2019/2020)

$pq_{bl\,mn} = p_{bm} \cdot q_{ln|bm}$, $b = 1,2,\ldots,v$,

$l = 1,2,\ldots,w$, $m = 1,2,\ldots,v$, $n = 1,2,\ldots,w$, (148)

where

p_{bm}, $b = 1,2,\ldots,v$, $m = 1,2,\ldots,v$, (149)

are transient probabilities between the operation states of the operation process $Z(t)$ defined in Section 4 and

$q_{ln|bm}$, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, $m = 1,2,\ldots,v$,

$n = 1,2,\ldots,w$, (150)

are conditional transient probabilities between the climate-weather states of the climate-weather change process $C(t)$ coressponding to those defined in Section 5 in case they are not conditional or by (Kołowrocki, 2019/2020)

$pq_{bl\,mn} = p_{bm|ln} \cdot q_{ln}$, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$,

$m = 1,2,\ldots,v$, $n = 1,2,\ldots,w$, (151)

where

q_{ln}, $l = 1,2,\ldots,w$, $n = 1,2,\ldots,w$, (152)

are transient probabilities beetwen the climate-weather states of the climate-weather change process $C(t)$ defined in Section 5 and

$p_{bm|ln}$, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, $m = 1,2,\ldots,v$,

$n = 1,2,\ldots,w$, (153)

are conditional transient probabilities between the operation states of the operation process $Z(t)$ coressponding to those defined in Section 4 in case they are not conditional.

The matrix of conditional distribution functions

$HC_{bl\,mn}(t) = P(\theta C_{bl\,mn} < t)$, $t \geq 0$,

$b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, $m = 1,2,\ldots,v$,

$n = 1,2,\ldots,w$, (154)

of the critical infrastructure operation process related to climate-weather change process $ZC(t)$ conditional sojourn times $\theta C_{bl\,mn}$, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, $m = 1,2,\ldots,v$, $n = 1,2,\ldots,w$, at the operation and climate-weather state zc_{bl}, $b = 1,2,\ldots,v$, $m = 1,2,\ldots,v$, when the next operation and climate-weather state is zc_{ln}, $l = 1,2,\ldots,w$, $n = 1,2,\ldots,w$, takes the following form (Kołowrocki, 2019/2020)

$$[HC_{bl\,mn}(t)]_{vw\times vw} =$$

$$\begin{bmatrix} HC_{11\;11}(t)\; HC_{11\;12}(t)\;...\; HC_{11\;1w}(t); HC_{11\;21}(t) HC_{11\;22}(t) ... HC_{11\;2w}(t); ...; HC_{11\;v1}(t) HC_{11\;v2}(t) ... HC_{11\;vw}(t) \\ HC_{12\;11}\,(t) HC_{12\;12}(t)\;...\; HC_{12\;1w}(t); HC_{12\;21}(t) HC_{12\;22}(t) ... HC_{12\;2w}(t); ...; HC_{12\;v1}(t) HC_{12\;v2}(t) ... HC_{12\;vw}(t) \\ ... \\ HC_{vw\;11}(t)\; HC_{vw\;12}(t)\;...\; HC_{vw\;1w}(t); HC_{vw\;21}(t) HC_{vw\;22}(t) ... HC_{vw\;2w}(t); ...; HC_{vw\;v1}(t) HC_{vw\;v2}(t) ... HC_{vw\;vw}(t) \end{bmatrix} \quad (155)$$

and the matrix of their corresponding conditional density functions

$$hc_{bl\,mn}(t) = \frac{d}{dt}[HC_{bl\,mn}(t)],\ t \geq 0, \quad (156)$$

where $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, $m = 1,2,\ldots,v$, $n = 1,2,\ldots,w$, has form

$$[hc_{bl\,mn}(t)]_{vw\times vw}$$

$$= \begin{bmatrix} hc_{11\;11}(t)\; hc_{11\;12}(t)\;...\; hc_{11\;1w}(t); hc_{11\;21}(t) hc_{11\;22}(t) ... hc_{11\;2w}(t); ...; hc_{11\;v1}(t) hc_{11\;v2}(t) ... hc_{11\;vw}(t) \\ hc_{12\;11}\,(t) hc_{12\;12}(t)\;...\; hc_{12\;1w}(t); hc_{12\;21}(t) hc_{12\;22}(t) ... hc_{12\;2w}(t); ...; hc_{12\;v1}(t) hc_{12\;v2}(t) ... hc_{12\;vw}(t) \\ ... \\ hc_{vw\;11}(t)\; hc_{vw\;12}(t)\;...\; hc_{vw\;1w}(t); hc_{vw\;21}(t) hc_{vw\;22}(t) ... hc_{vw\;2w}(t); ...; hc_{vw\;v1}(t) hc_{vw\;v2}(t) ... hc_{vw\;vw}(t) \end{bmatrix}. \quad (157)$$

In the case when the critical infrastructure operation process $Z(t)$ and the climate-weather change process $C(t)$ at its operating area are dependent, the distribution functions existing in the matrix (155) can be expressed either by (Kołowrocki, 2019/2020)

$$HC_{bl\,mn}(t) = P(\theta C_{bl\,mn} < t)$$

$$= P((\theta_{bm} < t) \cap (C_{ln} < t))$$

$$= H_{bm}(t) C_{ln|bm}(t),\ t \geq 0,\ b = 1,2,\ldots,v,$$

$$l = 1,2,\ldots,w,\ m = 1,2,\ldots,v,\ n = 1,2,\ldots,w, \quad (158)$$

where

$$H_{bm}(t),\ t \geq 0,\ b = 1,2,\ldots,v,\ m = 1,2,\ldots,v, \quad (159)$$

are distribution functions of the sojourn lifetimes at the operation states of the critical infrastructure operation process $Z(t)$ defined in Section 4 and

$$C_{ln|bm}(t) = P(C_{ln} < t | \theta_{bm} < t),\ t \geq 0,$$

$$b = 1,2,\ldots,v,\ l = 1,2,\ldots,w,\ m = 1,2,\ldots,v,$$

$$n = 1,2,\ldots,w, \quad (160)$$

are conditional distributions of the sojourn lifetimes at the climate-weather states of the climate-weather change process $C(t)$ at the critical infrastructure operating area coressponding to those defined in Section 5 in case they are not conditional or by

