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Identification of climate-weather change processes at Baltic Sea water area impacted on critical infrastructure safety and their accident consequences

Keywords

climate-weather change process, identification, critical infrastructure, Baltic Sea water area

Abstract

There are presented the methods of identification of the climate-weather change process. These are the methods and procedures for estimating the unknown basic parameters of the climate-weather change process semi-Markov model and identifying the distributions of the climate-weather change process conditional sojourn times at the climate-weather states. There are given the formulae estimating the probabilities of the climate-weather change process staying at the particular climate-weather states at the initial moment, the probabilities of the climate-weather change transitions between the climate-weather states and the parameters of the distributions suitable and typical for the description of the climate-weather change process conditional sojourn times at the particular climate-weather states. The proposed statistical methods applications for the unknown parameters identification of the climate-weather change process model determining the climate-weather change process parameters for the port oil piping transportation system and maritime ferry operating areas are presented.

1. Introduction

The critical infrastructures are often impacted by extreme weather hazards. In the face of the climate change, those hazards will be increasing. In aim to predict and mitigate their currently and future impact on the critical infrastructures safety and resilience, we should be able to determine or evaluate the unknown parameters of the climate-weather change process model proposed in (Kołowrocki et al., 2016b) and (Jakusik, Kołowrocki, Kuligowska et al., 2016a-f). Particularly, the probabilities of the climate-weather change process staying at the particular climate-weather states at the initial moment, the probabilities of the process transitions between the climate-weather states and the distributions of the conditional sojourn times of the process at the particular climate-weather states should be identified (Habibullah, Lumanpauw, Kołowrocki, 2004; Kołowrocki et al., 2009). It is possible using the statistical methods proposed in (Barbu & Limnios, 2006; Collet, 1996; Gamiz & Roman, 2008; Giudici & Figini, 2009; Grabski, 2014; Helvacioglu & Insel, 2008; Hryniewicz, 1995). The way of usage those methods to determine unknown parameters of the climate-weather change process model is presented in the paper on examples of climate-weather change processes related to critical infrastructures operating areas: the port oil piping transportation system – the part of the port oil terminal critical infrastructure and the maritime ferry – the part of shipping critical infrastructure.

Safety and Reliability of Systems and Processes, Summer Safety and Reliability Seminar 2021.

DOI: 10.26408/srsp-2021-06.

2. Identification of climate-weather change process

We assume, as in (Kołowrocki, Kuligowska & Torbicki, 2017), that the climate-weather change process for the critical infrastructure operating area is taking w, $w \in N$, different climate-weather states $c_1, c_2, ..., c_w$. Next, we mark by $C(t)$, $t \in \langle 0, +\infty)$ the climate-weather change process, that is a function of a continuous variable t, taking discrete values in the set $\{c_1, c_2, ..., c_w\}$ of the climate-weather states. We assume a semi-Markov model (Kołowrocki, 2014; Limnios & Oprisan, 2005; Limnios, Ouhbi & Sadek, 2005; Macci, 2008; Mercier, 2008), of the climate-weather change process $C(t)$ and we mark by C_{bl} its random conditional sojourn times at the climate-weather states c_b, when its next climate-weather state is c_l, $b, l = 1,2,\ldots,w$, $b \neq l$.

Under these assumptions, the climate-weather change process may be described by the vector $[q_b(0)]_{1\times w}$ of probabilities of the climate-weather change process staying at the particular climate-weather states at the initial moment $t = 0$, the matrix $[q_{bl}(t)]_{w\times w}$ of the probabilities of the climate-weather change process transitions between the climate-weather states and the matrix $[C_{bl}(t)]_{w\times w}$ of the distribution functions of the conditional sojourn times C_{bl} of the climate-weather change process at the climate-weather states or equivalently by the matrix $[c_{bl}(t)]_{w\times w}$ of the density functions of the conditional sojourn times C_{bl}, $b,l = 1,2,\ldots,w$, $b \neq l$, of the climate-weather change process at the climate-weather states. These all parameters of the climate-weather change process are unknown and before their use to the prognosis of this process characteristics have to be estimated on the basis of statistical data coming from practice.

2.1. Defining unknown parameters of climate-weather change process and data collection

To make the estimation of the unknown parameters of the climate-weather change process, the experiment delivering the necessary statistical data should be precisely planned (Kołowrocki, Kuligowska & Torbicki, 2017).

First, before the experiment, we should perform the following preliminary steps:

i) to analyze the climate-weather change process,

ii) to fix or to define the climate-weather change process following general parameters:
   - the number w of the climate-weather states of the process,
   - the climate-weather states $c_1, c_2, \ldots, c_w$, of the process,

iii) to fix the possible transitions between the climate-weather states,

iv) to fix the set of the unknown parameters of the climate-weather change process semi-Markov model.

Next, to estimate the unknown parameters of the process, based on the experiment, we should collect necessary statistical data performing the following steps:

i) to fix and to collect the following statistical data necessary to evaluate the probabilities $q_b(0)$ of the climate-weather change process staying at the climate-weather states at the initial moment $t = 0$:
   - the duration time Θ of the experiment,
   - the number $n(0)$ of the investigated realizations of the climate-weather change process,
   - the vector

$$[n_b(0)] = [n_1(0), n_2(0), ..., n_w(0)],$$

   of the realizations $n_b(0)$, $b = 1,2,..., w$, of the numbers of the process staying respectively at the climate-weather states $c_1, c_2, \ldots, c_w$, at the initial moment $t = 0$ of all $n(0)$ observed realizations of the process, where

$$n_1(0) + n_2(0) + n_w(0) = n(0),$$

ii) to fix and to collect the following statistical data necessary to evaluate the probabilities q_{bl} of the process transitions between the climate-weather states:
   - the matrix

$$[n_{bl}] = \begin{bmatrix} n_{11} & n_{12} & \ldots & n_{1w} \\ n_{21} & n_{22} & \ldots & n_{2w} \\ \vdots & \vdots & \ddots & \vdots \\ n_{w1} & n_{w2} & \ldots & n_{ww} \end{bmatrix},$$

of the realizations of the numbers n_{bl}, $b,l = 1,2,...,w$, $b \neq l$, of the transitions of the process from the climate-weather state c_b into the climate-weather state c_l at all observed realizations of the process, where $n_{bb} = 0$, for $b = 1,2,...,w$,

- the vector

$$[n_b] = [n_1, n_2, ..., n_w],$$

of the realizations of the numbers n_b, $b = 1,2,...,w$, of the process departures from the climate-weather states c_b, where

$$n_1 = n_{11} + n_{12} + ... + n_{1w},$$
$$n_2 = n_{21} + n_{22} + ... + n_{2w},$$
$$...$$
$$n_w = n_{w1} + n_{w2} + ... + n_{ww},$$

iii) to fix and to collect the following statistical data necessary to evaluate the unknown parameters of the distributions $C_{bl}(t)$ of the conditional sojourn times C_{bl} of the process at the particular climate-weather states:

- the numbers n_{bl}, $b,l = 1,2,...,w$, $b \neq l$, of realizations of the process conditional sojourn times C_{bl}, $b, l = 1,2,...,w$, $b \neq l$, at the climate-weather state c_b when the next transition is to the climate-weather state c_l during the observation time Θ,
- the realizations C_{bl}^k, $k = 1,2,\ldots,n_{bl}$, of the conditional sojourn times C_{bl} of the process at the climate-weather state c_b when the next transition is to the climate-weather state c_l during the observation time Θ, for each $b, l = 1,2,...,w$, $b \neq l$.

2.2. Estimating basic parameters of climate-weather change process

After collecting the statistical data, it is possible to estimate the unknown parameters of the climate-weather change process performing the following steps:

i) to determine the vector

$$[q(0)] = [q_1(0), q_2(0), \ldots, q_w(0)], \quad (1)$$

of the realizations of the probabilities $q_b(0)$, $b = 1,2,...,w$, of the climate-weather change process staying at the climate-weather states at the initial moment $t = 0$, according to the formula

$$q_b(0) = \frac{n_b(0)}{n(0)}, \text{ for } b = 1,2,...,w, \quad (2)$$

where

$$n(0) = \sum_{b=1}^{w} n_b(0), \quad (3)$$

is the number of the realizations of the climate-weather change process starting at the initial moment $t = 0$,

ii) to determine the matrix

$$[q_{bl}] = \begin{bmatrix} q_{11} & q_{12} & \cdots & q_{1w} \\ q_{21} & q_{22} & \cdots & q_{2w} \\ \vdots & \vdots & \ddots & \vdots \\ q_{w1} & q_{w2} & \cdots & q_{ww} \end{bmatrix}, \quad (4)$$

of the realizations of the probabilities q_{bl}, $b, l = 1,2,...,w$, of the climate-weather change process transitions from the climate-weather state c_b to the climate-weather state c_l according to the formula

$$q_{bl} = \frac{n_{bl}}{n_b}, \text{ for } b, l = 1,2,...,w, \ b \neq l,$$

$$q_{bb} = 0, \text{ for } b = 1,2,..., w, \quad (5)$$

where

$$n_b = \sum_{b \neq l}^{w} n_{bl}, \ b = 1,2,...,w, \quad (6)$$

is the realization of the total number of the process departures from the climate-weather state c_b, $b = 1,2,...,w$, during the experiment time Θ.

2.3. Estimating parameters of distributions of climate-weather change process conditional sojourn times at climate-weather states

Prior to estimating the parameters of the distributions of the climate-weather change process conditional sojourn times at the particular climate-weather states, we have to determine the following empirical characteristics of the realizations of the process conditional sojourn time at the particular climate-weather states:

- the realizations of the empirical mean values $\bar{C}_{bl}$ of the conditional sojourn times C_{bl} of the climate-weather change process at the climate-weather state c_b, $b = 1,2,...,w$, when the next transition is to the climate-weather state c_l, $l = 1,2,...,w$, according to the formula

$$\bar{C}_{bl} = \frac{1}{n_{bl}} \sum_{k=1}^{n_{bl}} C_{bl}^k, \; b, l = 1,2,...,w, \; b \neq l, \tag{7}$$

- the number $\bar{r}_{bl}$ of the disjoint intervals $I_j = \langle a_{bl}^j, b_{bl}^j)$, $j = 1,2,...,\bar{r}_{bl}$, that include the realizations C_{bl}^k, $k = 1,2,...,n_{bl}$, of the conditional sojourn times C_{bl} at the climate-weather state c_b when the next transition is to the climate-weather state c_l, according to the formula

$$\bar{r}_{bl} = \lceil n_{bl} \rceil,$$

- the length d_{bl} of the intervals $I_j = \langle a_{bl}^j, b_{bl}^j)$, $j = 1,2,...,\bar{r}_{bl}$, according to the formula

$$d_{bl} = \frac{\bar{R}_{bl}}{\bar{r}_{bl} - 1},$$

where $\bar{R}_{bl} = \max_{1 \le k \le n_{bl}} \theta_{bl}^k - \min_{1 \le k \le n_{bl}} \theta_{bl}^k$,

- the ends a_{bl}^j, b_{bl}^j, of the intervals $I_j = \langle a_{bl}^j, b_{bl}^j)$, $j = 1,2,...,\bar{r}_{bl}$, according to the formulae

$$a_{bl}^1 = \max\{\min_{1 \le k \le n_{bl}} \theta_{bl}^k - \frac{d_{bl}}{2}, 0\},$$

$$b_{bl}^j = a_{bl}^1 + j d_{bl}, \; j = 1,2,...,\bar{r}_{bl},$$

$$a_{bl}^j = b_{bl}^{j-1}, \; j = 2,3...,\bar{r}_{bl},$$

in such a way that

$$I_1 \cup I_2 \cup ... \cup I_{\bar{r}_{bl}} = \langle a_{bl}^1, b_{bl}^{\bar{r}_{bl}}),$$

$$I_i \cap I_j = \emptyset \text{ for all } i \neq j, \; i, j \in \{1,2,...,\bar{r}_{bl}\},$$

- the numbers n_{bl}^j of the realizations C_{bl}^k in the intervals $I_j = \langle a_{bl}^j, b_{bl}^j)$, $j = 1,2,..., \bar{r}_{bl}$, according to the formula

$$n_{bl}^j = \#\{k : C_{bl}^k \in I_j, k \in \{1,2,...,n_{bl}\}\}, \; j = 1,2,..., \bar{r}_{bl}$$

where $\sum_{j=1}^{\bar{r}_{bl}} n_{bl}^j = n_{bl}$,

whereas the symbol # means the number of elements of the set.

The estimates of the unknown parameters for the distribution functions of the process conditional sojourn times at the particular climate-weather states distinguished in (Jakusik, Kołowrocki, Kuligowska et al., 2016e) are presented in (Kołowrocki et al., 2017).

To formulate and next to verify the non-parametric hypothesis concerning the form of the distribution of the climate-weather change process conditional sojourn time C_{bl} at the climate-weather state c_b when the next transition is to the climate-weather state c_l, on the basis of at least 30 its realizations C_{bl}^k, $k = 1,2,\ldots,n_{bl}$, it is due to proceed according to the scheme presented in (Kołowrocki et al., 2016a). The typical distributions are uniform distribution, triangular distribution, exponential distribution, chimney distribution, Gamma distribution, double trapezium distribution and quazi trapezium distribution.

3. Statistical identification of climate-weather change processes – application

3.1. Ferry operation area

3.1.1. System description

The maritime ferry is a passenger ro-ro ship operating at the Baltic Sea between Gdynia and Karlskrona ports on regular everyday line. This operating area of the maritime ferry could be divided into four different areas: Gdynia Port, Baltic Sea restricted waters, Baltic Sea open waters, Karlskrona Port. The detailed maritime ferry

route is illustrated in Figure 1. More information about the maritime ferry, its assets and interconnections between them could be found in (Kołowrocki, Kuligowska & Soszyńska-Budny, 2016).
In following subsections, we will analyze the climate-weather change processes related to the maritime ferry operating at the Port Gdynia area (point 1 in Figure 1), the Baltic Sea restricted waters area (point 2 in Figure 1), the Baltic Sea open waters area between Gdynia bay and Karlskrona bay (points 3–6 in Figure 1) and the Karlskrona area (point 7 in Figure 1). In points 1–7 marked in the Figure 1 were obtained the climate-weather data (GMU, 2020).

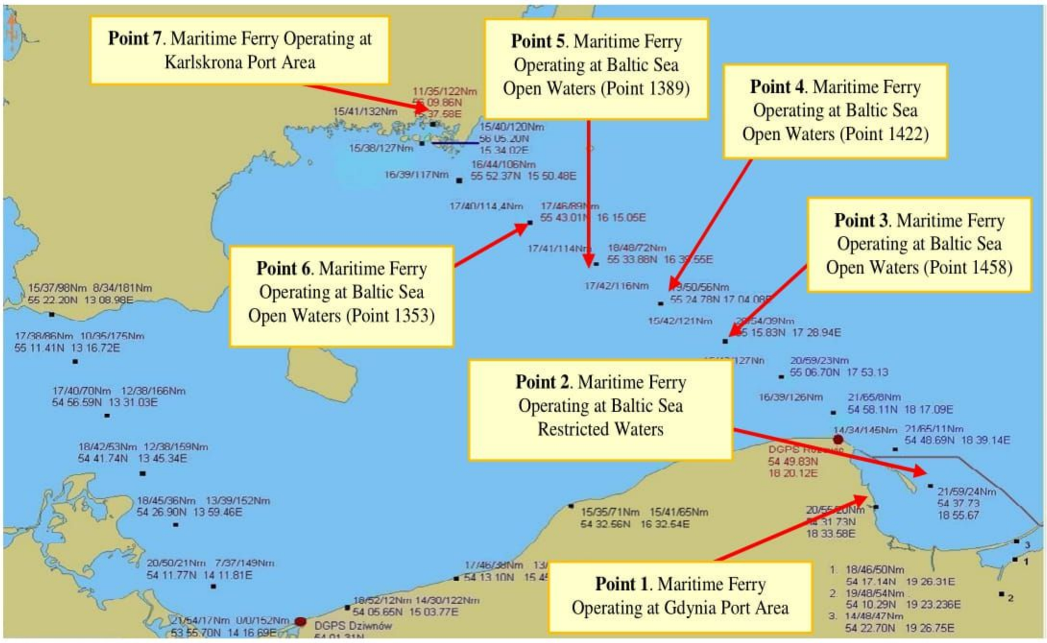


Figure 1. Maritime ferry route between Karlskrona and Gdynia ports.

3.1.2. Climate-weather change process at Gdynia Port area

Defining parameters and data collection of climate-weather change process at Gdynia Port area

Based on the expert opinion, there are distinguished the following $w = 6$ climate-weather states:

- the climate-weather state c_1 – the wave height from 0 m up to 2 m and the wind speed from 0 m/s up to 17.2 m/s,
- the climate-weather state c_2 – the wave height from 2 m up to 5.5 m and the wind speed from 0 m/s up to 17.2 m/s,
- the climate-weather state c_3 – the wave height from 5.5 m up to 14 m and the wind speed from 0 m/s up to 17.2 m/s,
- the climate-weather state c_4 – the wave height from 0 m up to 2 m and the wind speed from 17.2 m/s to 33 m/s,
- the climate-weather state c_5 – the wave height from 2 m up to 5.5 m and the wind speed from 17.2 m/s to 33 m/s,
- the climate-weather state c_6 – the wave height from 5.5 m up to 14 m and the wind speed from 17.2 m/s to 33 m/s.

The unknown parameters of the climate-weather change process semi-Markov model are:

- the initial probabilities $q_b(0)$, $b = 1,2,...,6$, of the climate-weather change process staying at the particular states c_b at the moment $t = 0$,
- the probabilities q_{bl}, $b,l = 1,2,...,6$, $b \neq l$, of the climate-weather change process transitions from the climate-weather state c_b into the climate-weather state c_l,
- the distributions of the climate-weather change conditional sojourn times C_{bl}, $b,l = 1,2,...,6$, $b \neq l$, at the particular climate-weather states and their mean values $M_{bl} = E[C_{bl}]$, $b,l = 1,2,...,6$, $b \neq l$.

To identify all these parameters of the climate-weather change process the statistical data about this process is needed.
The collected by the system operators statistical data necessary to evaluating the initial transient probabilities of the climate-weather change process at the particular states are:

- the climate-weather change process observation / experiment time $\Theta = 6$ years (1988–1993),
- the number of the climate-weather change process realizations $n(0) = 1886$,
- the vector of realizations of the numbers of the climate-weather change process staying at the particular climate-weather state c_b at the initial moment $t = 0$

$$[n_b(0)] = [770, 27, 848, 13, 228, 0].$$

The collected statistical data necessary to evaluating the probabilities of transitions of the process $C(t)$ between the climate-weather states are:

- the matrix of realizations of the numbers of climate-weather change process transitions from the state c_b into the state c_l during the experiment time

$$[n_{bl}] = \begin{bmatrix} 0 & 52 & 1512 & 9 & 197 & 0 \\ 58 & 0 & 7 & 14 & 0 & 0 \\ 1561 & 9 & 0 & 40 & 251 & 0 \\ 6 & 16 & 31 & 0 & 0 & 0 \\ 221 & 0 & 253 & 0 & 0 & 5 \\ 1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix},$$

- the vector of realizations of the total numbers of the climate-weather change process transitions from the climate-weather state c_b during the experiment time

$$[n_b] = [1770, 79, 1861, 53, 479, 2].$$

The statistical data for the conditional sojourn times C_{bl} at the climate-weather states c_b when the next climate-weather state is c_l, $b, l = 1,2,...,6$, $b \neq l$, related to the Gdynia Port area are given in the Figure 2.

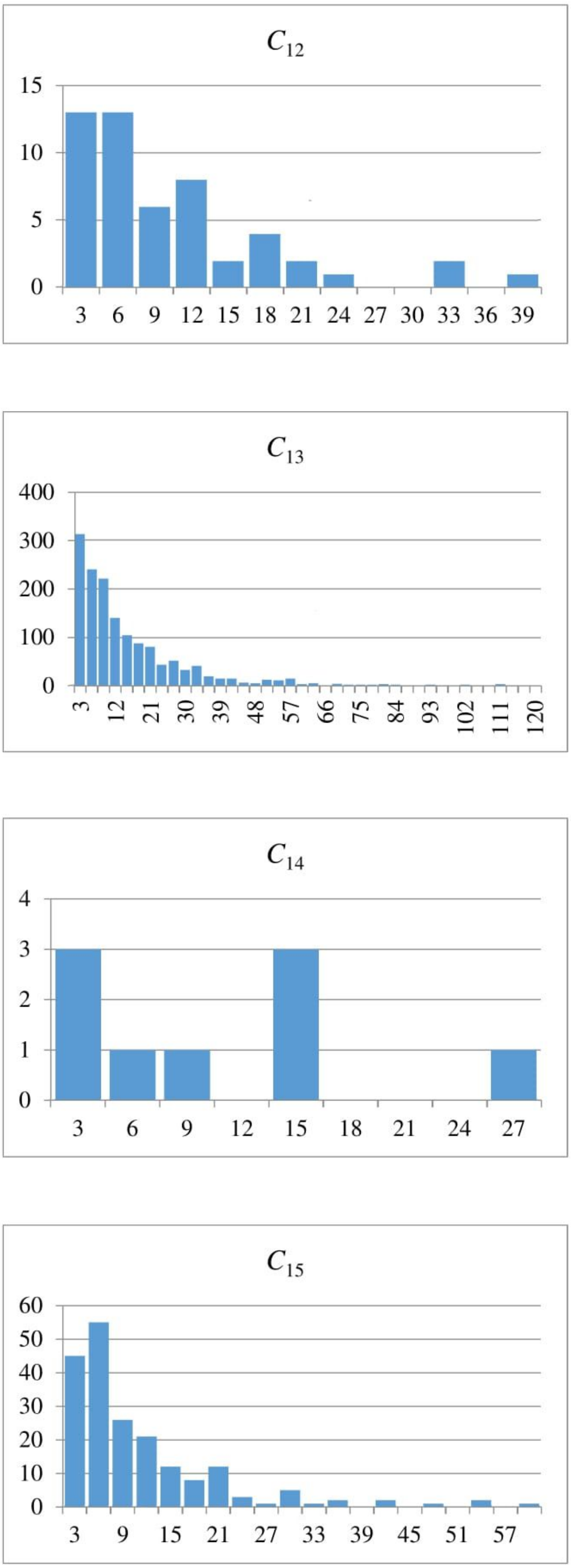


Figure 2. The graphical representation of the statistical data for the conditional sojourn times of the climate-weather change process at the Gdynia Port area.

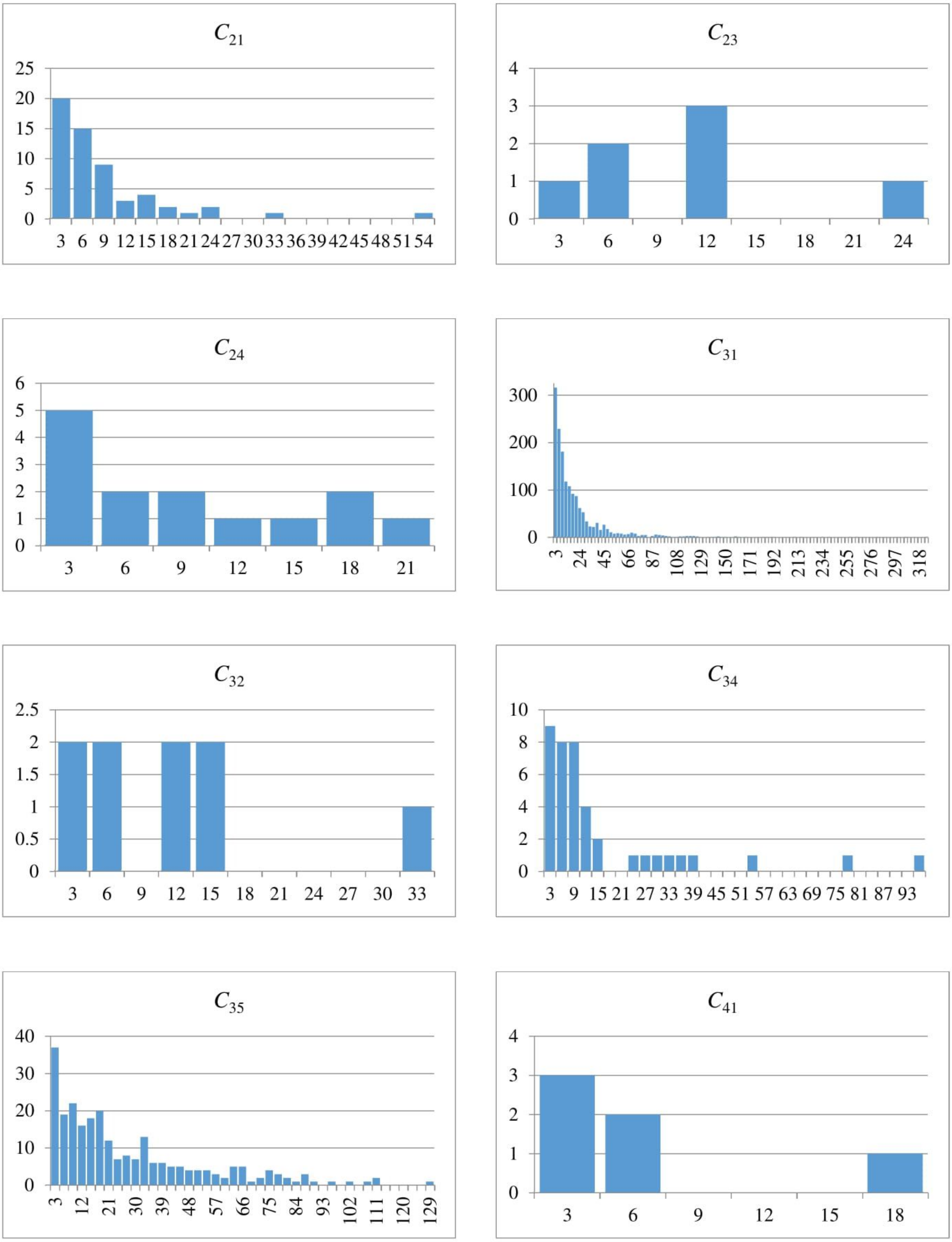


Figure 2 (*continuation*). The graphical representation of the statistical data for the conditional sojourn times of the climate-weather change process at the Gdynia Port area.

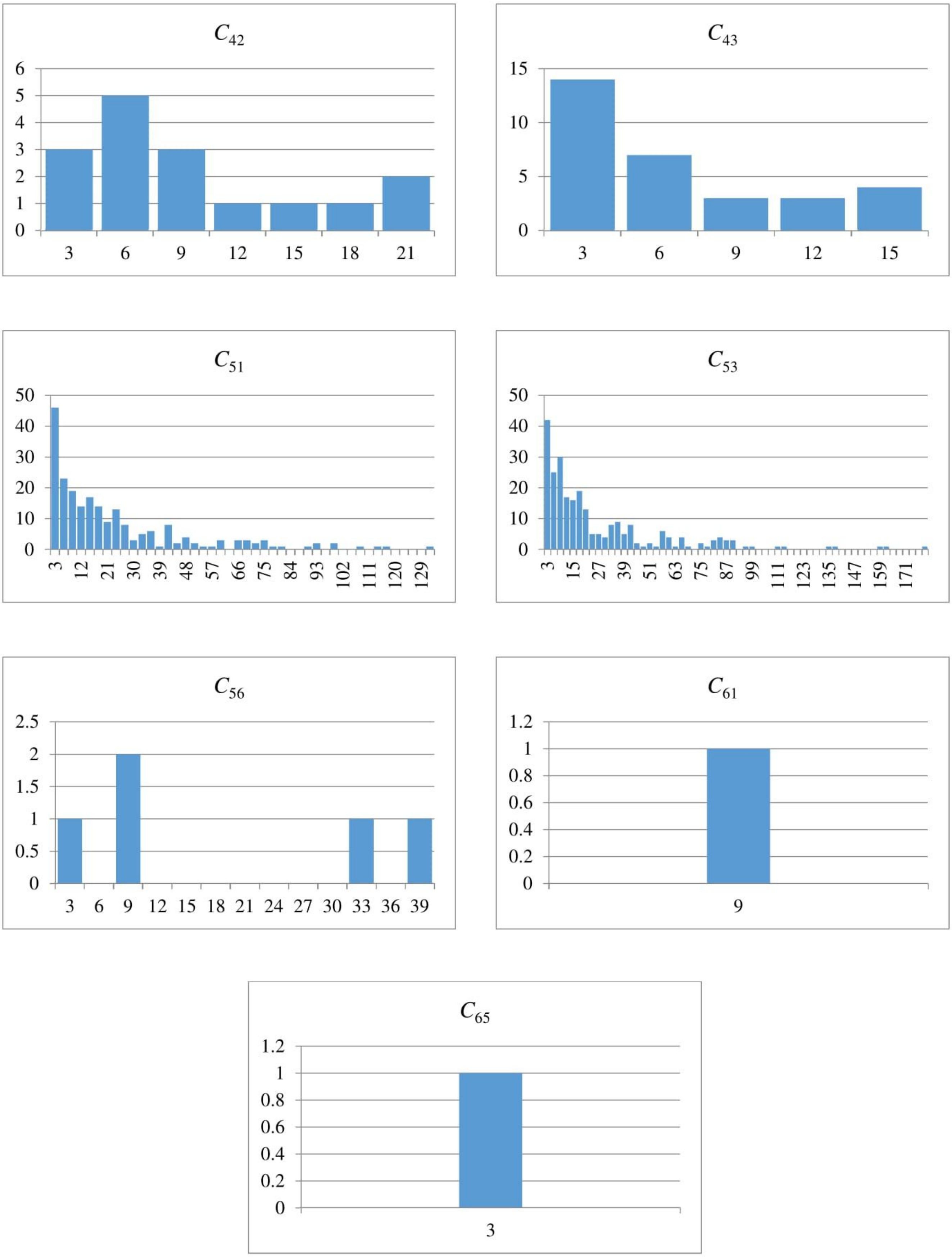


Figure 2 (*continuation*). The graphical representation of the statistical data for the conditional sojourn times of the climate-weather change process at the Gdynia Port area.

Evaluating basic parameters of climate-weather change process at Gdynia Port area

On the basis of the statistical data, it is possible to evaluate the following unknown basic parameters of the climate-weather change process at the Gdynia Port area:

- the vector

$$[q_b(0)] = [0.408, 0.014, 0.450, 0.007, 0.121, 0]$$

of the initial probabilities $q_b(0)$, $b,l = 1,2,...,6$, of the climate-weather change process staying at the particular states c_b at the $t = 0$,

- the matrix

$$[q_{bl}] = \begin{bmatrix} 0 & 0.029 & 0.855 & 0.005 & 0.111 & 0 \\ 0.734 & 0 & 0.089 & 0.177 & 0 & 0 \\ 0.839 & 0.005 & 0 & 0.021 & 0.135 & 0 \\ 0.113 & 0.302 & 0.585 & 0 & 0 & 0 \\ 0.461 & 0 & 0.529 & 0 & 0 & 0.01 \\ 0.05 & 0 & 0 & 0 & 0.5 & 0 \end{bmatrix},$$

of the probabilities q_{bl}, $b,l = 1,2,...,6$, of transitions of the climate-weather change process from the climate-weather state c_b into the climate-weather state c_l.

Identification of distribution functions of climate-weather change process at Gdynia Port area

Using the procedure given in (Kołowrocki, Kuligowska & Torbicki, 2017) and the statistical data and the results from section above, we may verify the hypotheses on the distributions of the climate-weather change process conditional sojourn times C_{bl}, $b,l = 1,2,...,6$, $b \neq l$, at the particular states. To do this, we need a sufficient number of realizations of these variables (Rice, 2007; Vercellis, 2009; Wilson, Graves, Hamada et al., 2006), namely, the sets of their realizations should contain at least 30 realizations coming from the experiment. This condition is not satisfied for the all statistical data we have in disposal.

To make the procedure familiar to the reader, we perform it for the conditional sojourn time C_{12} having sufficiently numerous set of realizations.

First, we have to determine the following empirical characteristics of the realizations of the conditional sojourn time C_{12}:

- the realization $\bar{C}_{12}$ of the defined by (7) mean value of the conditional sojourn time C_{12} of the climate-weather change process at the climate-weather state c_1 when the next transition is to the climate-weather state c_2

$$\bar{C}_{12} = \frac{1}{52}\sum_{k=1}^{52} C_{12}^k = \frac{540}{52} \cong 10.385,$$

- the number $\bar{r}_{12}$ of the disjoint intervals $I_j = \langle a_{12}^j, b_{12}^j)$, $j = 1,2,...,\bar{r}_{12}$, that include the realizations C_{12}^k, $k = 1,2,...,52$ of the conditional sojourn times C_{12} at the climate-weather state c_1 when the next transition is to the climate-weather state c_2

$$\bar{r}_{12} = \lceil \sqrt{52} \rceil = \lceil 7.2 \rceil = 8,$$

- the length d_{12} of the intervals $I_j = \langle a_{12}^j, b_{12}^j)$, $j = 1,2,\ldots,8$, that after considering

$$\bar{R}_{12} = \max_{1 \le k \le 52} C_{12}^k - \min_{1 \le k \le 52} C_{12}^k = 39 - 3 = 36,$$

is

$$d_{12} = \frac{\bar{R}_{12}}{\bar{r}_{12} - 1} = \frac{36}{7} \cong 5.14,$$

- the ends a_{12}^j, b_{12}^j, of the intervals $I_j = \langle a_{12}^j, b_{12}^j)$, $j = 1,2,\ldots,8$, that after considering

$$\min_{1 \le k \le 52} C_{12}^k - \frac{d_{12}}{2} = 3 - \frac{5.14}{2} = 0.43,$$

are

$$a_{12}^1 = \max\{0.43, 0\} = 0.43,$$
$$b_{12}^1 = a_{12}^1 + 5.14 = 0.43 + 5.14 = 5.57,$$

$$a_{12}^2 = b_{12}^1 = 5.57,$$
$$b_{12}^2 = a_{12}^1 + 2 \cdot 5.14 = 0.43 + 10.28 = 10.71,$$

$$a_{12}^3 = b_{12}^2 = 10.71,$$

$b_{12}^3 = a_{12}^1 + 3 \cdot 5.14 = 0.43 + 15.42 = 15.85,$

$a_{12}^4 = b_{12}^3 = 15.85,$
$b_{12}^4 = a_{12}^1 + 4 \cdot 5.14 = 0.43 + 20.56 = 20.99,$

$a_{12}^5 = b_{12}^4 = 20.99,$
$b_{12}^5 = a_{12}^1 + 5 \cdot 5.14 = 0.43 + 25.7 = 26.13,$

$a_{12}^6 = b_{12}^5 = 26.13,$
$b_{12}^6 = a_{12}^1 + 6 \cdot 5.14 = 0.43 + 30.84 = 31.27,$

$a_{12}^7 = b_{12}^6 = 31.27,$
$b_{12}^7 = a_{12}^1 + 7 \cdot 5.14 = 0.43 + 35.98 = 36.41,$

$a_{12}^8 = b_{12}^7 = 36.41,$
$b_{12}^8 = a_{12}^1 + 8 \cdot 5.14 = 0.43 + 41.12 = 41.55.$

- the numbers n_{12}^j of the realizations C_{12}^k in particular intervals $I_j = <a_{12}^j, b_{12}^j)$, $j = 1,2,\ldots,8$,

$n_{12}^1 = 13,\ n_{12}^2 = 19,\ n_{12}^3 = 10,\ n_{12}^4 = 4,$
$n_{12}^5 = 3,\ n_{12}^6 = 0,\ n_{12}^7 = 2,\ n_{12}^8 = 1.$

The realization $\bar{c}_{12}(t)$ of the histogram of the climate-weather change process conditional sojourn time C_{12}, is presented in Table 1 and illustrated in Figure 3.

Table 1. The realization of the histogram of the climate-weather change process conditional sojourn time C_{12}

	Histogram of the conditional sojourn time C_{12}							
$I_j = \langle a_{12}^j, b_{12}^j)$	⟨0.43, 5.57)	⟨5.57, 10.71)	⟨10.71, 15.85)	⟨15.85, 20.99)	⟨20.99, 26.13)	⟨26.13, 31.27)	⟨31.27, 36.41)	⟨36.41, 41.55)
n_{12}^j	13	19	10	4	3	0	2	1
$\bar{h}_{12}(t) = n_{12}^j / n_{12}$	13/52	19/52	10/52	4/52	3/52	0/52	2/52	1/52

After analyzing and comparing the realization $\bar{c}_{12}(t)$ of the histogram with the graphs of the density functions $c_{bl}(t)$ of the previously distinguished in (Jakusik, Kołowrocki, Kuligowska et al., 2016e) distributions, we formulate the null hypothesis H_0 in the following form:

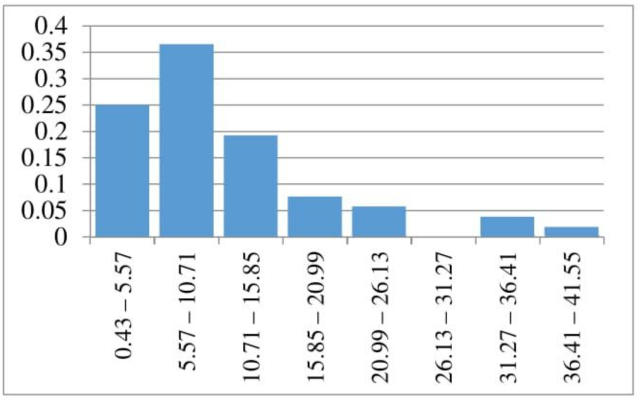


Figure 3. The graph of the histogram of the climate-weather change process conditional sojourn time C_{12}.