$$HC_{bl\,mn}(t) = P(\theta C_{bl\,mn} < t)$$

$$= P((\theta_{bm} < t) \cap (C_{ln} < t))$$

$$= H_{bm|ln}(t) C_{ln}(t),\ t \in \langle 0,\infty),\ t \geq 0,$$

$$b = 1,2,\ldots,v,\ l = 1,2,\ldots,w,\ m = 1,2,\ldots,v,$$

$$n = 1,2,\ldots,w, \quad (161)$$

where

$$C_{ln}(t),\ l = 1,2,\ldots,w,\ n = 1,2,\ldots,w \quad (162)$$

are distribution functions of the sojourn lifetimes at the climate-weather states of the climate-weather change process $C(t)$ at the critical infrastructure operating area defined in Section 5 and

$$H_{bm|ln}(t) = P(\theta_{bm} < t | C_{ln} < t),\ t \geq 0,$$

$$b = 1,2,\ldots,v,\ l = 1,2,\ldots,w,\ m = 1,2,\ldots,v,$$

$$n = 1,2,\ldots,w, \quad (163)$$

are conditional distributions of the sojourn lifetimes at the operation states of the critical infrastructure operation process $Z(t)$ coresponding

to those defined in Section 4 in case they are not conditional.
Hence, the density functions existing in the matrix (157) can be expressed either by (Kołowrocki, 2019/2020)

$$hc_{bl\,mn}(t) = \frac{d}{dt}[HC_{bl\,mn}(t)]$$

$$= \frac{d}{dt}\left[H_{bm}(t)C_{ln|bm}(t)\right]$$

$$= h_{bm}(t)C_{ln|bm}(t) + H_{bm}(t)c_{ln|bm}(t),$$

$t \geq 0$, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, $m = 1,2,\ldots,v$,

$n = 1,2,\ldots,w$, (164)

where

$H_{bm}(t)$, $t \geq 0$, $b = 1,2,\ldots,v$, $m = 1,2,\ldots,v$, (165)

are conditional distribution functions given by (159) and

$C_{ln|bm}(t)$, $t \geq 0$, $l = 1,2,\ldots,w$, $n = 1,2,\ldots,w$,(166)

are conditional distribution functions given by (160) and

$h_{bm}(t), t \geq 0$, $b = 1,2,\ldots,v$, $m = 1,2,\ldots,v$,

and

$c_{ln|bm}(t)$, $t \geq 0$, $l = 1,2,\ldots,w$, $n = 1,2,\ldots,w$,

are their derivatives or by

$$hc_{bl\,mn}(t) = \frac{d}{dt}[HC_{bl\,mn}(t)]$$

$$= \frac{d}{dt}\left[H_{bm|ln}(t)C_{ln}(t)\right]$$

$$= h_{bm|ln}(t)C_{ln}(t) + H_{bm|ln}(t)c_{ln}(t),$$

$t \geq 0$, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, $m = 1,2,\ldots,v$,

$n = 1,2,\ldots,w$, (167)

where

$C_{ln}(t)$, $t \geq 0$, $l = 1,2,\ldots,w$, $n = 1,2,\ldots,w$, (168)

are distribution functions given by (162) and

$H_{bm|ln}(t)$, $b = 1,2,\ldots,v$, $m = 1,2,\ldots,v$, (169)

are conditional distribution functions given by (163) and

$c_{ln}(t)$, $t \geq 0$, $l = 1,2,\ldots,w$, $n = 1,2,\ldots,w$,

and

$h_{bm|ln}(t)$, $t \geq 0$, $b = 1,2,\ldots,v$, $m = 1,2,\ldots,v$, (170)

are their derivatives.
We assume that the suitable and typical distributions to describe the critical infrastructure operation process related to climate-weather change process $ZC(t)$, $t \geq 0$, conditional sojourn times $\theta C_{bl\,mn}$, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, $m = 1,2,\ldots,v$, $n = 1,2,\ldots,w$, at the particular operation and climate-weather states are of the same kind as those distinguished for the critical infrastructure operation process $Z(t)$ conditional sojourn times θ_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$.

6.2. Critical infrastructure operation process related to climate-weather change process

Assuming that we have identified the unknown parameters of the critical infrastructure operation process related to climate-weather change process $ZC(t)$, $t \in \langle 0,\infty)$ that can take vw, $v, w \in N$, different operation and climate-weather states $zc_{11}, zc_{12}, \ldots, zc_{vw}$, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, defined in Section 6.1.1 and Section 6.1.2 and described by:

- the vector $[pq_{bl}(0)]_{1\times vw}$ of initial probabilities of the critical infrastructure operation process related to climate-weather change process $ZC(t)$ staying at the initial moment $t = 0$ at the operation and climate-weather states zc_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$,
- the matrix $[pq_{bl\,mn}(0)]_{vw\times vw}$ of the probabilities of transitions of the critical infrastructure operation process related to climate-weather change process $ZC(t)$ between the operation and climate-weather states zc_{bl}, and zc_{mn}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, $m = 1,2,\ldots,v$, $n = 1,2,\ldots,w$,

- the matrix $[HC_{bl\,mn}(0)]_{vw\times vw}$ of conditional distribution functions of the critical infrastructure operation process related to climate-weather change process $ZC(t)$, $t \geq 0$, conditional sojourn times $\theta C_{bl\,mn}$, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, $m = 1,2,\ldots,v$, $n = 1,2,\ldots,w$, at the operation and climate-weather state zc_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, when the next operation and climate-weather state is zc_{mn}, $m = 1,2,\ldots,v$, $n = 1,2,\ldots,w$,

we can predict this process basic characteristics.

6.2.1. Critical infrastructure operation process related to climate-weather change process characteristics in case of independent critical infrastructure operation process and climate-weather change process