H_0: The climate-weather change process conditional sojourn time C_{12} at the climate-weather state c_1 when the next transition is to the climate-weather state c_2, has exponential distribution with the density function defined by (4.5) in (Kołowrocki et al., 2016b) of the form

$$c_{12}(t) = \begin{cases} 0, & t < x_{12} \\ \alpha_{12} \exp[-\alpha_{12}(t - x_{12})], & t \ge x_{12}. \end{cases} \quad (8)$$

We estimate the unknown parameters of the density function of the hypothetical exponential distribution using the formulae (2.13) in (Kołowrocki et al., 2016a) and we obtain the following results

$x_{12} = a_{12}^1 = 0.43,$

$$\alpha_{12} = \frac{1}{\bar{C}_{12} - x_{12}} = \frac{1}{10.385 - 0.43} \cong 0.1005.$$

Next, we join the intervals defined in the realization of the histogram $\bar{h}_{12}(t)$ that have the numbers n_{12}^j, of realizations less than 4 into new intervals and we perform the following steps:

- we fix the new number of intervals $\bar{\bar{r}}_{12} = 5$,
- we determine the new intervals and we fix the numbers of realizations in the new intervals (Table 2),

Table 2. The numbers of the conditional sojourn time C_{12} realizations in the intervals $\bar{I}_j$

$\bar{I}_j = \langle a_{12}^j, b_{12}^j)$	⟨0.43, 5.57)	⟨5.57, 10.71)	⟨10.71, 15.85)	⟨15.85, 20.99)	⟨20.99, +∞)
$\bar{n}_{12}^j$	13	19	10	4	6

- we calculate the hypothetical probabilities that the variable C_{12} takes values from the new intervals

$$p_1 = P(C_{12} \in \bar{I}_1) = P(0.43 \le C_{12} < 5.57) = C_{12}(5.57) - C_{12}(0.43) = 0.4 - 0 = 0.4,$$

$$p_2 = P(C_{12} \in \bar{I}_2) = P(5.57 \le C_{12} < 10.71) = C_{12}(10.71) - C_{12}(5.57) = 0.64 - 0.4 = 0.24,$$

$$p_3 = P(C_{12} \in \bar{I}_3) = P(10.71 \le C_{12} < 15.85) = C_{12}(15.85) - C_{12}(10.71) = 0.79 - 0.64 = 0.15,$$

$$p_4 = P(C_{12} \in \bar{I}_4) = P(15.85 \le C_{12} < 20.99) = C_{12}(20.99) - C_{12}(15.85) = 0.87 - 0.79 = 0.08,$$

$$p_5 = P(C_{12} \in \bar{I}_5) = P(20.99 \le C_{12} < +\infty) = C_{12}(+\infty) - C_{12}(20.99) = 1 - 0.87 = 0.13.$$

- we calculate the realization of the χ^2 (chi-square)-Pearson's statistics

$$u_{12} = \sum_{j=1}^{3} \frac{(\bar{n}_{12}^j - n_{12} p_j)^2}{n_{12} p_j} = \frac{(13-52\cdot 0.4)^2}{52\cdot 0.4} + \frac{(19-52\cdot 0.24)^2}{52\cdot 0.24} + \frac{(10-52\cdot 0.15)^2}{52\cdot 0.15} + \frac{(4-52\cdot 0.08)^2}{52\cdot 0.08} + \frac{(6-52\cdot 0.13)^2}{52\cdot 0.13} \cong 2.93 + 3.41 + 0.62 + 0.01 + 0.09 = 7.06,$$

- we assume the significance level $\alpha = 0.05$,
- we fix the number of degrees of freedom

$$\bar{\bar{r}}_{12} - l - 1 = 5 - 1 - 1 = 3,$$

- we read from the tables of the χ^2 –Pearson's distribution the value χ^2_α for the fixed values of the significance level $\alpha = 0.05$ and the number of degrees of freedom $\bar{\bar{r}}_{12} - l - 1 = 3$, such that, according to (4.48) in (Kołowrocki & Soszyńska-Budny, 2011), the following equality holds

$$P(U_{12} > \chi^2_\alpha) = \alpha = 0.05,$$

that amounts $\chi^2_\alpha = 7.81$ and we determine the critical domain in the form of the interval $(7.81, +\infty)$ and the acceptance domain in the form of the interval $\langle 0, 7.81 \rangle$.

- we compare the obtained value $u_{12} = 7.06$ of the realization of the statistics U_{12} with the read from the tables critical value $\chi^2_\alpha = 7.81$ of the chi-square random variable and since the value $u_{12} = 7.06$ does not belong to the critical domain, i.e.

$$u_{12} = 7.06 \le \chi^2_\alpha = 7.81,$$

then we do not reject the hypothesis H_0, that the sojourn time C_{12} has the exponential distribution with the density function given by (8).

For the remaining cases, when the realizations of conditional sojourn times C_{bl}, $b,l = 1,2,...,6$, $b \ne l$, at the particular climate-weather states are more than 30, proceeding afterwards in an analogous way as in the case of the conditional sojourn time C_{12}, we can get the distributions of the climate-weather change process conditional sojourn times with the parameters given in Table 3.

For the distributions identified in this section, by application either the general formulae for the mean value given by (2.12) or the particular formulae (2.13)–(2.19) in (Kołowrocki & Soszyńska-Budny, 2011), the mean values $M_{bl} = E[\theta_{bl}]$, $b,l = 1,2,...,6$, $b \ne l$, of the climate-weather change process conditional sojourn times at the particular climate-weather states related to the Gdynia Port area can be determined (Table 3).

Because of the lack of sufficient numbers of realizations of the climate-weather change process conditional sojourn times at the climate-weather states, it is not possible to identify statistically their distributions.

In those cases of not identified distributions it is possible to find the approximate empirical values of the mean values $M_{bl} = E[C_{bl}]$ of the conditional sojourn times at the particular climate-weather states that are given in Table 4.

Table 3. Distribution functions of conditional sojourn times when their realizations are more than 30

Distribution	Sojourn time	Parameters	Mean
exponential	C_{12}	$x_{12} = 0.43$ $\alpha_{12} = 0.1005$	10.38
	C_{21}	$x_{21} = 0$ $\alpha_{21} = 0.1118$	8.945
	C_{35}	$x_{35} = 0$ $\alpha_{35} = 0.0361$	27.701
	C_{43}	$x_{43} = 2$ $\alpha_{43} = 0.2138$	6.677
	C_{51}	$x_{51} = 0$ $\alpha_{51} = 0.0422$	23.697
double trapezium	C_{13}	$x_{13} = 1.5$ $y_{13} = 121.5$ $z_{13} = 15.9087$ $w_{13} = 0.0002$ $q_{13} = 0.069$	49.53
	C_{15}	$x_{15} = 0.97$ $y_{15} = 62.02$ $z_{15} = 11.0711$ $w_{15} = 0.0012$ $q_{15} = 0.0561$	28.842
	C_{31}	$x_{31} = 0$ $y_{31} = 329.23$ $z_{31} = 21.7226$ $w_{31} = 0.0001$ $q_{31} = 0.0435$	126.017
	C_{53}	$x_{53} = 0$ $y_{53} = 188.02$ $z_{53} = 27.2846$ $w_{53} = 0.0004$ $q_{53} = 0.0347$	83.552
chimney	C_{34}	$A_{34} = 0$ $K_{34} = 0.775$ $D_{34} = 0.225$ $x_{34} = 0$ $y_{34} = 108.5$ $z1_{34} = 0$ $z2_{34} = 15.5$	19.956

Table 4. Distribution functions of conditional sojourn times when their realizations are less than 30

Distribution	Sojourn time	Parameters, intervals	Mean
empirical	C_{14}	{0, 3/9, 4/9, 5/9, 8/9, 1}; {(-∞, 3⟩, (3, 6⟩, (6, 9⟩, (9, 15⟩, (15, 27⟩, (27, +∞)}	10.667
	C_{23}	{0, 1/7, 3/7, 6/7, 1}; {(-∞, 3⟩, (3, 6⟩, (6, 12⟩, (12, 24⟩, (24, +∞)}	10.714
	C_{24}	{0, 5/14, 7/14, 9/14, 10/14, 11/14, 13/14, 1}; {(-∞, 3⟩, (3, 6⟩, (6, 9⟩, (9, 12⟩,(12, 15⟩, (15, 18⟩, (18, 21⟩, (21, +∞)}	9.214
	C_{32}	{0, 2/9, 4/9, 6/9, 8/9, 1}; {(-∞, 3⟩, (3, 6⟩, (6, 12⟩, (12, 15⟩, (15, 33⟩, (33, +∞)}	11.667
	C_{41}	{0, 3/6, 5/6, 1}; {(-∞, 3⟩, (3, 6⟩, (6, 18⟩, (18, +∞)}	6.5
	C_{42}	{0, 3/16, 8/16, 11/16, 12/16, 13/16, 14/16, 1}; {(-∞, 3⟩, (3, 6⟩, (6, 9⟩, (9, 12⟩, (12, 15⟩, (15, 18⟩, (18, 21⟩, (21, +∞)}	9.562
	C_{56}	{0, 1/5, 3/5, 4/5, 1}; {(-∞, 3⟩, (3, 9⟩, (9, 33⟩, (33, 39⟩, (39, +∞)}	18.6
	C_{61}	{0, 1}; {(-∞, 9⟩, (9, +∞)}	9
	C_{65}	{0, 1}; {(-∞, 3⟩, (3, +∞)}	3

As there are no realizations of the rest climate-weather change process conditional sojourn times at the climate-weather states then it is impossible to estimate their empirical conditional mean values.

3.1.3. Climate-weather change process at Baltic Sea restricted waters area

Defining parameters and data collection of climate-weather change process at Baltic Sea restricted waters area

Based on the expert opinion, there are distinguished the following $w = 6$ climate-weather states:

- the climate-weather state c_1 – the wave height from 0 m up to 2 m and the wind speed from 0 m/s up to 17.2 m/s,
- the climate-weather state c_2 – the wave height from 2 m up to 5.5 m and the wind speed from 0 m/s up to 17.2 m/s,
- the climate-weather state c_3 – the wave height from 5.5 m up to 14 m and the wind speed from 0 m/s up to 17.2 m/s,
- the climate-weather state c_4 – the wave height from 0 m up to 2 m and the wind speed from 17.2 m/s to 33 m/s,
- the climate-weather state c_5 – the wave height from 2 m up to 5.5 m and the wind speed from 17.2 m/s to 33 m/s,

- the climate-weather state c_6 – the wave height from 5.5 m up to 14 m and the wind speed from 17.2 m/s to 33 m/s.

The unknown parameters of the climate-weather change process semi-Markov model are:

- the initial probabilities $q_b(0)$, $b = 1,2,...,6$, of the climate-weather change process staying at the particular states c_b at the moment $t = 0$,
- the probabilities q_{bl}, $b,l = 1,2,...,6$, $b \neq l$ of the climate-weather change process transitions from the climate-weather state c_b into the climate-weather state c_l,
- the distributions of the climate-weather change conditional sojourn times C_{bl}, $b,l = 1,2,...,6$, $b \neq l$ at the particular climate-weather states and their mean values $M_{bl} = E[C_{bl}]$, $b,l = 1,2,...,6$, $b \neq l$.

To identify all these parameters of the climate-weather change process the statistical data about this process is needed.

The collected by the system operators statistical data necessary to evaluating the initial transient probabilities of the climate-weather change process at the particular states are:

- the climate-weather change process observation / experiment time

$$\Theta = 6 \text{ years (1988–1993)},$$

- the number of the climate-weather change process realizations $n(0) = 1886$,
- the vector of realizations of the numbers of the climate-weather change process staying at the particular climate-weather state c_b at the initial moment $t = 0$

$$[n_b(0)] = [1501, 335, 6, 0, 20, 24].$$

The collected statistical data necessary to evaluating the probabilities of transitions of the climate-weather state change process $C(t)$ between the climate-weather states are:

- the matrix of realizations of the numbers of climate-weather change process transitions from the state c_b into the state c_l during the experiment time

$$[n_{bl}] = \begin{bmatrix} 0 & 1735 & 0 & 0 & 19 & 0 \\ 554 & 0 & 10 & 0 & 130 & 4 \\ 0 & 53 & 0 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 46 & 4 & 0 & 0 & 48 \\ 0 & 1 & 69 & 0 & 1 & 0 \end{bmatrix},$$

- the vector of realizations of the total numbers of the climate-weather change process transitions from the climate-weather state c_b during the experiment time

$$[n_b] = [1754, 698, 60, 0, 98, 71].$$

The statistical data for the conditional sojourn times C_{bl} at the climate-weather states c_b when the next climate-weather state is c_l, $b,l = 1,2,...,6$, $b \neq l$, related to the Baltic Sea restricted waters area are given in Figure 4.

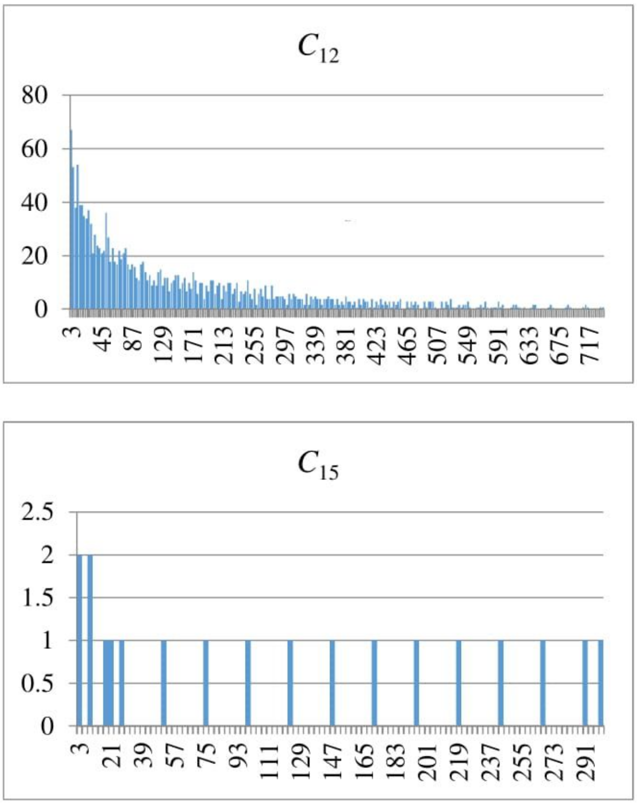


Figure 4. The graphical representation of the statistical data for the conditional sojourn times of the climate-weather change process at the Baltic Sea restricted waters area.

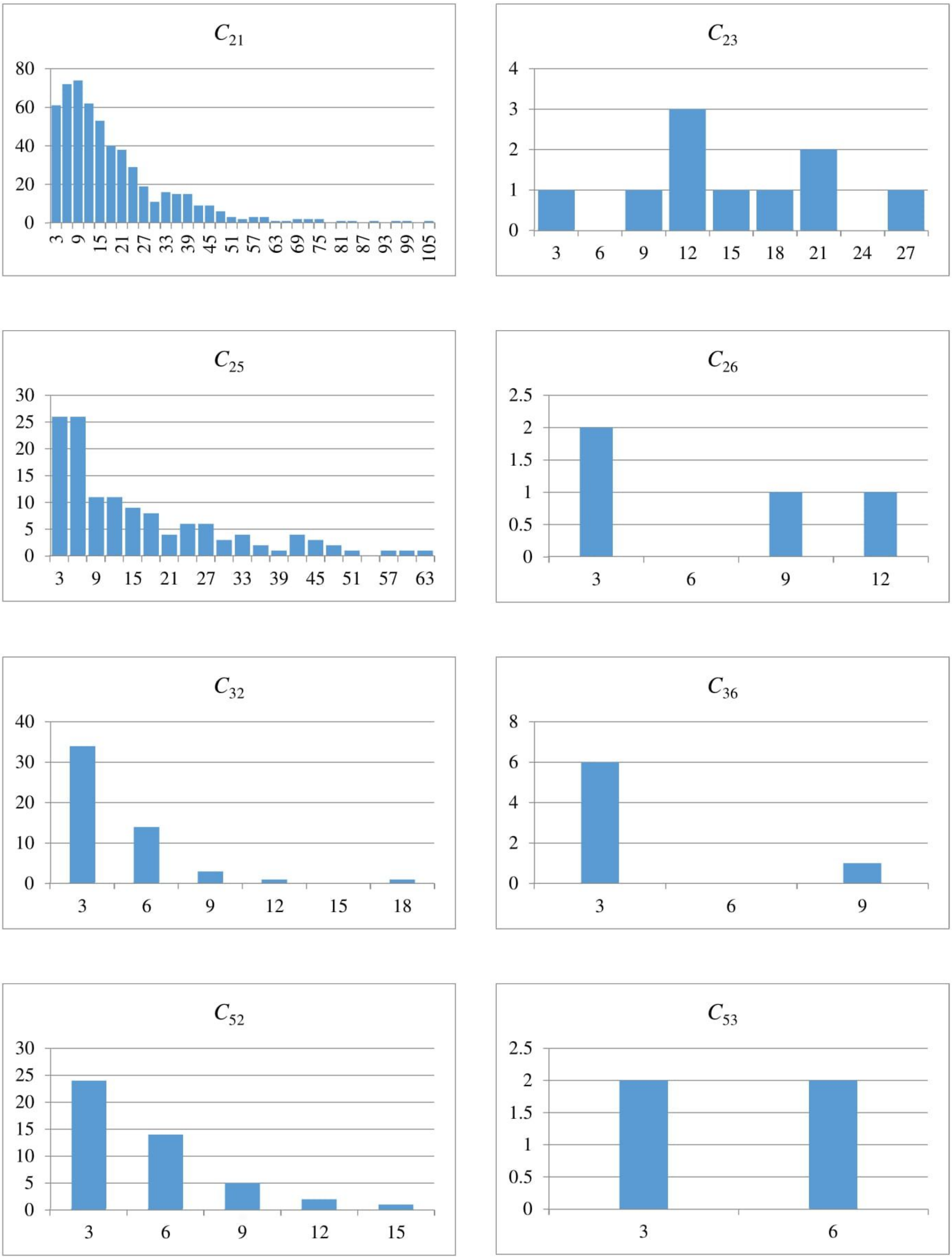


Figure 4 (*continuation*). The graphical representation of the statistical data for the conditional sojourn times of the climate-weather change process at the Baltic Sea restricted waters area.

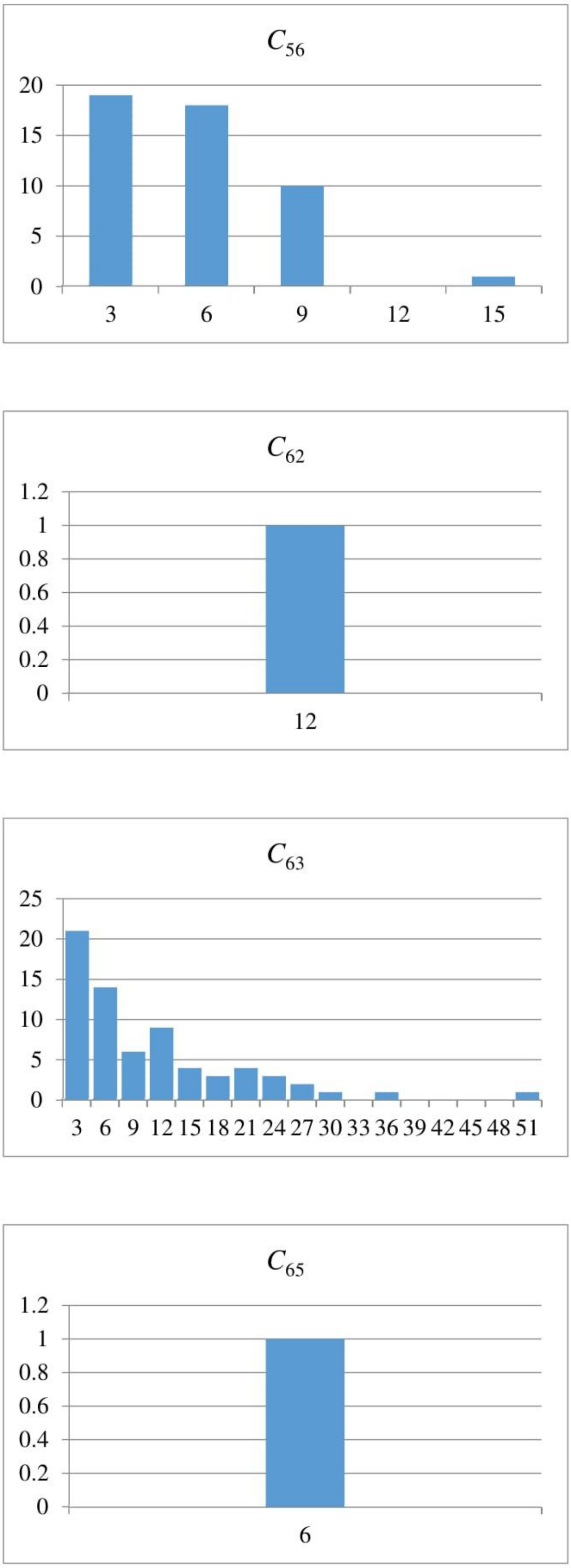


Figure 4 (*continuation*). The graphical representation of the statistical data for the conditional sojourn times of the climate-weather change process at the Baltic Sea restricted waters area.

Evaluating basic parameters of climate-weather change process at Baltic Sea restricted waters area

On the basis of the statistical data from section above, it is possible to evaluate the following unknown basic parameters of the climate-weather change process:

- the vector

$$[q_b(0)] = [0.795, 0.178, 0.003, 0, 0.011, 0.013]$$

  of the initial probabilities $q_b(0)$, $b,l = 1,2,...,6$, of the climate-weather change process staying at the particular states c_b at the $t = 0$,
- the matrix

$$[q_{bl}] = \begin{bmatrix} 0 & 0.989 & 0 & 0 & 0.011 & 0 \\ 0.794 & 0 & 0.014 & 0 & 0.186 & 0.006 \\ 0 & 0.883 & 0 & 0 & 0 & 0.117 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.469 & 0.041 & 0 & 0 & 0.49 \\ 0 & 0.014 & 0.972 & 0 & 0.014 & 0 \end{bmatrix},$$

  of the probabilities q_{bl}, $b,l = 1,2,...,6$, of transitions of the climate-weather change process from the climate-weather state c_b into the climate-weather state c_l.

Identification of distribution functions of climate-weather change process at Baltic Sea restricted waters area

Using the procedure given in (Kołowrocki, Kuligowska & Torbicki, 2017), the results and the statistical data from section above, we may verify the hypotheses on the distributions of the climate-weather change process conditional sojourn times C_{bl}, $b,l = 1,2,...,6$, $b \neq l$, at the particular states. To do this, we need a sufficient number of realizations of these variables (Rice, 2007; Vercellis, 2009; Wilson, Graves, Hamada et al., 2006), namely, the sets of their realizations should contain at least 30 realizations coming from the experiment. This condition is not satisfied for the all statistical data we have in disposal. However, when the realizations of conditional sojourn times

C_{bl}, $b,l = 1,2,...,6$, $b \neq l$, at the particular climate-weather states are more than 30, we can get the distributions of the climate-weather change process conditional sojourn times with the parameters given in Table 5.

For the distributions identified in this section, by application either the general formulae for the mean value given by (2.12) or the particular formulae (2.13)–(2.19) in (Kołowrocki & Soszyńska-Budny, 2011), the mean values $M_{bl} = E[C_{bl}]$, $b, l = 1,2,...,6$, $b \neq l$, of the climate-weather change process conditional sojourn times at the particular climate-weather states related to the Baltic Sea restricted waters area can be determined (Table 5).

Table 5. Distribution functions of conditional sojourn times when their realizations are more than 30

Distribution	Sojourn Time	Parameters	Mean
exponential	C_{25}	$x_{25} = 0.28$ $\alpha_{25} = 0.0631$	16.128
	C_{32}	$x_{32} = 1.93$ $\alpha_{32} = 0.3766$	4.585
	C_{63}	$x_{63} = 0$ $\alpha_{63} = 0.092$	10.87
double trapezium	C_{12}	$x_{12} = 0$ $y_{12} = 749.49$ $z_{12} = 149.7476$ $w_{12} = 0$ $q_{12} = 0.0083$	299.746
	C_{21}	$x_{21} = 0.88$ $y_{21} = 107.13$ $z_{21} = 18.6444$ $w_{21} = 0.0004$ $q_{21} = 0.0259$	46.263
	C_{52}	$x_{52} = 2.15$ $y_{52} = 15.83$ $z_{52} = 5.2174$ $w_{52} = 0.0127$ $q_{52} = 0.3051$	11.741
	C_{56}	$x_{56} = 2.15$ $y_{56} = 15.83$ $z_{56} = 5.625$ $w_{56} = 0.0122$ $q_{56} = 0.2315$	11.613

Because of the lack of sufficient numbers of realizations of the climate-weather change process conditional sojourn times at the climate-weather states, it is not possible to identify statistically their distributions. In those cases when the number of the conditional sojourn times realizations is non-zero it is possible to find the approximate empirical values of the mean values $M_{bl} = E[C_{bl}]$ of the conditional sojourn times at the particular climate-weather states that are given in Table 6.

Table 6. Distribution functions of conditional sojourn times when their realizations are less than 30

Distribution	Sojourn Time	Parameters, intervals	Mean
empirical	C_{15}	{0, 2/19, 4/19, 5/19, 6/19, 7/19, 8/19, 9/19, 10/19, 11/19, 12/19, 13/19, 14/19, 15/19, 16/19, 17/19, 18/19, 1}; {(-∞, 3⟩, (3, 9⟩, (9, 18⟩, (18, 21⟩, (21, 27⟩, (27, 51⟩, (51, 75⟩, (75, 99⟩, (99, 123⟩, (123, 147⟩, (147, 171⟩, (171, 195⟩, (195, 219⟩, (219, 243⟩, (243, 267⟩, (267, 291⟩, (291, 300⟩, (300, +∞)}	119.526
	C_{23}	{0, 1/10, 2/10, 5/10, 6/10, 7/10, 9/10, 1}; {(-∞, 3⟩, (3, 9⟩, (9, 12⟩, (12, 15⟩, (15, 18⟩, (18, 21⟩, (21, 27⟩, (27, +∞)}	15
	C_{26}	{0, 2/4, 3/4, 1}; {(-∞, 3⟩, (3, 9⟩, (9, 12⟩, (12, +∞)}	6.75
	C_{36}	{0, 6/7, 1}; {(-∞, 3⟩, (3, 9⟩, (9, +∞)}	3.857
	C_{53}	{0, 2/4, 1}; {(-∞, 3⟩, (3, 6⟩, (6, +∞)}	4.5
	C_{62}	{0, 1}; {(-∞, 12⟩, (12, +∞)}	12
	C_{65}	{0, 1}; {(-∞, 6⟩, (6, +∞)}	6

As there are no realizations of the rest climate-weather change process conditional sojourn times at the climate-weather states then it is impossible to estimate their empirical conditional mean values.

3.1.4. Climate-weather change process at Baltic Sea open waters area

Defining parameters and data collection of climate-weather change process at Baltic Sea open waters area

Based on the expert opinion, there are distinguished the following $w = 6$ climate-weather states:

- the climate-weather state c_1 – the wave height from 0 m up to 2 m and the wind speed from 0 m/s up to 17.2 m/s,
- the climate-weather state c_2 – the wave height from 2 m up to 5.5 m and the wind speed from 0 m/s up to 17.2 m/s,
- the climate-weather state c_3 – the wave height from 5.5 m up to 14 m and the wind speed from 0 m/s up to 17.2 m/s,
- the climate-weather state c_4 – the wave height from 0 m up to 2 m and the wind speed from 17.2 m/s to 33 m/s,
- the climate-weather state c_5 – the wave height from 2 m up to 5.5 m and the wind speed from 17.2 m/s to 33 m/s,
- the climate-weather state c_6 – the wave height from 5.5 m up to 14 m and the wind speed from 17.2 m/s to 33 m/s.

The unknown parameters of the climate-weather change process semi-Markov model are:

- the initial probabilities $q_b(0)$, $b = 1,2,...,6$, of the climate-weather change process staying at the particular states c_b at the moment $t = 0$,
- the probabilities q_{bl}, $b,l = 1,2,...,6$, $b \neq l$, of the climate-weather change process transitions from the climate-weather state c_b into the climate-weather state c_l,
- the distributions of the climate-weather change conditional sojourn times C_{bl}, $b,l = 1,2,...,6$, $b \neq l$, at the particular climate-weather states and their mean values $M_{bl} = E[C_{bl}]$, $b,l = 1,2,...,6$, $b \neq l$.

To identify all these parameters of the climate-weather change process the statistical data about this process is needed.

The collected by the system operators statistical data necessary to evaluating the initial transient probabilities of the climate-weather change process at the particular states are:

- the climate-weather change process observation / experiment time

$$\Theta = 6 \text{ years } (1988–1993),$$

- the number of the climate-weather change process realizations $n(0) = 7544$,
- the vector of realizations of the numbers of the climate-weather change process staying at the particular climate-weather state c_b at the initial moment $t = 0$

$$[n_b(0)] = [5747, 1567, 1, 0, 176, 53].$$

The collected statistical data necessary to evaluating the probabilities of transitions of the climate-weather state change process $C(t)$ between the climate-weather states are:

- the matrix of realizations of the numbers of climate-weather change process transitions from the state c_b into the state c_l during the experiment time

$$[n_{bl}] = \begin{bmatrix} 0 & 6762 & 0 & 0 & 1 & 0 \\ 2540 & 0 & 0 & 0 & 735 & 0 \\ 0 & 17 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 506 & 4 & 0 & 0 & 128 \\ 0 & 54 & 21 & 0 & 75 & 0 \end{bmatrix},$$

- the vector of realizations of the total numbers of the climate-weather change process transitions from the climate-weather state c_b during the experiment time

$$[n_b] = [6873, 3275, 19, 1, 638, 150].$$

The statistical data for the conditional sojourn times C_{bl} at the climate-weather states c_b when the next climate-weather state is c_l, $b,l = 1,2,...,6$, $b \neq l$, related to the Baltic Sea open waters area are given in Figure 5.

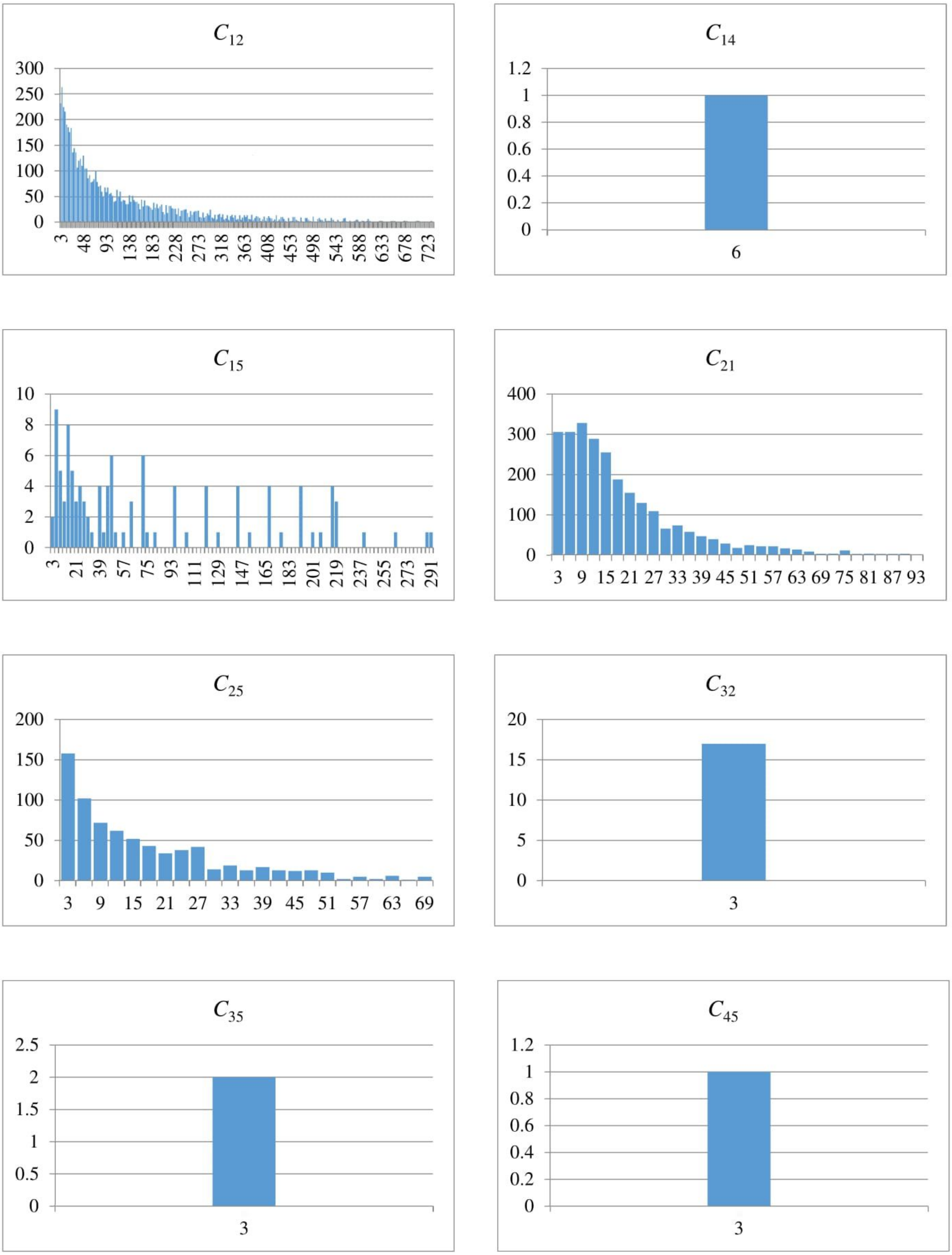


Figure 5. The graphical representation of the statistical data for the conditional sojourn times of the climate-weather change process at the Baltic Sea open waters area.

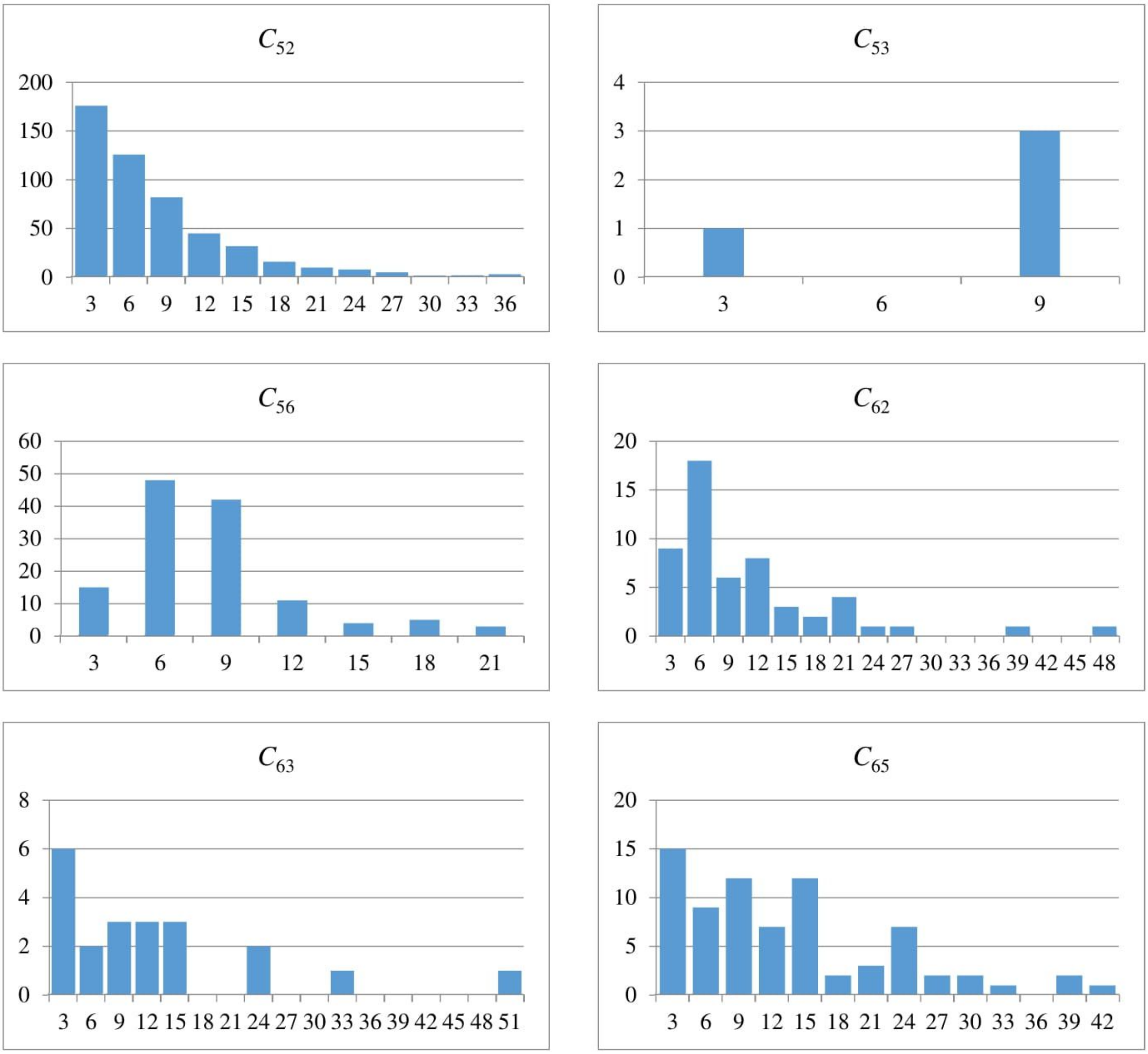


Figure 5 (*continuation*). The graphical representation of the statistical data for the conditional sojourn times of the climate-weather change process at the Baltic Sea open waters area.