The mean values of the conditional sojourn times $\theta C_{bl\,mn}$, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, $m = 1,2,\ldots,v$, $n = 1,2,\ldots,w$, at the operation and climate-weather state zc_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, when the next operation and climate-weather state is zc_{mn}, $m = 1,2,\ldots,v$, $n = 1,2,\ldots,w$, are defined by (Holden et al., 2013; Kołowrocki, 2019/2020; Kołowrocki & Kuligowska, 2018)

$$MN_{bl\,mn} = E[\theta C_{bl\,mn}] = \int_0^\infty t dHC_{bl\,mn}(t)$$

$$= \int_0^\infty t hc_{bl\,mn}(t)dt, \; b = 1,2,\ldots,v,$$

$$l = 1,2,\ldots,w, \; m = 1,2,\ldots,v, \; n = 1,2,\ldots,w, \quad (171)$$

where $HC_{bl\,mn}(t)$, $t \geq 0$, and $hc_{bl\,mn}(t)$, $t \geq 0$, are respectively defined by (125) and (126).
In the case when the processess $Z(t)$ and $C(t)$ are independent, according to (129), the expessions (171) takes the form (Kołowrocki, 2019/2020)

$$MN_{bl\,mn} = E[\theta C_{bl\,mn}]$$

$$= \int_0^\infty t[h_{bm}(t)C_{ln}(t) + H_{bm}(t)c_{ln}(t)]dt,$$

$$b = 1,2,\ldots,v, \; l = 1,2,\ldots,w, \; m = 1,2,\ldots,v,$$

$$n = 1,2,\ldots,w, \quad (172)$$

where the distribution functions $H_{bm}(t)$, $t \geq 0$, and $C_{ln}(t)$, $t \geq 0$, are defined by (130) and the density functions $h_{bm}(t)$, $c_{ln}(t)$, $t \geq 0$, are defined by (131).
From the formula for total probability, it follows that the unconditional distribution functions of the conditional sojourn times θC_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, of the critical infrastructure operation process related to climate-weather change process $ZC(t)$ at the operation and climate-weather states state zc_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, are given by

$$HC_{bl}(t) = \sum_{m=1}^{v}\sum_{n=1}^{w} pq_{bl\,mn}HC_{bl\,mn}(t),$$

$$t \geq 0, \; b = 1,2,\ldots,v, \; l = 1,2,\ldots,w, \quad (173)$$

where the probabilities of transitions between operation and climate-weather states are given by (118)–(119) and the distributions $HC_{bl\,mn}(t)$, $t \geq 0$, are defined by (124).
In the case when the processes $Z(t)$ and $C(t)$ are independent, according to (121)–(122) and (128) the expressions (173) takes the form (Kołowrocki, 2019/2020)

$$HC_{bl}(t) = \sum_{m=1}^{v}\sum_{n=1}^{w} p_{bm}q_{ln}H_{bm}(t)C_{ln}(t),$$

$$t \geq 0, \; b = 1,2,\ldots,v, \; l = 1,2,\ldots,w, \quad (174)$$

where the probabilities of transitions p_{bm}, q_{ln} are defined by (122) and the distribution functions $H_{bm}(t)$ and $C_{ln}(t)$ are defined by (130).
From (173) it follows that the mean values $E[\theta C_{bl}]$ of the unconditional distribution functions of the conditional sojourn times θC_{bl}, of the critical infrastructure operation process related to climate-weather change process $ZC(t)$, $t \geq 0$, at the operation and climate-weather states zc_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, are given by

$$MN_{bl}(t) = E[\theta C_{bl}]$$

$$= \sum_{m=1}^{v}\sum_{n=1}^{w} pq_{bl\,mn}MN_{bl\,mn}(t),$$

$$b = 1,2,\ldots,v, \; l = 1,2,\ldots,w, \quad (175)$$

where MN_{bl} are given by the formula (171).
In the case when the processess $Z(t)$ and $C(t)$ are independent, considering (121) and (174) the expession (175) takes the form (Kołowrocki, 2019/2020)

$$MN_{bl}(t) = E[\theta C_{bl}]$$

$$= \sum_{m=1}^{v}\sum_{n=1}^{w} p_{bm}q_{ln}MN_{bl\,mn}(t), \; t \geq 0,$$

$b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, (176)

where the probabilities of transitions p_{bm}, q_{ln} are defined by (122) and the mean values MN_{bl}, are given by the formula (172).

The transient probabilities of the critical infrastructure operation process related to climate-weather change process $ZC(t),\ t \geq 0$, at the operation and climate-weather states zc_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, can be defined by

$$pq_{bl}(t) = P(ZC(t) = zc_{bl}), t \geq 0,$$

$$b = 1,2,\ldots,v,\ l = 1,2,\ldots,w. \quad (177)$$

In the case when the processess $Z(t)$ and $C(t)$ are independent the expession (177) for the transient probabilities can be expressed in the following way (Kołowrocki, 2019/2020)

$$pq_{bl}(t) = P(ZC(t) = zc_{bl})$$

$$= P(Z(t) = z_b \cap C(t) = c_l)$$

$$= P(Z(t) = z_b) \cdot P(C(t) = c_l) = p_b(t) \cdot q_l(t),$$

$$t \geq 0, b = 1,2,\ldots,v,\ l = 1,2,\ldots,w, \quad (178)$$

$$p_b(t) = P(Z(t) = z_b), t \geq 0,$$

$$b = 1,2,\ldots,v, \quad (179)$$

are the transient probabilities at the operation states of the operation process $Z(t)$ defined in Section 4 and

$$q_l(t) = P(C(t) = c_l), t \geq 0, l = 1,2,\ldots,w, \quad (180)$$

are the transient probabilities at the climate-weather states of the climate-weather change process $C(t)$ defined in Section 5.

The limit values of transient probabilities $P(ZC(t) = zc_{bl})$, $t \geq 0$, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, of the critical infrastructure operation process related to climate-weather change process $ZC(t)$ at the operation and climate-weather states zc_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, can be found from the formula (Kołowrocki, 2019/2020)

$$pq_{bl} = \lim_{t\to\infty} P(ZC(t) = zc_{bl})$$

$$= \frac{\pi_{bl} MN_{bl}}{\sum_{m=1}^{v}\sum_{n=1}^{w}\pi_{mn} MN_{mn}}, t \geq 0,$$

$$b = 1,2,\ldots,v,\ l = 1,2,\ldots,w, \quad (181)$$

where MN_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, are given by (176), while the steady probabilities π_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, of the vector $[\pi_{bl}]_{1\times vw}$ satisfy the system of equations

$$\begin{cases} [\pi_{bl}] = [\pi_{bn}][p_{bl\,mn}] \\ \sum_{b=1}^{v}\sum_{l=1}^{w}\pi_{bl} = 1, \end{cases} \quad (182)$$

where pq_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, $m = 1,2,\ldots,v$, $n = 1,2,\ldots,w$, are given by (118)–(119).

In the case of a periodic system operation process, the limit transient probabilities pq_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, at the operation and climate-weather states given by (181), are the long term proportions of the critical infrastructure operation process related to climate-weather change process $ZC_{bl}(t)$ sojourn times at the particular operation and climate-weather states zc_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$.

Other interesting characteristics of the critical infrastructure operation process related to climate-weather change process $ZC_{bl}(t)$ possible to obtain are its total sojourn times $\widehat{\theta C}_{bl}$, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,$w, at the particular operation and climate-weather states zc_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, during the fixed system opetation time. It is well known (Holden et al., 2013) that the system operation process related to climate-weather change process total sojourn times $\widehat{\theta C}_{bl}$, at the particular operation and climate-weather states zc_{bl}, for sufficiently large operation time θ, have approximately normal distributions with the mean values given by (Kołowrocki, 2019/2020)

$$\widehat{MN}_{bl} = E[\widehat{\theta C}_{bl}] = pq_{bl}\theta,$$

$$b = 1,2,\ldots,v,\ l = 1,2,\ldots,w, \quad (183)$$

where pq_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, are given by (181).

From (178) it follows that in case of the independent processes $Z(t)$ and $C(t)$ the formula (181) for limit values of transient probabilities takes simpler following form

$$pq_{bl} = p_b \cdot q_l, b = 1,2,\ldots,v,\ l = 1,2,\ldots,w, \quad (184)$$

where p_b, $b = 1,2,\ldots,v$, are the limit values of the transient probabilities $P(Z(t) = z_b)$, $b = 1,2,\ldots,v$, of the operation process $Z(t)$, $t \geq 0$, defined in Section 4 by (34) and q_l, $l = 1,2,\ldots,w$, are the limit values of the transient probabilities $P(C(t) = q_l)$, $l = 1,2,\ldots,w$, of the climate-weather change process $C(t)$, $t \geq 0$, defined in Section 5 by (75) and consequently the formula for total sojourn times $\widehat{\theta C}_{bl}$, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, at the particular operation and climate-weather states zc_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, for sufficiently large operation time θ, takes simplified following form (Kołowrocki, 2019/2020)

$$\widehat{MN}_{bl} = E[\widehat{\theta C}_{bl}] = p_b \cdot q_l \cdot \theta,$$

$$b = 1,2,\ldots,v,\ l = 1,2,\ldots,w, \quad (185)$$

where p_b, $b = 1,2,\ldots,v$, q_l , $l = 1,2,\ldots,w$, are respectively given by (34) in Section 4 and by (75) in Section 5.

6.2.2. Critical infrastructure operation process related to climate-weather change process characteristics in case of dependent critical infrastructure operation process and climate-weather change process

The mean values of the conditional sojourn times $\theta C_{bl\,mn}$, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, $m = 1,2,\ldots,v$, $n = 1,2,\ldots,w$, at the operation and climate-weather state zc_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, when the next operation and climate-weather state is zc_{mn}, $m = 1,2,\ldots,v$, $n = 1,2,\ldots,w$, are defined by (Kołowrocki, 2019/2020))

$$MN_{bl\,mn} = E[\theta C_{bl\,mn}] = \int_0^\infty t dHC_{bl\,mn}(t)$$

$$= \int_0^\infty t hc_{bl\,mn}(t)dt,$$

$$b = 1,2,\ldots,v,\ l = 1,2,\ldots,w,\ m = 1,2,\ldots,v,$$

$$n = 1,2,\ldots,w, \quad (186)$$

where $HC_{bl\,mn}(t)$, $t \geq 0$, and $hc_{bl\,mn}(t)$, $t \geq 0$, are respectively defined by (125) and (126).
Since from the formula for total probability, it follows that the unconditional distribution functions of the conditional sojourn times θC_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, of the critical infrastructure operation process related to climate-weather change process $ZC(t), t \geq 0$, at the operation and climate-weather state zc_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, are given by (Kołowrocki, 2019/2020)

$$HC_{bl}(t) = \sum_{m=1}^{v}\sum_{n=1}^{w} pq_{bl\,mn} HC_{bl\,mn}(t),$$

$$t \geq 0,\ b = 1,2,\ldots,v,\ l = 1,2,\ldots,w. \quad (187)$$

Then, the mean values $E[\theta C_{bl}]$ of the unconditional distribution functions of the conditional sojourn times θC_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, of the critical infrastructure operation process related to climate-weather change process $ZC(t)$, $t \geq 0$, at the operation and climate-weather states zc_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, are given by (Kołowrocki, 2019/2020)

$$MN_{bl} = E[\theta C_{bl}] = \sum_{m=1}^{v}\sum_{n=1}^{w} pq_{bl\,mn} MN_{bl\,mn},$$

$$b = 1,2,\ldots,v,\ l = 1,2,\ldots,w, \quad (188)$$

where probabilities of transitions between the operation and climate-weather states are defined by (148) and the mean values MN_{bl} are defined by the formula (186).
The transient probabilities of the critical infrastructure operation process related to climate-weather change process $ZC(t)$, $t \geq 0$, at the operation and climate-weather states zc_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, can be defined by (Kołowrocki, 2019/2020)

$$pq_{bl}(t) = P(ZC(t) = zc_{bl}), t \geq 0,$$

$$b = 1,2,\ldots,v,\ l = 1,2,\ldots,w. \quad (189)$$

In the case when the processess $Z(t)$ and $C(t)$ are dependent the transient probabilities can be expresed either by (Kołowrocki, 2019/2020)

$$pq_{bl}(t) = P(ZC(t) = zc_{bl})$$

$$= P(Z(t) = z_b \cap C(t) = c_l)$$

$$= P(Z(t) = z_b) \cdot P(C(t) = c_l | Z(t) = z_b)$$

$$= p_b(t) \cdot q_{l|b}(t), t \geq 0,$$

$$b = 1,2,\ldots,v,\ l = 1,2,\ldots,w, \quad (190)$$

where

$p_b(t) = P(Z(t) = z_b)$, $t \in \langle 0,\infty)$ $b = 1,2,\ldots,v$, (191)

are transient probabilities at the operation state of the operation process $Z(t)$ defined in Section 4 and

$q_{l|b}(t) = P(C(t) = c_l | Z(t) = z_b)$, $t \geq 0$,

$b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, (192)

are conditional transient probabilities at the climate-weather states of the climate-weather change process $C(t)$ corresponding to those defined in Section 5 in case they are not conditional or by

$pq_{bl}(t) = P(ZC(t) = zc_{bl})$

$= P(Z(t) = z_b \cap C(t) = c_l)$

$= P(Z(t) = z_b | C(t) = c_l) \cdot P(C(t) = c_l)$

$= p_{b|l}(t) \cdot q_l(t)$, $t \geq 0$,

$b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, (193)

where

$q_l(t) = P(C(t) = c_l)$, $t \geq 0$, $l = 1,2,\ldots,w$, (194)

are transient probabilities at the climate-weather state of the climate-weather change process $C(t)$ defined in Section 5 and

$p_{b|l}(t) = P(Z(t) = z_b | C(t) = c_l)$, $t \geq 0$,

$b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, (195)

are conditional transient probabilities at operation state of the operation proces process $Z(t)$ corresponding to those defined in Section 4 in case they are not conditional.