Evaluating basic parameters of climate-weather change process at Baltic Sea open waters area

On the basis of the statistical data from section above, it is possible to evaluate the following unknown basic parameters of the climate-weather change process:

- the vector

$$[q_b(0)] = [0.762, 0.208, 0, 0, 0.023, 0.007]$$

of the initial probabilities $q_b(0)$, $b,l = 1,2,...,6$, of the climate-weather change process staying at the particular states c_b at the $t = 0$,

- the matrix

$$[q_{bl}] = \begin{bmatrix} 0 & 0.984 & 0 & 0 & 0.016 & 0 \\ 0.776 & 0 & 0 & 0 & 0.224 & 0 \\ 0 & 0.895 & 0 & 0 & 0.105 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0.793 & 0.006 & 0 & 0 & 0.201 \\ 0 & 0.36 & 0.14 & 0 & 0.5 & 0 \end{bmatrix},$$

of the probabilities q_{bl}, $b,l = 1,2,...,6$, of transitions of the climate-weather change process from the climate-weather state c_b into the climate-weather state c_l.

Identification of distribution functions of climate-weather change process at Baltic Sea open waters area

Using the procedure given in (Kołowrocki, Kuligowska & Torbicki, 2017), the results and the statistical data from section above, we may verify the hypotheses on the distributions of the climate-weather change process conditional sojourn times C_{bl}, $b,l = 1,2,...,6,\ b \neq l$, at the particular states. To do this, we need a sufficient number of realizations of these variables (Rice, 2007; Vercellis, 2009; Wilson, Graves, Hamada et al., 2006), namely, the sets of their realizations should contain at least 30 realizations coming from the experiment.

This condition is not satisfied for the all statistical data we have in disposal. However, when the realizations of conditional sojourn times C_{bl}, $b,l = 1,2,...,6,\ b \neq l$, at the particular climate-weather states are more than 30, we can get the distributions of the climate-weather change process conditional sojourn times with the parameters given in Table 7.

For the distributions identified in this section, by application either the general formulae for the mean value given by (2.12) or the particular formulae (2.13)–(2.19) in Kołowrocki & Soszyńska-Budny, 2011), the mean values $M_{bl} = E[C_{bl}]$, $b,l = 1,2,...,6,\ b \neq l$, of the climate-weather change process conditional sojourn times at the particular climate-weather states related to the Baltic Sea open waters area can be determined (Table 7).

Because of the lack of sufficient numbers of realizations of the climate-weather change process conditional sojourn times at the climate-weather states, it is not possible to identify statistically their distributions. In those cases when the number of the conditional sojourn times realizations is non-zero it is possible to find the approximate empirical values of the mean values $M_{bl} = E[C_{bl}]$, $b,l = 1,2,...,6,\ b \neq l$, of the conditional sojourn times at the particular climate-weather states that are given in Table 8.

As there are no realizations of the rest climate-weather change process conditional sojourn times at the climate-weather states then it is impossible to estimate their empirical conditional mean values.

Table 7. Distribution functions of conditional sojourn times when their realizations are more than 30

Distribution	Sojourn Time	Parameters	Mean
exponential	C_{52}	$x_{52} = 2.25$ $\alpha_{52} = 0.1726$	8.044
	C_{62}	$x_{62} = 0$ $\alpha_{62} = 0.0918$	10.893
	C_{65}	$x_{65} = 0.84$ $\alpha_{65} = 0.0796$	13.403
double trapezium	C_{12}	$x_{12} = 0$ $y_{12} = 737.87$ $z_{12} = 124.1575$ $w_{12} = 0$ $q_{12} = 0.0082$	287.343
	C_{15}	$x_{15} = 0$ $y_{15} = 316.8$ $z_{15} = 79.0091$ $w_{15} = 0.0003$ $q_{15} = 0.0133$	157.027
	C_{21}	$x_{21} = 2.1$ $y_{21} = 93.9$ $z_{21} = 18.3803$ $w_{21} = 0.0002$ $q_{21} = 0.0669$	40.775
	C_{25}	$x_{25} = 1.78$ $y_{25} = 70.1$ $z_{25} = 16.9959$ $w_{25} = 0.0028$ $q_{25} = 0.0881$	42.468
	C_{56}	$x_{56} = 2.18$ $y_{56} = 21.86$ $z_{56} = 8.25$ $w_{56} = 0.0143$ $q_{56} = 0.0715$	17.356

Table 8. Distribution functions of conditional sojourn times when their realizations are less than 30

Distribution	Sojourn Time	Parameters, intervals	Mean
empirical	C_{14}	{0, 1}; {(-∞, 6⟩, (6, +∞)}	6
	C_{32}	{0, 1}; {(-∞, 3⟩, (3, +∞)}	3
	C_{35}	{0, 1}; {(-∞, 3⟩, (3, +∞)}	3
	C_{45}	{0, 1}; {(-∞, 3⟩, (3, +∞)}	3
	C_{53}	{0, 1/4, 1}; {(-∞, 3⟩, (3, 9⟩, (9, +∞)}	7.5
	C_{63}	{0, 6/21, 8/21, 11/21, 14/21, 17/21, 19/21, 20/21, 1}; {(-∞, 3⟩, (3, 6⟩, (6, 9⟩, (9, 12⟩, (12, 15⟩, (15, 24⟩, (24, 33⟩, (33, 51⟩, (51, +∞)}	12.857

3.1.5. Climate-weather change process at Karlskrona Port area

Defining parameters and data collection of climate-weather change process at Karlskrona Port area

Based on the expert opinion, there are distinguished the following $w = 6$ climate-weather states:

- the climate-weather state c_1 – the wave height from 0 m up to 2 m and the wind speed from 0 m/s up to 17.2 m/s,
- the climate-weather state c_2 – the wave height from 2 m up to 5.5 m and the wind speed from 0 m/s up to 17.2 m/s,
- the climate-weather state c_3 – the wave height from 5.5 m up to 14 m and the wind speed from 0 m/s up to 17.2 m/s,
- the climate-weather state c_4 – the wave height from 0 m up to 2 m and the wind speed from 17.2 m/s to 33 m/s,
- the climate-weather state c_5 – the wave height from 2 m up to 5.5 m and the wind speed from 17.2 m/s to 33 m/s,
- the climate-weather state c_6 – the wave height from 5.5 m up to 14 m and the wind speed from 17.2 m/s to 33 m/s.

The unknown parameters of the climate-weather change process semi-Markov model are:

- the initial probabilities $q_b(0)$, $b = 1,2,...,6$, of the climate-weather change process staying at the particular states c_b at the moment $t = 0$,
- the probabilities q_{bl}, $b,l = 1,2,...,6$, $b \neq l$, of the climate-weather change process transitions from the climate-weather state c_b into the climate-weather state c_l,
- the distributions of the climate-weather change conditional sojourn times C_{bl}, $b,l = 1,2,...,6$, $b \neq l$, at the particular climate-weather states and their mean values $M_{bl} = E[C_{bl}]$, $b,l = 1,2,...,6$, $b \neq l$.

To identify all these parameters of the climate-weather change process the statistical data about this process is needed.

The collected by the system operators statistical data necessary to evaluating the initial transient probabilities of the climate-weather change process at the particular states are:

- the climate-weather change process observation / experiment time

$$\Theta = 6 \text{ years } (1988\text{–}1993),$$

- the number of the climate-weather change process realizations $n(0) = 1886$,
- the vector of realizations of the numbers of the climate-weather change process staying at the particular climate-weather state c_b at the initial moment $t = 0$

$$[n_b(0)] = [647, 9, 793, 42, 391, 4].$$

The collected statistical data necessary to evaluating the probabilities of transitions of the climate-weather state change process $C(t)$ between the climate-weather states are:

- the matrix of realizations of the numbers of climate-weather change process transitions from the state c_b into the state c_l during the experiment time

$$[n_{bl}] = \begin{bmatrix} 0 & 66 & 803 & 19 & 283 & 0 \\ 19 & 0 & 5 & 21 & 0 & 0 \\ 962 & 3 & 0 & 96 & 363 & 8 \\ 3 & 2 & 107 & 0 & 0 & 2 \\ 211 & 0 & 453 & 0 & 0 & 7 \\ 1 & 0 & 2 & 0 & 6 & 0 \end{bmatrix},$$

- the vector of realizations of the total numbers of the climate-weather change process transitions from the climate-weather state c_b during the experiment time

$$[n_b] = [1171, 45, 1432, 114, 671, 9].$$

The statistical data for the conditional sojourn times C_{bl} at the climate-weather states c_b when the next climate-weather state is c_l, $b,l = 1,2,...,6$, $b \neq l$, related to the Karlskrona Port area are given in Figure 6.

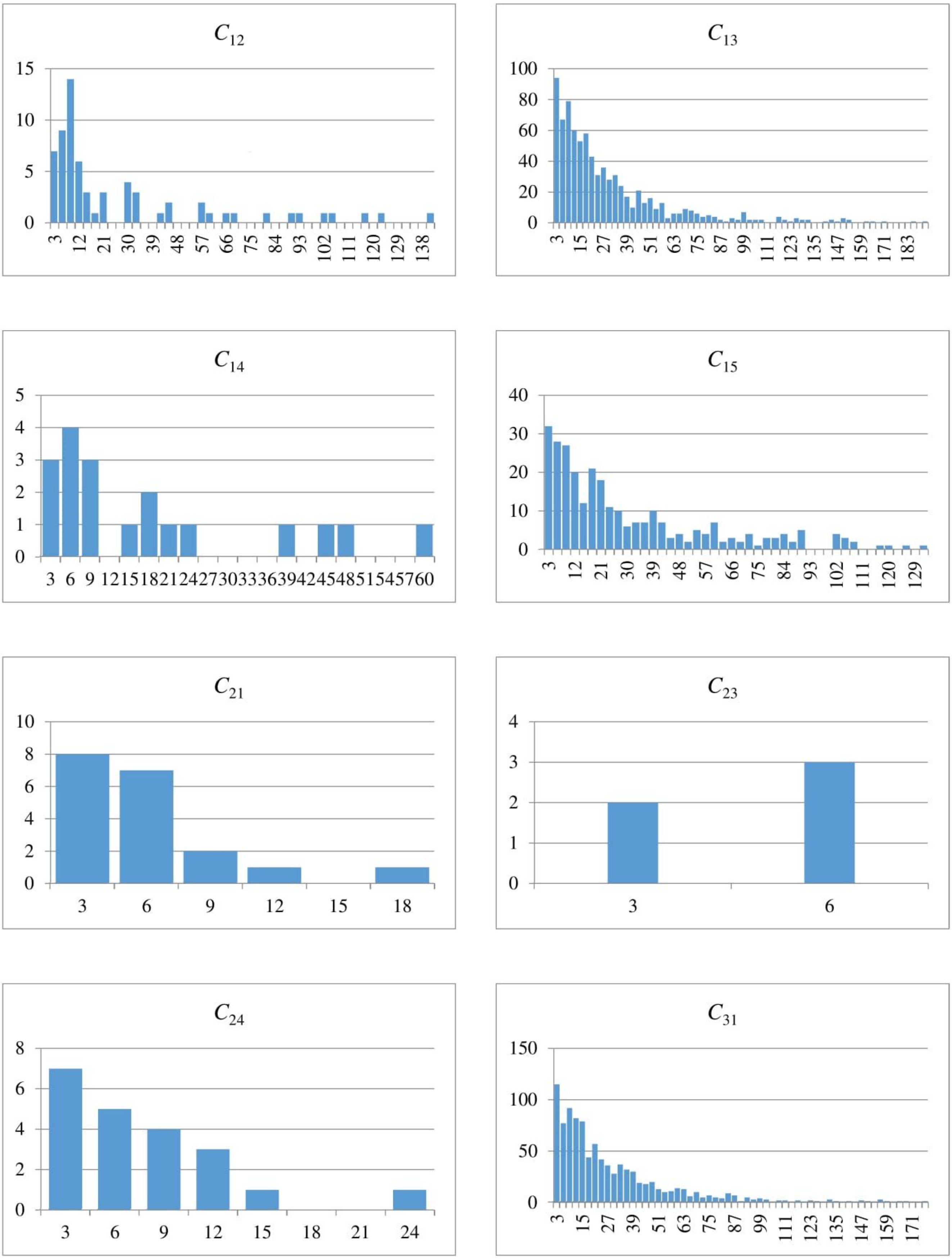


Figure 6. The graphical representation of the statistical data for the conditional sojourn times of the climate-weather change process at the Karlskrona Port area.

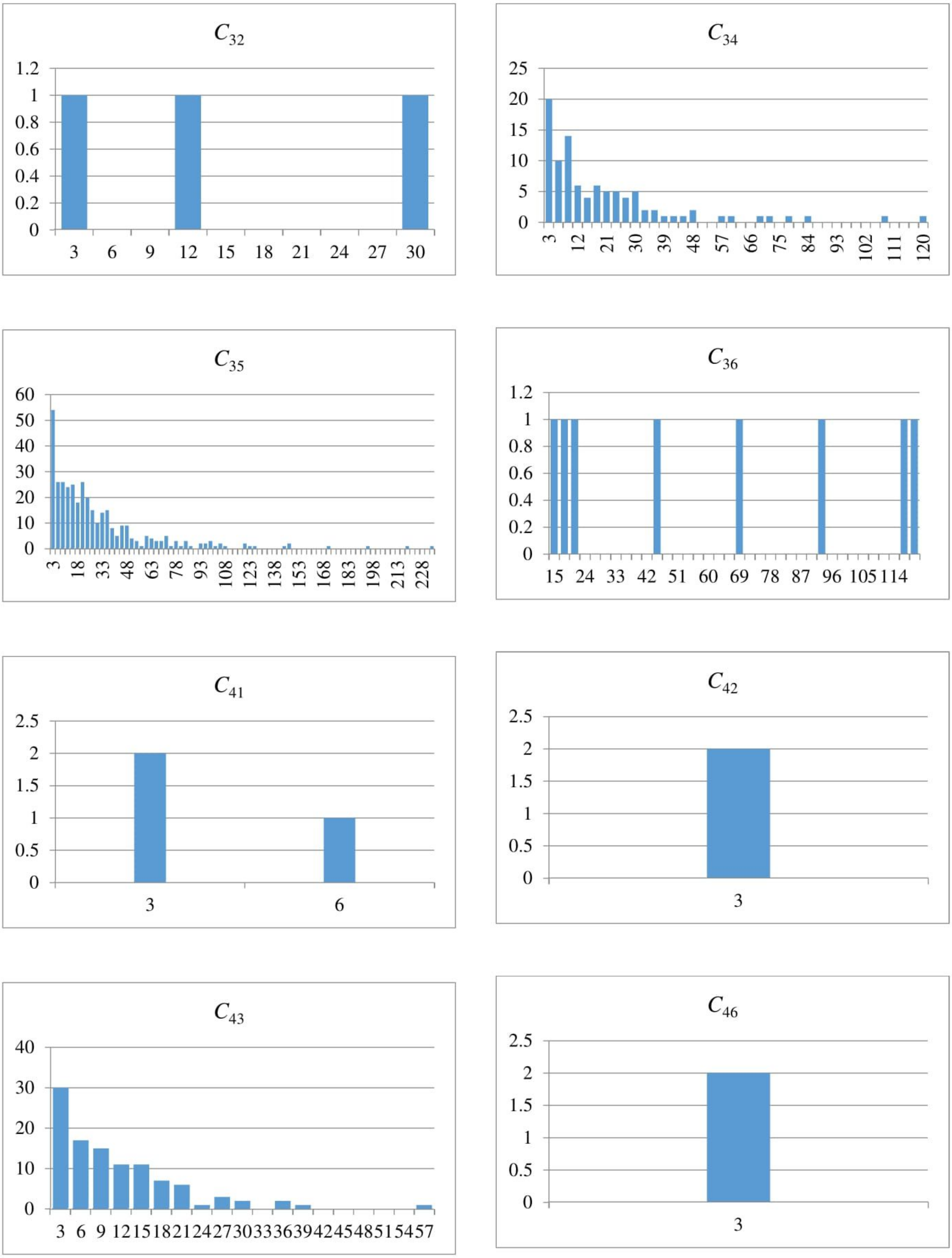


Figure 6 (*continuation*). The graphical representation of the statistical data for the conditional sojourn times of the climate-weather change process at the Karlskrona Port area.

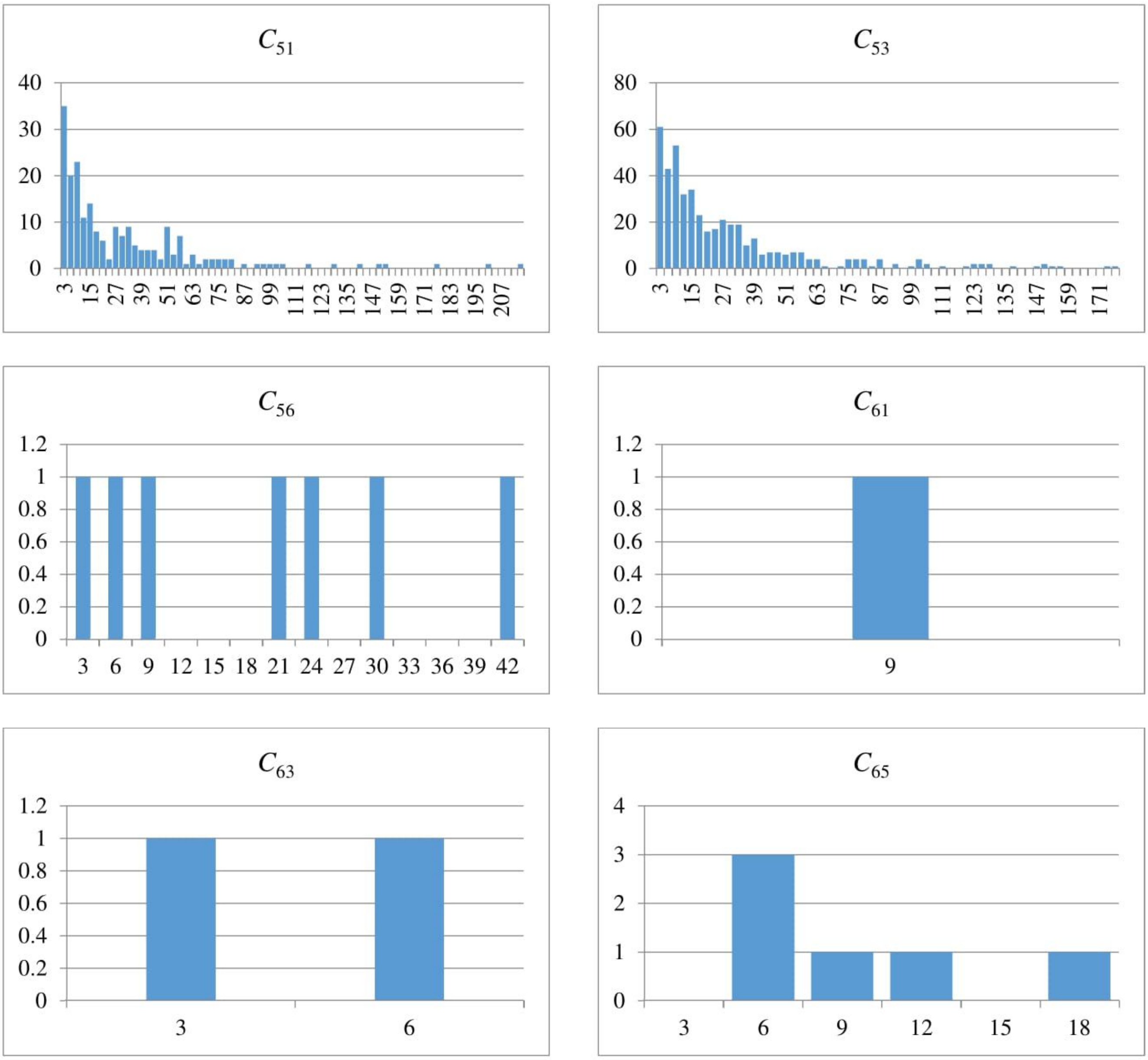


Figure 6 (*continuation*). The graphical representation of the statistical data for the conditional sojourn times of the climate-weather change process at the Karlskrona Port area.

Evaluating basic parameters of climate-weather change process at Karlskrona Port area

On the basis of the statistical data from section above, it is possible to evaluate the following unknown basic parameters of the climate-weather change process:

- the vector

$$[q_b(0)] = [0.343, 0.005, 0.421, 0.022, 0.207, 0.002]$$

of the initial probabilities $q_b(0)$, $b,l = 1,2,...,6,$ of the climate-weather change process staying at the particular states c_b at the $t = 0$,

- the matrix

$$[q_{bl}] = \begin{bmatrix} 0 & 0.056 & 0.686 & 0.016 & 0.242 & 0 \\ 0.422 & 0 & 0.111 & 0.467 & 0 & 0 \\ 0.672 & 0.002 & 0 & 0.067 & 0.253 & 0.006 \\ 0.026 & 0.018 & 0.938 & 0 & 0 & 0.018 \\ 0.314 & 0 & 0.676 & 0 & 0 & 0.01 \\ 0.111 & 0 & 0222 & 0 & 0.667 & 0 \end{bmatrix},$$

of the probabilities q_{bl}, $b,l = 1,2,...,6$, of transitions of the climate-weather change process from the climate-weather state c_b into the climate-weather state c_l.

Identification of distribution functions of climate-weather change process at Karlskrona Port area

Using the procedure given in (Kołowrocki, Kuligowska & Torbicki, 2017), the results and the statistical data from section above, we may verify the hypotheses on the distributions of the climate-weather change process conditional sojourn times C_{bl}, $b,l = 1,2,...,6$, $b \neq l$, at the particular states. To do this, we need a sufficient number of realizations of these variables (Rice, 2007; Vercellis, 2009; Wilson, Graves, Hamada et al., 2006), namely, the sets of their realizations should contain at least 30 realizations coming from the experiment. This condition is not satisfied for the all statistical data we have in disposal. However, when the realizations of conditional sojourn times C_{bl}, $b,l = 1,2,...,6$, $b \neq l$, at the particular climate-weather states are more than 30, we can get the distributions of the climate-weather change process conditional sojourn times with the parameters given in Table 9.

For the distributions identified in this section, by application either the general formulae for the mean value given by (2.12) or the particular formulae (2.13)–(2.19) in Kołowrocki & Soszyńska-Budny, 2011), the mean values $M_{bl} = E[C_{bl}]$, $b,l = 1,2,...,6$, $b \neq l$, of the climate-weather change process conditional sojourn times at the particular climate-weather states related to the Karlskrona Port area can be determined (Table 9).

Because of the lack of sufficient numbers of realizations of the climate-weather change process conditional sojourn times at the climate-weather states, it is not possible to identify statistically their distributions.

In those cases when the number of the conditional sojourn times realizations is non-zero it is possible to find the approximate empirical values of the mean values $M_{bl} = E[C_{bl}]$, $b,l = 1,2,...,6$, $b \neq l$, of the conditional sojourn times at the particular climate-weather states that are given in Table 10.

Table 9. Distribution functions of conditional sojourn times when their realizations are more than 30

Distribution	Sojourn Time	Parameters	Mean
Gamma	C_{12}	$\alpha_{12} = 0.7294$ $\beta_{12} = 39.6949$	28.953
	C_{43}	$\alpha_{43} = 1.3738$ $\beta_{43} = 8.0062$	11.299
exponential	C_{34}	$x_{34} = 0$ $\alpha_{34} = 0.0486$	20.576
	C_{35}	$x_{35} = 0$ $\alpha_{35} = 0.0332$	30.12
	C_{51}	$x_{51} = 0$ $\alpha_{51} = 0.0321$	31.153
double trapezium	C_{13}	$x_{13} = 0$ $y_{13} = 195.75$ $z_{13} = 29.4471$ $w_{13} = 0.0002$ $q_{13} = 0.0297$	81.452
	C_{15}	$x_{15} = 0$ $y_{15} = 136.62$ $z_{15} = 30.4134$ $w_{15} = 0.0005$ $q_{15} = 0.0279$	63.455
	C_{31}	$x_{31} = 0.2$ $y_{31} = 179.72$ $z_{31} = 28.1757$ $w_{31} = 0.0002$ $q_{31} = 0.0213$	74.811
	C_{53}	$x_{53} = 0$ $y_{53} = 182.38$ $z_{53} = 28.0795$ $w_{53} = 0.0003$ $q_{53} = 0.0277$	78.469

Table 10. Distribution functions of conditional sojourn times when their realizations are less than 30

Distribution	Sojourn Time	Parameters, intervals	Mean
empirical	C_{14}	{0, 3/19, 7/19, 10/19, 11/19, 13/19, 14/19, 15/19, 16/19, 17/19, 18/19, 1}; {(-∞, 3⟩, (3, 6⟩, (6, 9⟩, (9, 15⟩, (15, 18⟩, (18, 21⟩, (21, 24⟩, (24, 39⟩, (39, 45⟩, (45, 48⟩, (48, 60⟩, (60, +∞)}	18.316
	C_{21}	{0, 8/19, 15/19, 17/19, 18/19, 1}; {(-∞, 3⟩, (3, 6⟩, (6, 9⟩, (9, 12⟩, (12, 18⟩, (18, +∞)}	6
	C_{23}	{0, 2/5, 1}; {(-∞, 3⟩, (3, 6⟩, (6, +∞)}	4.8

Table 10 (*continuation*). Distribution functions of conditional sojourn times when their realizations are less than 30

Distribution	Sojourn Time	Parameters, intervals	Mean
empirical	C_{24}	{0, 7/21, 12/21, 16/21, 19/21, 20/21, 1}; {(-∞, 3⟩, (3, 6⟩, (6, 9⟩, (9, 12⟩, (12, 15⟩, (15, 24⟩, (24, +∞)}	7.714
	C_{32}	{0, 1/3, 2/3, 1}; {(-∞, 3⟩, (3, 12⟩, (12, 30⟩, (30, +∞)}	15
	C_{36}	{0, 1/8, 2/8, 3/8, 4/8, 5/8, 6/8, 7/8, 1}; {(-∞, 15⟩, (15, 18⟩, (18, 21⟩, (21, 45⟩, (45, 69⟩, (69, 93⟩, (93, 117⟩, (117, 120⟩, (120, +∞)}	62.25
	C_{41}	{0, 2/3, 1}; {(-∞, 3⟩, (3, 6⟩, (6, +∞)}	4
	C_{42}	{0, 1}; {(-∞, 3⟩, (3, +∞)}	3
	C_{46}	{0, 1}; {(-∞, 3⟩, (3, +∞)}	3
	C_{56}	{0, 1/7, 2/7, 3/7, 4/7, 5/7, 6/7, 1}; {(-∞, 3⟩, (3, 6⟩, (6, 9⟩, (9, 21⟩, (21, 24⟩, (24, 30⟩, (30, 42⟩, (42, +∞)}	19.286
	C_{61}	{0, 1}; {(-∞, 9⟩, (9, +∞)}	9
	C_{63}	{0, 1/2, 1}; {(-∞, 3⟩, (3, 6⟩, (6, +∞)}	4.5
	C_{65}	{0, 3/6, 4/6, 5/6, 1}; {(-∞, 6⟩, (6, 9⟩, (9, 12⟩, (12, 18⟩, (18, +∞)}	9.5

As there are no realizations of the rest climate-weather change process conditional sojourn times at the climate-weather states then it is impossible to estimate their empirical conditional mean values.

3.2. Port oil piping transportation system operation area

3.2.1. System description

The port oil piping transportation system is operating at one of the Baltic Oil Terminals. It is designed for transporting oil products such like petrol and oil between the pier of Gdynia Port and Oil Terminal in Dębogórze.

The considered terminal is composed of four parts *A*, *B* and *C*, linked by the piping transportation system with Pier, and a post *PB*. The scheme of this terminal is presented in Figure 7. More information about the port oil transportation system, its assets and interconnections between them could be found in (Drzazga, Kołowrocki, Soszyńska-Budny & Torbicki, 2016). Moreover, in Figure 8 is shown where particular parts of the considered terminal are located.

In following subsections, we will analyze the climate-weather change process of the port oil piping transportation system operating at the underwater Baltic Sea area (Figure 9) and at the land Baltic seaside area (Figure 8). In points 1-4 marked in above figures were obtained the climate-weather data (GMU, 2020).

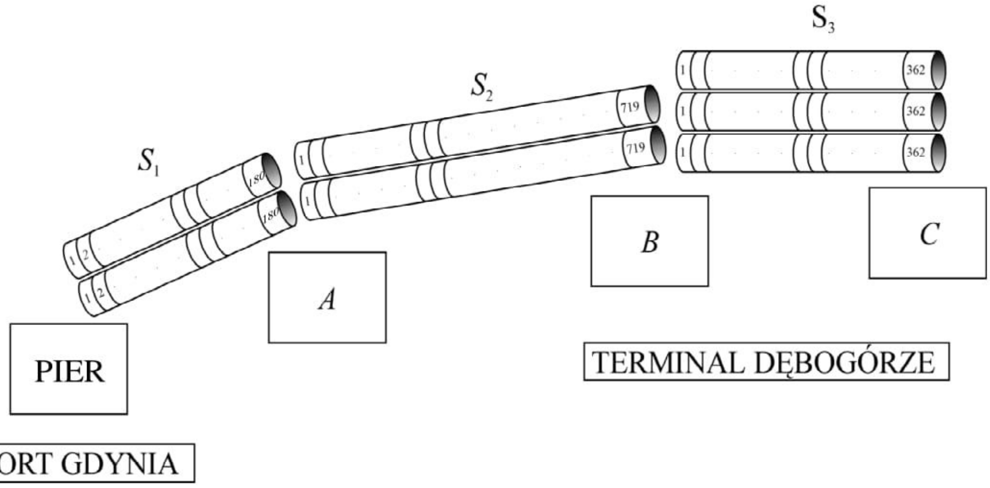


Figure 7. The scheme of the port oil transportation system.

Figure 8. The port oil piping transportation system operating between the Gdynia Port and the Terminal in Dębogórze.

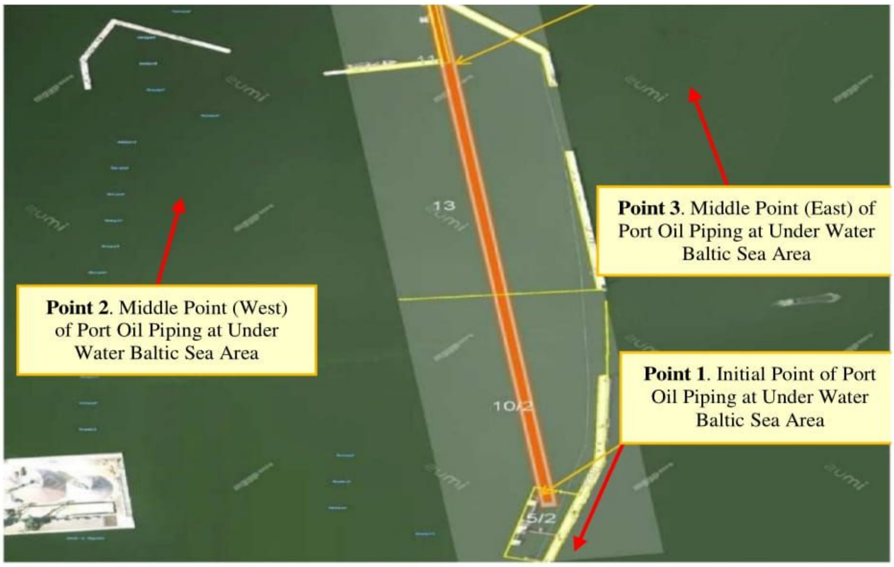


Figure 9. The port oil piping transportation system alignment in the Gdynia Port.

3.2.2. Climate-weather change process at under water Baltic Sea area

Defining parameters and data collection of climate-weather change process at under water Baltic Sea area

Based on the expert opinions, there are distinguished the following $w = 6$ climate-weather states:

- the climate-weather state c_1 – the wave height from 0 up to 2 m and the wind speed from 0 m/s up to 17 m/s,
- the climate-weather state c_2 – the wave height from 2 m up to 5 m and the wind speed from 0 m/s up to 17 m/s,
- the climate-weather state c_3 – the wave height from 5 m up to 14 m and the wind speed from 0 m/s up to 17 m/s,
- the climate-weather state c_4 – the wave height from 0 up to 2 m and the wind speed from 17 m/s up to 33 m/s,
- the climate-weather state c_5 – the wave height from 2 m up to 5 m and the wind speed from 17 m/s up to 33 m/s,

- the climate-weather state c_6 – the wave height from 5 m up to 14 m and the wind speed from 17 m/s up to 33 m/s.

The unknown parameters of the climate-weather change process semi-Markov model are:

- the initial probabilities $q_b(0)$, $b = 1,2,...,6$, of the climate-weather change process staying at the particular states c_b at the moment $t = 0$,
- the probabilities q_{bl}, $b,l = 1,2,...,6$, $b \neq l$, of the climate-weather change process transitions from the climate-weather state c_b into the climate-weather state c_l,
- the distributions of the climate-weather change conditional sojourn times C_{bl}, $b,l = 1,2,...,6$, $b \neq l$, at the particular climate-weather states and their mean values $M_{bl} = E[C_{bl}]$, $b,l = 1,2,...,6$, $b \neq l$.

To identify all these parameters of the climate-weather change process the statistical data about this process is needed.

The collected by the system operators statistical data necessary to evaluating the initial transient probabilities of the climate-weather change process at the particular states are:

- the climate-weather change process observation / experiment time

$$\Theta = 6 \text{ years } (1988–1993),$$

- the number of the climate-weather change process realizations $n(0) = 5658$,
- the vector of realizations of the numbers of the climate-weather change process staying at the particular climate-weather state c_b at the initial moment $t = 0$

$$[n_b(0)] = [4633, 885, 9, 4, 89, 38].$$

The collected statistical data necessary to evaluating the probabilities of transitions of the climate-weather state change process $C(t)$ between the climate-weather states are:

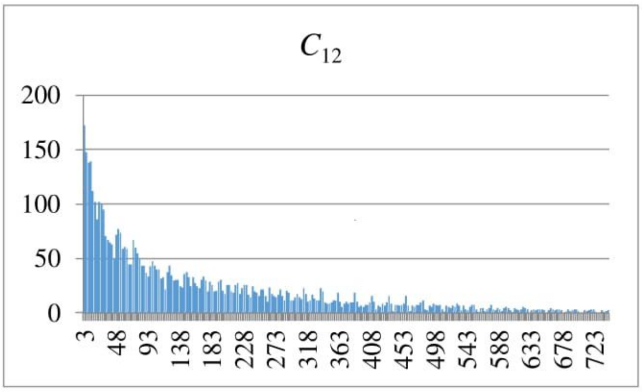
- the matrix of realizations of the numbers of climate-weather change process transitions from the state c_b into the state c_l during the experiment time

$$[n_{bl}] = \begin{bmatrix} 0 & 5131 & 0 & 12 & 52 & 0 \\ 1530 & 0 & 8 & 1 & 361 & 2 \\ 0 & 54 & 0 & 0 & 0 & 5 \\ 2 & 0 & 0 & 0 & 8 & 0 \\ 3 & 241 & 12 & 1 & 0 & 71 \\ 0 & 15 & 65 & 0 & 20 & 0 \end{bmatrix},$$

- the vector of realizations of the total numbers of the climate-weather change process transitions from the climate-weather state c_b during the experiment time

$$[n_b] = [5195, 1902, 59, 10, 328, 100].$$

The statistical data for the conditional sojourn times C_{bl} at the climate-weather states c_b when the next climate-weather state is c_l, $b,l = 1,2,...,6$, $b \neq l$, related to the under water Baltic Sea area are given in Figure 10.