The limit values of the critical infrastructure operation process related to climate-weather change process $ZC(t)$, $t \geq 0$, at the operation and climate-weather state zc_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, can be found from (Kołowrocki, 2019/2020; Kołowrocki et al., 2018)

$$pq_{bl} = \lim_{t\to\infty} P(ZC(t) = zc_{bl})$$

$$= \frac{\pi_{bl} MN_{bl}}{\sum_{m=1}^{v}\sum_{n=1}^{w}\pi_{mn} MN_{mn}},$$

$b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, (196)

where MN_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, are given by (188), while the steady probabilities π_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, of the vector $[\pi_{bl}]_{1\times vw}$ satisfy the system of equations

$$\begin{cases}[\pi_{bl}] = [\pi_{bn}][p_{bl\,mn}]\\ \sum_{b=1}^{v}\sum_{l=1}^{w}\pi_{bl} = 1,\end{cases}$$

where $pq_{bl\,mn}$, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, $m = 1,2,\ldots,v$, $n = 1,2,\ldots,w$, are given by (118)–(119).

In the case of a periodic system operation process, the limit transient probabilities pq_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, at the operation and climate-weather states given by (196), are the long term proportions of the critical infrastructure operation process related to climate-weather change process $ZC_{bl}(t)$ sojourn times at the particular operation and climate-weather states zc_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$.

Other interesting characteristics of the critical infrastructure operation process related to climate-weather change $ZC_{bl}(t)$, $t \geq 0$, possible to obtain are its total sojourn times $\widehat{\theta C}_{bl}$, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, at the particular operation and climate-weather states zc_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, during the fixed system opetation time. It is well known that the system operation process related to climate-weather change process total sojourn times $\widehat{\theta C}_{bl}$, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, at the particular operation and climate-weather states zc_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, for sufficiently large operation time θ, have approximately normal distributions with the expected value given by (Kołowrocki, 2019/2020)

$\widehat{MN}_{bl} = E[\widehat{\theta C}_{bl}] = pq_{bl}\theta$, $b = 1,2,\ldots,v$,

$l = 1,2,\ldots,w$, (197)

where pq_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, are given by (196).

6.3. Safety and resilience indicators of critical infrastructure impacted by operation process related to climate-weather change process

We denote by $[T^3(u)]^{(bl)}$, $u = 1,2,...,z$, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, the critical infrastructure conditional lifetime in the safety state subset $\{u, u+1,\ldots,z\}$, $u = 1,2,...,z$, while its operation process related to the climate-weather change process $ZC(t)$, $t \geq 0$, is at the operation and climate-weather state zc_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, and we introduce the conditional safety function of the critical infrastructure impacted by the operation process related to the climate-weather change process $ZC(t)$, $t \geq 0$, defined by the vector (Kołowrocki, 2019/2020; Kołowrocki et al., 2018)

$$[\boldsymbol{S}^3(t,\cdot)]^{(bl)} = [[\boldsymbol{S}^3(t,1)]^{(bl)},..., [\boldsymbol{S}^3(t,z)]^{(bl)}],$$

$$t \in \langle 0,\infty), b = 1,2,\ldots,v, l = 1,2,\ldots,w, \quad (198)$$

with the coordinates

$$[\boldsymbol{S}^3(t,u)]^{(bl)} = P([T^3(u)]^{bl} > t | ZC(t) = zc_{bl}),$$

$$t \geq 0, u = 1,2,...,z, b = 1,2,\ldots,v, l = 1,2,\ldots,w. \quad (199)$$

The safety function $[\boldsymbol{S}^3(t,u)]^{(bl)}$, $t \geq 0$, $u = 1,2,...,z$, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, is the conditional probability that the critical infrastructure impacted by the operation process related to the climate-weather change process $ZC(t)$, $t \geq 0$, lifetime $[T^3(u)]^{bl}$, $u = 1,2,\ldots,z$, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, in the safety state subset $\{u, u+1,\ldots,z\}$, $u = 1,2,...,z$, is greater than t, while the operation process related to the climate-weather change process $ZC(t)$, $t \geq 0$, is at the operation and climate-weather state zc_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$.

Next, we denote by $T^3(u)$, $u = 1,2,\ldots,z$, the critical infrastructure impacted by the operation process related to the climate-weather change process $ZC(t)$, $t \geq 0$, unconditional lifetime in the safety state subset $\{u, u+1,\ldots,z\}$, $u = 1,2,...,z$, and the unconditional safety function (SafI1) of the critical infrastructure impacted by the operation process related to the climate-weather change process $ZC(t)$, $t \geq 0$, by the vector

$$\boldsymbol{S}^3(t,\cdot) = [\boldsymbol{S}^3(t,1),..., \boldsymbol{S}^3(t,z)], t \geq 0, \quad (200)$$

with the coordinates defined by

$$\boldsymbol{S}^3(t,u) = P(T^3(u) > t) \text{ for } t \geq 0,$$

$$u = 1,2,...,z. \quad (201)$$

In the case when the critical infrastructure operation time θ is large enough, the coordinates of the unconditional safety function of the critical infrastructure impacted by the operation process related to the climate-weather change process $ZC(t)$, $t \geq 0$, defined by (201), are given by (Kołowrocki, 2019/2020; Kołowrocki et al., 2018)

$$\boldsymbol{S}^3(t,u) \cong \sum_{b=1}^{v} \sum_{l=1}^{w} pq_{bl} [\boldsymbol{S}^3(t,u)]^{(bl)},$$

$$t \geq 0, u = 1,2,...,z, \quad (202)$$

where

$$[\boldsymbol{S}^3(t,u)]^{(bl)}, t \geq 0, u = 1,2,...,z, b = 1,2,\ldots,v,$$

$$l = 1,2,\ldots,w,$$

are the coordinates of the critical infrastructure impacted by the operation process related to the climate-weather change process $ZC(t)$, $t \geq 0$, conditional safety functions defined by (198)–(199) and pq_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, are the operation process related to the climate-weather change process $ZC(t)$, $t \geq 0$, at the critical infrastructure operating area limit transient probabilities at the operation and climate-weather states zc_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, given either by (181) or (196).