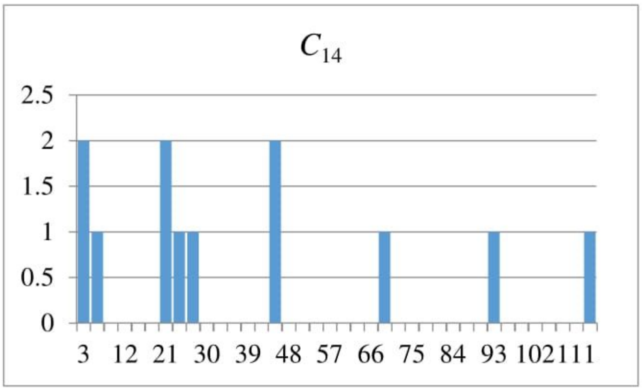


Figure 10. The graphical representation of the statistical data for the conditional sojourn times of the climate-weather change process at the under water Baltic Sea area.

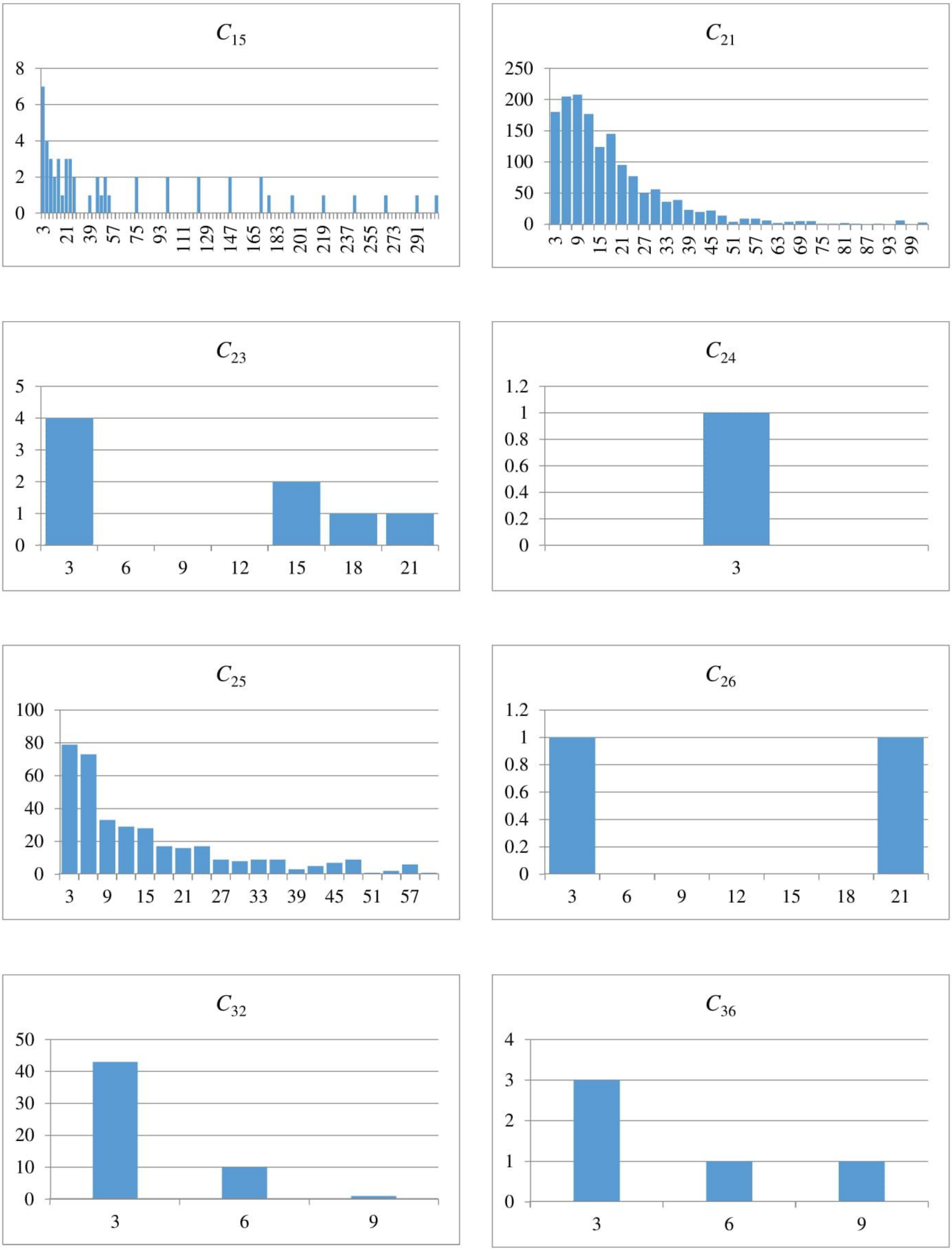


Figure 10 (*continuation*). The graphical representation of the statistical data for the conditional sojourn times of the climate-weather change process at the under water Baltic Sea area.

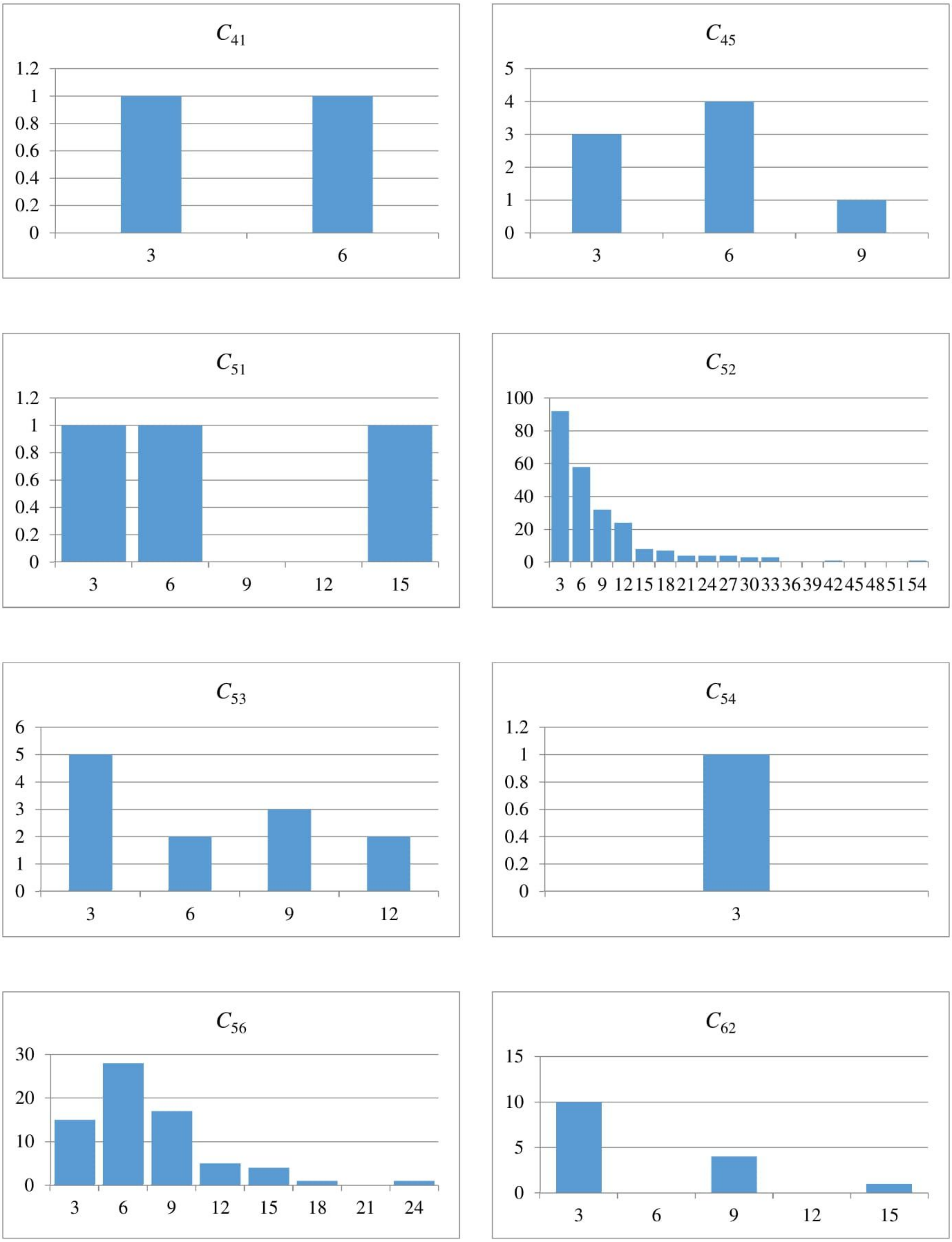
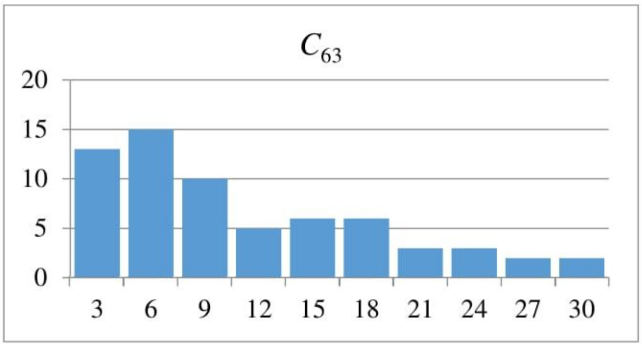


Figure 10 (*continuation*). The graphical representation of the statistical data for the conditional sojourn times of the climate-weather change process at the under water Baltic Sea area.



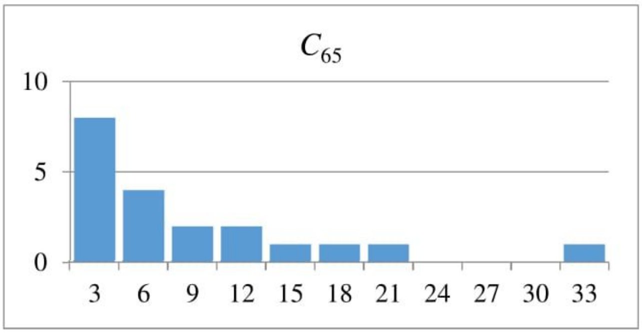


Figure 10 (*continuation*). The graphical representation of the statistical data for the conditional sojourn times of the climate-weather change process at the under water Baltic Sea area.

Evaluating basic parameters of climate-weather change process at under water Baltic Sea area

On the basis of the statistical data from Section 3.2.1., it is possible to evaluate the following unknown basic parameters of the climate-weather change process:

- the vector

  $[q_b(0)]$ = [0.818, 0.156, 0.002, 0.001, 0.016, 0.007]

  of the initial probabilities $q_b(0)$, $b,l = 1,2,...,6$, of the climate-weather change process staying at the particular states c_b at the $t = 0$,

- the matrix

$$[q_{bl}] = \begin{bmatrix} 0 & 0.988 & 0 & 0.002 & 0.01 & 0 \\ 0.804 & 0 & 0.004 & 0.001 & 0.19 & 0.001 \\ 0 & 0.915 & 0 & 0 & 0 & 0.085 \\ 0.2 & 0 & 0 & 0 & 0.8 & 0 \\ 0.009 & 0.735 & 0.037 & 0.003 & 0 & 0.216 \\ 0 & 0.15 & 0.65 & 0 & 0.2 & 0 \end{bmatrix}$$

  of the probabilities q_{bl}, $b,l = 1,2,...,6$, of transitions of the climate-weather change process from the climate-weather state c_b into the climate-weather state c_l.

Identification of distribution functions of climate-weather change process at under water Baltic Sea area

Using the procedure given in (Kołowrocki, Kuligowska & Torbicki, 2017), the results and the statistical data from Section 3.2.1., we may verify the hypotheses on the distributions of the climate-weather change process conditional sojourn times C_{bl}, $b,l = 1,2,...,6$, $b \neq l$, at the particular states. To do this, we need a sufficient number of realizations of these variables (Rice, 2007; Vercellis, 2009; Wilson, Graves, Hamada et al. 2006), namely, the sets of their realizations should contain at least 30 realizations coming from the experiment. This condition is not satisfied for the all statistical data we have in disposal and that are presented in Section 3.2.1. However, when the realizations of conditional sojourn times C_{bl}, $b,l = 1,2,...,6$, $b \neq l$, at the particular climate-weather states are more than 30, we can get the distributions of the climate-weather change process conditional sojourn times with the parameters given in Table 11.

For the distributions identified in this section, by application either the general formulae for the mean value given by (2.12) or the particular formulae (2.13)–(2.19) in (Kołowrocki & Soszyńska-Budny, 2011), the mean values $M_{bl} = E[C_{bl}]$, $b,l = 1,2,...,6$, $b \neq l$, of the climate-weather change process conditional sojourn times at the particular climate-weather states related to the under water Baltic Sea area can be determined (Table 11).

Because of the lack of sufficient numbers of realizations of the climate-weather change process conditional sojourn times at the climate-weather states, it is not possible to identify statistically their distributions. In those cases when the number of the conditional sojourn times realizations is non-zero it is possible to find the approximate empirical values of the mean values $M_{bl} = E[C_{bl}]$ of the conditional sojourn times at the particular climate-weather states that are given in Table 12.

Table 11. Distribution functions of conditional sojourn times when their realizations are more than 30

Distribution	Sojourn Time	Parameters	Mean
Gamma	C_{63}	$\alpha_{63} = 1.6899$ $\beta_{63} = 5.834$	11.169
exponential	C_{15}	$x_{15} = 0$ $\alpha_{15} = 0.0144$	69.444
	C_{25}	$x_{25} = 1.5$ $\alpha_{25} = 0.0724$	15.312
	C_{52}	$x_{52} = 1.41$ $\alpha_{52} = 0.1435$	8.379
double trapezium	C_{12}	$x_{12} = 0$ $y_{12} = 748.25$ $z_{12} = 166.0513$ $w_{12} = 0.0001$ $q_{12} = 0.0087$	351.424
	C_{21}	$x_{21} = 1.73$ $y_{21} = 103.33$ $z_{21} = 17.7314$ $w_{21} = 0.0008$ $q_{21} = 0.0463$	48.758
chimney	C_{32}	$A_{32} = 0$ $K_{32} = 0.7963$ $D_{32} = 0.2037$ $x_{32} = 2.57$ $y_{32} = 9.45$ $z1_{32} = 2.57$ $z2_{32} = 3.43$	3.701
	C_{56}	$A_{56} = 0$ $K_{56} = 0.8451$ $D_{56} = 0.1549$ $x_{56} = 1.69$ $y_{56} = 25.36$ $z1_{56} = 1.69$ $z2_{56} = 9.58$	7.468

As there are no realizations of the rest climate-weather change process conditional sojourn times at the climate-weather states then it is impossible to estimate their empirical conditional mean values.

Table 12. Distribution functions of conditional sojourn times when their realizations are less than 30

Distribution	Sojourn Time	Parameters, intervals	Mean
empirical	C_{14}	{0, 2/12, 3/12, 5/12, 6/12, 7/12, 9/12, 10/12, 11/12, 1} {(-∞, 3〉, (3, 6〉, (6, 21〉, (21, 24〉, (24, 27〉, (27, 45〉, (45, 69〉, (69, 93〉, (93,114〉 (114, +∞)}	39.25
	C_{23}	{0, 4/8, 6/8, 7/8, 1}; {(-∞, 3〉, (3, 15〉, (15, 18〉, (18, 21〉, (21, +∞)}	10.125
	C_{24}	{0, 1}; {(-∞, 3〉, (3, +∞)}	3
	C_{26}	{0, 1/2, 1}; {(-∞, 3〉, (3, 21〉, (21, +∞)}	12
	C_{36}	{0, 3/5, 4/5, 1}; {(-∞, 3〉, (3, 6〉, (6, 9〉, (9, +∞)}	4.8
	C_{41}	{0, 1/2, 1}; {(-∞, 3〉, (3, 6〉, (6, +∞)}	4.5
	C_{45}	{0, 3/8, 7/8, 1}; {(-∞, 3〉, (3, 6〉, (6, 9〉, (9, +∞)}	5.25
	C_{51}	{0, 1/3, 2/3, 1}; {(-∞, 3〉, (3, 6〉, (6, 15〉, (15, +∞)}	8
	C_{53}	{0, 5/12, 7/12, 10/12, 1}; {(-∞, 3〉, (3, 6〉, (6, 9〉, (9, 12〉, (12, +∞)}	6.5
	C_{54}	{0, 1}; {(-∞, 3〉, (3, +∞)}	3
	C_{62}	{0, 10/15, 14/15, 1}; {(-∞, 3〉, (3 ,9〉, (9, 15〉, (15, +∞)}	5.4
	C_{65}	{0, 8/20, 12/20, 14/20, 16/20, 17/20, 18/20, 19/20, 1}; {(-∞, 3〉, (3, 6〉, (6, 9〉, (9, 12〉, (12, 15〉, (15, 18〉, (18, 21〉, (21, 33〉, (33, +∞)}	8.85

3.2.3. Climate-weather change process at land Baltic seaside area

Defining parameters and data collection of climate-weather change process at land Baltic seaside area

Based on the expert opinion, there are distinguished the following $w = 16$ climate-weather states:

- the climate-weather state c_1 – the air temperature from –25°C up to –15°C and the soil temperature from –30°C up to –5°C,
- the climate-weather state c_2 – the air temperature from –15°C up to –5°C and the soil temperature from –30°C up to –5°C,
- the climate-weather state c_3 – the air temperature from 5°C up to 25°C and the soil temperature from –30°C up to –5°C,

- the climate-weather state c_4 – the air temperature from 25°C up to 35°C and the soil temperature from –30°C up to –5°C,
- the climate-weather state c_5 – the air temperature from –25°C up to –15°C and the soil temperature from –5°C up to 5°C,
- the climate-weather state c_6 – the air temperature from –15°C up to –5°C and the soil temperature from –5°C up to 5°C,
- the climate-weather state c_7 – the air temperature from 5°C up to 25°C and the soil temperature from –5°C up to 5°C,
- the climate-weather state c_8 – the air temperature from 25°C up to 35°C and the soil temperature from –5°C up to 5°C,
- the climate-weather state c_9 – the air temperature from –25°C up to –15°C and the soil temperature from 5°C up to 20°C,
- the climate-weather state c_{10} – the air temperature from –15°C up to 5°C and the soil temperature from 5°C up to 20°C,
- the climate-weather state c_{11} – the air temperature from 5°C up to 25°C and the soil temperature from 5°C up to 20°C,
- the climate-weather state c_{12} – the air temperature from 25°C up to 35°C and the soil temperature from 5°C up to 20°C,
- the climate-weather state c_{13} – the air temperature from –25°C up to –15°C and the soil temperature from 20°C up to 37°C,
- the climate-weather state c_{14} – the air temperature from 15°C up to 5°C and the soil temperature from 20°C up to 37°C,
- the climate-weather state c_{15} – the air temperature from 5°C up to 25°C and the soil temperature from 20°C up to 37°C,
- the climate-weather state c_{16} – the air temperature from 25°C up to 35°C and the soil temperature from 20°C up to 37°C.