If r is the critical safety state, then the second safety indicator of the critical infrastructure impacted by the operation process related to the climate-weather change process $ZC(t)$, $t \geq 0$, the risk function (SafI2)

$$\boldsymbol{r}^3(t) = P(s(t) < r \mid s(0) = z) = P(T^3(r) \leq t), t \geq 0, \quad (203)$$

is defined as a probability that the critical infrastructure impacted by the operation process related to the climate-weather change process $ZC(t)$, $t \in \langle 0,\infty)$ is in the subset of safety states worse than the critical safety state r, $r \in \{1,...,z\}$ while it was in the best safety state z at the moment $t = 0$

and given by (Kołowrocki, 2019/2020; Kołowrocki et al., 2018)

$$\boldsymbol{r}^3(t) = 1 - \boldsymbol{S}^3(t,r),\ t \in \langle 0,\infty),\tag{204}$$

where $\boldsymbol{S}^3(t,r)$ is the coordinate of the critical infrastructure impacted by the operation process related to the climate-weather change process $ZC(t)$, $t \geq 0$, unconditional safety function given by (202) for $u = r$.

The graph of the critical infrastructure risk function $\boldsymbol{r}^3$(t), $t \geq 0$, defined by (204), is the safety indicator called the fragility curve (SafI3) of the critical infrastructure impacted by the operation process related to the climate-weather change process $ZC(t)$, $t \in \langle 0,\infty)$,

Other practically useful safety indicators of the critical infrastructure impacted by the operation process related to the climate-weather change process $ZC(t)$, $t \geq 0$, are (Kołowrocki, 2019/2020):

- the mean value of the critical infrastructure unconditional lifetime $T^3(r)$ up to exceeding critical safety state r (SafI4) given by

$$\boldsymbol{\mu}^3(r) = \int_0^\infty \boldsymbol{S}^3(t,r)dt$$

$$\cong \sum_{b=1}^{v}\sum_{l=1}^{w} pq_{bl}[\boldsymbol{\mu}^3(r)]^{(bl)},\tag{205}$$

where $[\boldsymbol{\mu}^3(r)]^{bl}$, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, are the mean values of the critical infrastructure conditional lifetimes $[T^3(r)]^{(bl)}$, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, in the safety state subset $\{r, r+1,\ldots,z\}$ at the operation and climate-weather state zc_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, of the operation process related to the climate-weather change process $ZC(t)$, $t \geq 0$, given by

$$[\boldsymbol{\mu}^3(r)]^{(bl)} = \int_0^\infty [\boldsymbol{S}^3(t,r)]^{(bl)}dt,$$

$$b = 1,2,\ldots,v,\ l = 1,2,\ldots,w,\tag{206}$$

and

$$[\boldsymbol{S}^3(t,r)]^{(bl)},\ t \geq 0,\ b = 1,2,\ldots,v,\ l = 1,2,\ldots,w,$$

are defined by (198)–(199) and pq_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, are given either by (181) or by (196),

- the standard deviation of the critical infrastructure lifetime $T^3(r)$ up to the exceeding the critical safety state r (SafI5) given by

$$\boldsymbol{\sigma}^3(r) = \sqrt{\boldsymbol{n}^3(r) - [\boldsymbol{\mu}^3(r)]^2},\tag{207}$$

where

$$\boldsymbol{n}^3(r) = 2\int_0^\infty t\boldsymbol{S}3(t,r)dt,\tag{208}$$

and $\boldsymbol{S}^3(t,r)$, $t \geq 0$, is defined by (201) for $u = r$, and $\boldsymbol{\mu}^3(r)$ is given by (205),

- the moment $\boldsymbol{\tau}^3$, of exceeding acceptable value of critical infrastructure risk function level d (SafI6) given by

$$\boldsymbol{\tau}^3 = (\boldsymbol{r}^3)^{-1}(\delta),\tag{209}$$

where $(\boldsymbol{r}^3)^{-1}(t)$, $t \geq 0$, is the inverse function of the risk function $\boldsymbol{r}^3$(t) given by (203),

- the mean lifetime of the critical infrastructure in the safety state subsets $\{u, u+1,\ldots,z\}$, $u = 1,2,...,z$, (SafI7), given by

$$\boldsymbol{\mu}^3(u) = \int_0^\infty [\boldsymbol{S}^3(t,u)]dt$$

$$\cong \sum_{b=1}^{v}\sum_{l=1}^{w} p\,q_{bl}[\boldsymbol{\mu}^3(u)]^{(bl)},$$

$$u = 1,2,\ldots,z\tag{210}$$

where $[\boldsymbol{\mu}^3(u)]^{(bl)}$, $u = 1,2,\ldots,z$, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, are the mean values of the critical infrastructure conditional lifetimes $[T^3(u)]^{(bl)}$, $u = 1,2,\ldots,z$, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, in the safety state subset $\{u, u+1,\ldots,z\}$, at the critical infrastructure operation process related to the climate-weather change process state zc_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, given by

$$[\boldsymbol{\mu}^3(u)]^{(bl)} = \int_0^\infty [\boldsymbol{S}^3(t,u)]^{(bl)}dt,$$

$$u = 1,2,\ldots,z,\ b = 1,2,\ldots,v,\ l = 1,2,\ldots,w,\tag{211}$$

and $[\boldsymbol{S}^3(t,u)]^{(bl)}$, $u = 1,2,\ldots,z$, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, are defined by (198)–(199) and pq_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, are given by (181) or by (196),

- the standard deviations of the critical infrastructure lifetimes in the safety state subsets $\{u, u+1,\ldots,z\}$, $u = 1,2,...,z$, (SafI8), given by

$$\boldsymbol{\sigma}^3(u) = \sqrt{\boldsymbol{n}^3(u) - [\boldsymbol{\mu}^3(u)]^2},\ u = 1,2,...,z,\tag{212}$$

where

$$\boldsymbol{n}^3(u) = 2\int_0^\infty t\boldsymbol{S}^3(t,u)dt,\ u = 1,2,...,z, \quad (213)$$

- the mean lifetimes $\bar{\boldsymbol{\mu}}^3(u)$, $u = 1,2,...,z$, of the critical infrastructure in the particular safety states (SafI9)

$$\bar{\boldsymbol{\mu}}^3(u) = \boldsymbol{\mu}^3(u) - \boldsymbol{\mu}^3(u+1),$$

$$u = 0,1,\ldots,z-1,\ \bar{\boldsymbol{\mu}}^3(z) = \boldsymbol{\mu}^3(z), \quad (214)$$

- the intensities of degradation of the critical infrastructure / the intensities of critical infrastructure departure from the safety state subset $\{u, u+1,\ldots,z\}$, $u = 1,2,...,z$, (SafI10), i.e. the coordinates of the vector

$$\boldsymbol{\lambda}^3(t,\cdot) = [\boldsymbol{\lambda}^3(t,1),\ \ldots,\ \boldsymbol{\lambda}^3(t,z)],\ t \ge 0, \quad (215)$$

where

$$\boldsymbol{\lambda}^3(t,u) = -\frac{d\boldsymbol{S}^3(t,u)}{dt}\cdot\frac{1}{\boldsymbol{S}^3(t,u)},$$

$$t \ge 0,\ u = 1,2,...,z, \quad (216)$$

- the coefficients of the operation process related to the climate-weather change process impact on the critical infrastructure intensities of degradation / the coefficients of the operation process related to the climate-weather change process impact on critical infrastructure intensities of departure from the safety state subset $\{u, u+1,\ldots,z\}$ (ResI1), i.e. the coordinates of the vector

$$\boldsymbol{\rho}^3(t,\cdot) = [\boldsymbol{\rho}^3(t,1),\ \ldots,\ \boldsymbol{\rho}^3(t,z)],\ t \ge 0, \quad (217)$$

where

$$\boldsymbol{\lambda}^3(t,u) = \boldsymbol{\rho}^3(t,u)\cdot\boldsymbol{\lambda}^0(t,u),\ t \ge 0,$$

$$u = 1,2,...,z, \quad (218)$$

i.e.

$$\boldsymbol{\rho}^3(t,u) = \frac{\boldsymbol{\lambda}^3(t,u)}{\boldsymbol{\lambda}^0(t,u)},\ t \ge 0,\ u = 1,2,...,z, \quad (219)$$

and $\boldsymbol{\lambda}^0(t,u)$, $t \ge 0$, $u = 1,2,...,z$, defined by (21), are the intensities of degradation of the critical infrastructure without of the operation process related to the climate-weather change process impact, i.e. the coordinate of the vector

$$\boldsymbol{\lambda}^0(t,\cdot) = [\boldsymbol{\lambda}^0(t,1),\ \ldots,\boldsymbol{\lambda}^0(t,z)],\ t \ge 0, \quad (220)$$

and $\boldsymbol{\lambda}^3(t,u)$, $t \in \langle 0,\infty)$, $u = 1,2,...,z$, defined by (216), are the intensities of degradation of the critical infrastructure with of the operation process related to the climate-weather change process impact, i.e. the coordinate of the vector

$$\boldsymbol{\lambda}^3(t,\cdot) = [\boldsymbol{\lambda}^3(t,1),\ \ldots,\ \boldsymbol{\lambda}^3(t,z)],\ t \ge 0, \quad (221)$$

- the indicator of critical infrastructure resilience to operation process related to climate-weather change process impact (ResI2) defined by

$$\boldsymbol{RI}^3(t,r) = \frac{1}{\boldsymbol{\rho}^3(t,r)},\ t \ge 0, \quad (222)$$

where $\boldsymbol{\rho}^3(t,r)$, $t \ge 0$, is the coefficients of operation process related to the climate-weather change process impact on the critical infrastructure intensities of degradation given by (219) for $u = r$.

In the case, the critical infrastructure have the piecewise exponential safety functions, i.e.

$$\boldsymbol{S}^3(t,\cdot) = [\boldsymbol{S}^3(t,1),\ \ldots,\boldsymbol{S}^3(t,z)],\ t \ge 0, \quad (223)$$

where

$$\boldsymbol{S}^3(t,u) = \exp[-\boldsymbol{\lambda}^3(u)t],\ t \ge 0,$$

$$\boldsymbol{\lambda}^3(u) \ge 0,\ u = 1,2,\ldots,z, \quad (224)$$

the critical infrastructure safety indicators defined by (215)–(222) take forms (Kołowrocki, 2019/2020):

- the intensities of degradation of the critical infrastructure related to the operation process related to climate-weather change process impact, i.e. the coordinates of the vector
-

$$\boldsymbol{\lambda}^3(\cdot) = [\boldsymbol{\lambda}^3(1),\ \ldots,\ \boldsymbol{\lambda}^3(z)], \quad (225)$$

are constant and

$$\boldsymbol{\lambda}^3(u) = \frac{1}{\boldsymbol{\mu}^3(u)},\ u = 1,2,...,z, \quad (226)$$

where $\boldsymbol{\mu}^3(u)$ are given by (210),

- the coefficients of the operation process related to the climate-weather change process impact on the critical infrastructure intensities of degradation / the coefficients of the operation process related to the climate-weather change process impact on critical infrastructure intensities of departure from the safety state subset $\{u, u+1, \ldots, z\}$ (ResII) i.e. the coordinates of the vector

$$\boldsymbol{\rho}^3(\cdot) = [\boldsymbol{\rho}^3(1), \ldots, \boldsymbol{\rho}^3(z)], \tag{227}$$

where

$$\boldsymbol{\rho}^3(u) = \frac{\boldsymbol{\lambda}^3(u)}{\boldsymbol{\lambda}^0(u)} = \frac{\boldsymbol{\mu}^0(u)}{\boldsymbol{\mu}^3(u)}, u = 1,2,...,z, \tag{228}$$

and $\boldsymbol{\lambda}^0(u)$, $u = 1,2,...,z$, defined by (25) are the intensities of degradation of the critical infrastructure without of the operation process related to the climate-weather change process impact, i.e. the coordinate of the vector

$$\boldsymbol{\lambda}^0(\cdot) = [\boldsymbol{\lambda}^0(1), \ldots, \boldsymbol{\lambda}^0(z)], \tag{229}$$

and $\boldsymbol{\lambda}^3(u)$, $u = 1,2,...,z$, defined by (126) are the intensities of degradation of the critical infrastructure related to the operation process and the climate-weather change process impact, i.e. the coordinates of the vector

$$\boldsymbol{\lambda}^3(\cdot) = [\boldsymbol{\lambda}^3(1), \ldots, \boldsymbol{\lambda}^3(z)], \tag{230}$$

- the indicator of critical infrastructure resilience to the operation process related to the climate-weather change process impact (ResI2) defined by

$$\boldsymbol{RI}^3(r) = \frac{1}{\boldsymbol{\rho}^3(r)}, \tag{231}$$

where $\boldsymbol{\rho}^3(r)$ is the coefficient of the operation process related to the climate-weather-change process impact on the critical infrastructure intensities of degradation given by (228) for $u = r$.

The assets safety parameters of the critical infrastructure impacted by the operation process and the climate-weather change process can by introduced in an analogous way (Kołowrocki, 2019/2020; Kołowrocki et al., 2018).

7. Conclusion

In this chapter, the comprehensive approach to the safety analysis, evaluation and prediction of the critical infrastructure impacted by its operation process and the climate climate-weather change process at its operating area is presented. The proposed approach to ageing multistate system safety analysis (Dąbrowska, 2020; Kołowrocki, 2020a; Kołowrocki & Kuligowska, 2018; Kołowrocki & Magryta-Mut, 2020; Magryta-Mut, 2021; Torbicki, 2019a, 2019b, 2019c) based on the two-state and multistate system reliability modelling (Brunelle & Kapur, 1999; Kołowrocki, 2000, 2003, 2005, 2008, 2014; Li & Pham, 2005; Lisnianski, 2010; Natvig, 2007; Ouyang, 2014; Xue, 1985; Xue & Yang, 1995a, 1995b) is introduced and widely developed for safety analysis of such system. First, the proposed approach is transformed for modelling safety of the ageing multistate critical infrastructure without outside impacts. Next, the approach is transformed and developed to safety analysis of critical infrastructure impacted, separately and jointly, by its operation process and by its operation process related to the climate-weather change at its operating area.

Thus, starting from the critical infrastructure simplest safety model, defined as a multistate ageing system without considering outside impacts, this safety model is combined with the model of critical infrastructure operation process, in order to create the integrated model. That integrated model is used for to safety modelling and prediction of critical infrastructure impacted by its operation process and to extend the set of safety and resilience indicators practically useful in the critical infrastructure examination. The next created integrated critical infrastructure safety model is related to the climate-weather change process at its operating area influence on its safety, linking its multistate safety model and the model of climate-weather change process at its operating area. That allows to create the critical infrastructure climate-weather impact safety model with other, practically significant, critical infrastructure safety and resilience indicators. The most general critical infrastructure safety model that simultaneously considers the operation process and the climate-weather change process influence on the safety of a critical infrastructure is proposed. It is a safety model of a critical infrastructure influenced by the operation process, which itself is related to climate-weather change at its operating area. That general model links the multistate safety model and the joint model of operation process related to climate-weather change at its operating area, to create the critical infrastructure joint operation and climate-weather impact safety model. This safety model considers variable critical infrastructure safety structures and its components safety parameters, impacted by climate-weather states at different operation states and introduces other useful critical infrastructure safety and resilience indicators.

All proposed critical infrastructure safety and resilience indicators are defined for any critical infrastructures with varying in time their safety structures and assets safety parameters, which are influenced by, changing in time, operation and climate-weather conditions at their operating areas. These models application and validation, can be realized through examination of real critical infrastructures of various kinds.

The results obtained may play the role of a universal tools necessary in safety evaluation of real complex technical systems, both during design phase and during their operation. To make the results and the proposed methods an easy and useful tool for practitioners their usage should be illustrated by practical application to the evaluation of the real critical infrastructure safety characteristics and indicators. All proposed methods and models can be applied to the safety examination of the critical infrastructures changing their its safety structures and their assets safety parameters depending on their operation states and the climate-weather states. Those all tools can also be useful in safety, availability and maintenance optimization and operation cost analysis (Kołowrocki & Magryta, 2020b, 2021; Magryta-Mut 2020, 2021). They can be applied also to very wide class of real technical systems in varying operation and climate-weather conditions (Kołowrocki & Kuligowska, 2018; Torbicki, 2019a, 2019b, 2019c) that have influence on their changing safety structures and their components safety characteristics.

The path we should follow in our future research activity is to investigate and solve problems of safety and resilience strengthening of critical infrastructure impacted by operation and climate-weather change. This activity will lead to establishing of elaborate models of business continuity for critical infrastructure under operation and climate pressures. It will allow to solve the critical infrastructure safety optimization (Kołowrocki & Magryta, 2020) as well as its degradation (Kołowrocki, 2003, 2008) and accident consequences identification and mitigation (Bogalecka, 2020).

All presented models are the basis for procedures, which are easy to use by the practitioners and operators of the critical infrastructures in their operation and safety analysis. The created models, and procedures based on them, can be modified and developed for other problems of safety of critical infrastructure analysis. In this context, modelling and prediction of critical infrastructure safety presented in this paper, further developed by considering inner dependences between the critical infrastructure assets (Kołowrocki, 2020b), will be a very important broadening to real practice in critical infrastructure safety examination. It will allow also for building of the model which considers simultaneously the critical infrastructure ageing, its inside dependences and outside impacts (Kołowrocki, 2020b, 2021). All the proposed indicators, and other safety and resilience tools, can be validated through their practical application to the real critical infrastructures (Kołowrocki, 2020a).

Further research activities could concentrate on investigating and solving of optimization problems for critical infrastructure safety (Kołowrocki & Magryta, 2020a). This research should include finding of optimal values of safety and resilience indictors, as well as analysis of resilience and strengthening of critical infrastructure against climate-weather change. This activity will result in elaboration of business continuity models for critical infrastructure under the operation with climate-weather pressures, cost-effectiveness analysis and modelling, critical infrastructure degradation and accident consequences analysis and mitigation (Bogalecka, 2020).

In the paper, the approaches to the safety analysis of aging multistate critical infrastructures impacted by their operation and climate-weather conditions at their operating areas that consider their subsystems and components' independency are presented. In the future research, there should be proposed an innovative approach for the joint safety analysis of ageing multistate systems (Kołowrocki, 2000, 2003, 2005, 2008, 2014) that considers also their components' dependency

(Kołowrocki, 2020b) and their varying safety parameters. In the next steps of research the problem of dependence and cascading effect in complex systems and critical infrastructure networks, defined as a problem of failure dependency among components and subsystems (Kołowrocki, 2020b) should be developed to multistate, aging critical infrastructures (Kołowrocki, 2000, 2003, 2005, 2008, 2014). Thus, as a consequence of the above analysis, the further research could be focused on safety analysis of critical infrastructure networks, considering their ageing, inside dependencies and outside impacts (Dąbrowska, 2020; Holden et al., 2013; Kołowrocki, 2019/2020, 2020b, 2021), and use of achieved results to improve their safety, strengthen their resilience and mitigate the effects of their degradation (Bogalecka, 2020).

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