The unknown parameters of the climate-weather change process semi-Markov model are:

- the initial probabilities $q_b(0)$, $b = 1,2,...,16$, of the climate-weather change process staying at the particular states c_b at the moment $t = 0$,
- the probabilities q_{bl}, $b,l = 1,2,...,16$, $b \neq l$, of the climate-weather change process transitions from the climate-weather state c_b into the climate-weather state c_l,
- the distributions of the climate-weather change conditional sojourn times C_{bl}, $b,l = 1,2,...,16$, $b \neq l$, at the particular climate-weather states and their mean values $M_{bl} = E[C_{bl}]$, $b,l = 1,2,...,16$, $b \neq l$.

To identify all these parameters of the climate-weather change process the statistical data about this process is needed.

The collected by the system operators statistical data necessary to evaluating the initial transient probabilities of the climate-weather change process at the particular states are:

- the climate-weather change process observation / experiment time

$$\Theta = 9 \text{ years } (2007\text{–}2015),$$

- the number of the climate-weather change process realizations $n(0) = 3287$,
- the vector of realizations of the numbers of the climate-weather change process staying at the particular climate-weather state c_b at the initial moment $t = 0$

$$[n_b(0)] = [32873, 98, 0, 0, 0, 1008, 162, 0, 0, 2, 1975, 0, 0, 0, 39, 0].$$

The collected statistical data necessary to evaluating the probabilities of transitions of the climate-weather state change process $C(t)$ between the climate-weather states are:

- the matrix of realizations of the numbers of climate-weather change process transitions from the state c_b into the state c_l during the experiment time

$$[n_{bl}] = \begin{bmatrix}
0 & 8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
11 & 0 & 0 & 0 & 0 & 204 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 340 & 0 & 0 & 0 & 0 & 400 & 0 & 0 & 731 & 349 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 488 & 0 & 0 & 0 & 1 & 439 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 182 & 2 & 0 & 0 & 0 & 222 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 175 & 1182 & 0 & 0 & 95 & 0 & 0 & 0 & 0 & 2439 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1171 & 0 & 0 & 0 & 0 & 96 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 79 & 0
\end{bmatrix}.$$

- the vector of realizations of the total numbers of the climate-weather change process transitions from the climate-weather state c_b during the experiment time

$$[n_b] = [8, 215, 0, 0, 0, 1820, 928, 0, 0, 406, 3892, 0, 0, 0, 1267, 81].$$

The statistical data for the conditional sojourn times C_{bl} at the climate-weather states c_b when the next climate-weather state is c_l, $b,l = 1,2,...,16,$ $b \neq l$, related to the land Baltic seaside area are given in the Figure 11.

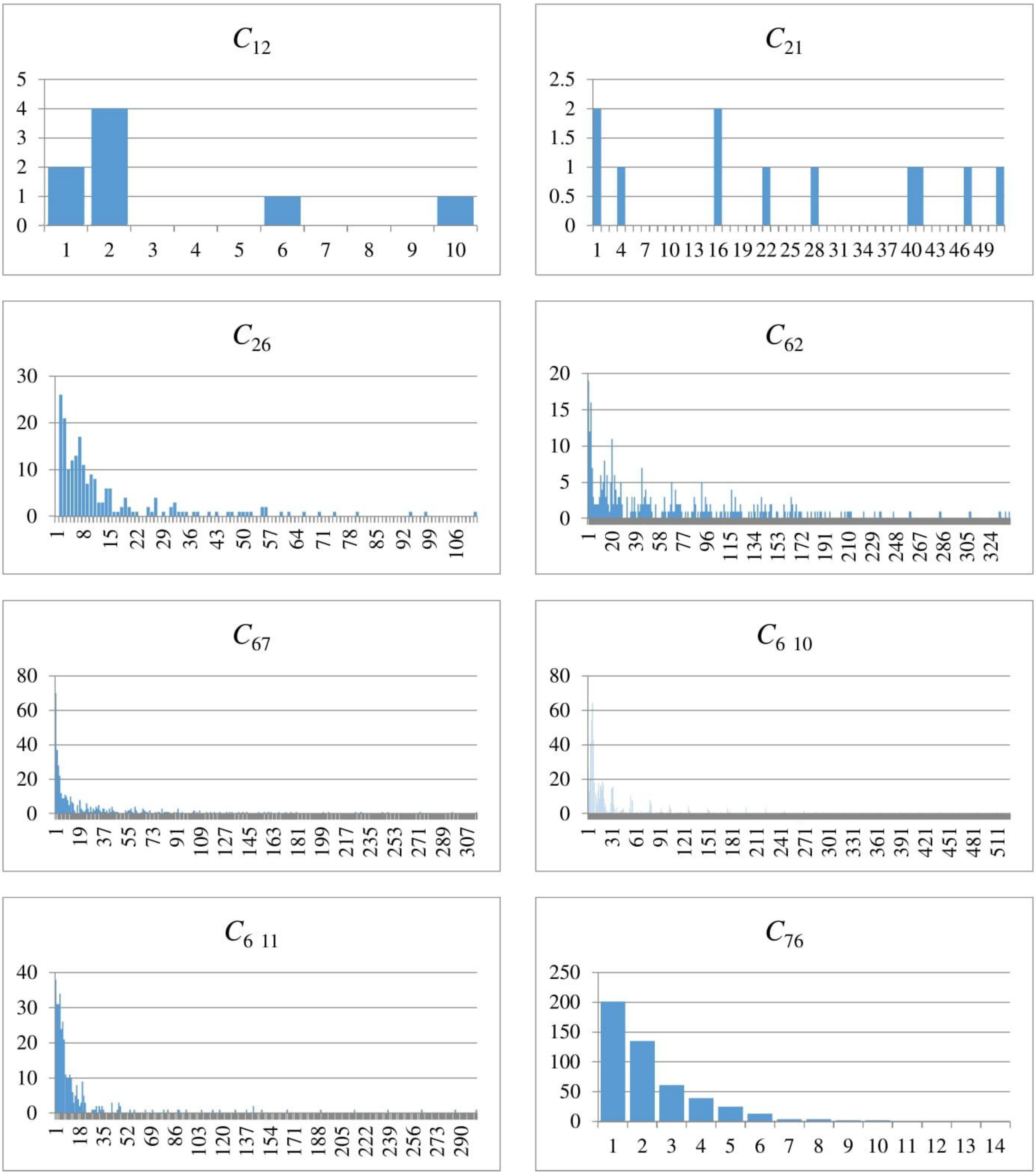


Figure 11. The graphical representation of the statistical data for the conditional sojourn times of the climate-weather change process at the land Baltic seaside area.

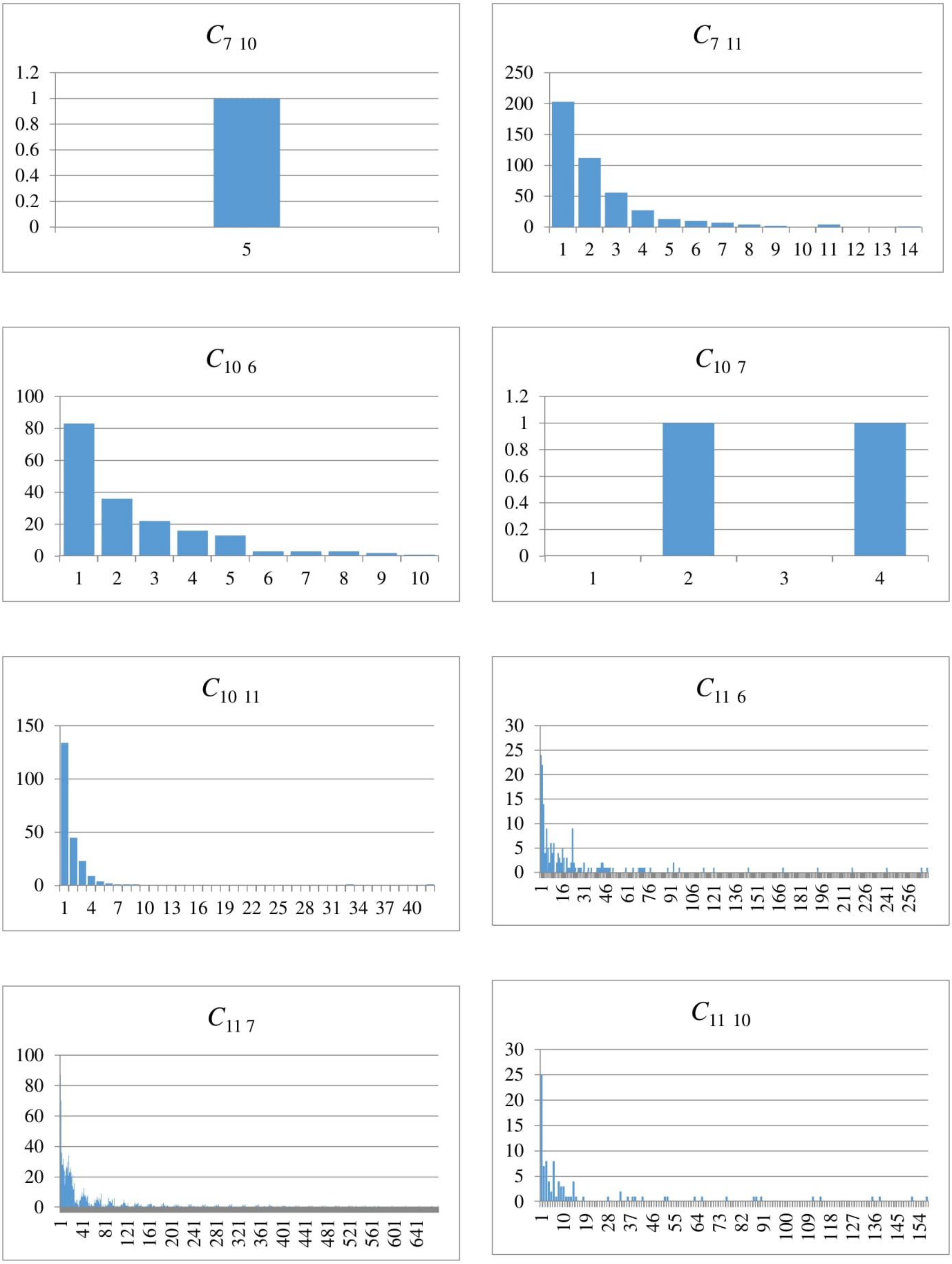


Figure 11 (*continuation*). The graphical representation of the statistical data for the conditional sojourn times of the climate-weather change process at the land Baltic seaside area.

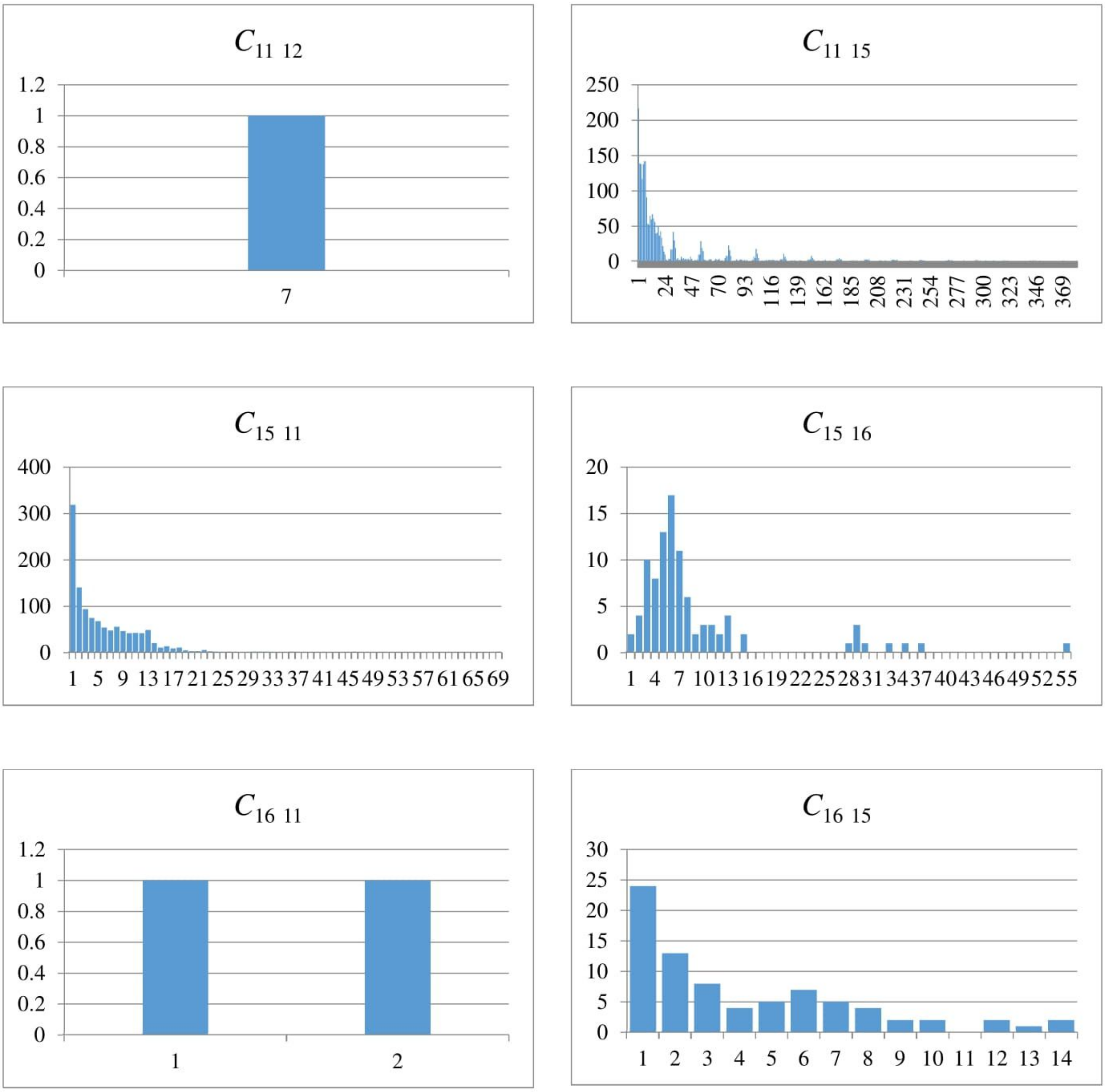


Figure 11 (*continuation*). The graphical representation of the statistical data for the conditional sojourn times of the climate-weather change process at the land Baltic seaside area.

Evaluating basic parameters of climate-weather change process at land Baltic seaside area

On the basis of the statistical data from Section 3.2.2., it is possible to evaluate the following unknown basic parameters of the climate-weather change process:

- the vector

$$[q_b(0)] = [0.001, 0.03, 0, 0, 0, 0.307, 0.049, 0, 0, 0.001, 0.6, 0, 0, 0, 0.012, 0]$$

of the initial probabilities $q_b(0)$, b, $l = 1,2,...,16$, of the climate-weather change process staying at the particular states c_b at the $t = 0$,

- the matrix

$[q_{bl}] =$

$$
\begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.051 & 0 & 0 & 0 & 0 & 0.949 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.187 & 0 & 0 & 0 & 0 & 0.22 & 0 & 0 & 0.401 & 0.192 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.526 & 0 & 0 & 0 & 0.001 & 0.473 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.448 & 0.005 & 0 & 0 & 0 & 0.547 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.045 & 0.304 & 0 & 0 & 0.024 & 0 & 0 & 0 & 0 & 0.627 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.924 & 0 & 0 & 0 & 0 & 0.076 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.025 & 0 & 0 & 0 & 0.975 & 0
\end{bmatrix}
$$

of the probabilities q_{bl}, $b,l = 1,2,...,16$, of transitions of the climate-weather change process from the climate-weather state c_b into the climate-weather state c_l.

Identification of distribution functions of climate-weather change process at land Baltic seaside area

Using the procedure given in (Kołowrocki, Kuligowska & Torbicki, 2017), the results and the statistical data from Section 3.2.2., we may verify the hypotheses on the distributions of the climate-weather change process conditional sojourn times C_{bl}, $b,l = 1,2,...,16$, $b \neq l$, at the particular states. To do this, we need a sufficient number of realizations of these variables (Rice, 2007; Vercellis, 2009; Wilson, Graves, Hamada et al., 2006), namely, the sets of their realizations should contain at least 30 realizations coming from the experiment. This condition is not satisfied for the all statistical data we have in disposal and that are presented in Section 3.2.2. However, when the realizations of conditional sojourn times C_{bl}, $b,l = 1,2,...,16$, $b \neq l$, at the particular climate-weather states are more than 30, we can get the distributions of the climate-weather change process conditional sojourn times with the parameters given in Table 13.

Table 13. Distribution functions of conditional sojourn times when their realizations are more than 30

Distribution	Sojourn Time	Parameters	Mean
Gamma	C_{26}	$\alpha_{2\,6} = 0.5303$ $\beta_{2\,6} = 27.6389$	14.657
	C_{67}	$\alpha_{6\,7} = 0.3686$ $\beta_{6\,7} = 80.8599$	29.805
	$C_{11\,6}$	$\alpha_{11\,6} = 0.316$ $\beta_{11\,6} = 81.7737$	25.84

Table 13 (*continuation*). Distribution functions of conditional sojourn times when their realizations are more than 30

Distribution	Sojourn Time	Parameters	Mean
Gamma	$C_{11\,7}$	$\alpha_{11\,7} = 0.3471$ $\beta_{11\,7} = 174.827$	60.682
	$C_{11\,10}$	$\alpha_{11\,10} = 0.3327$ $\beta_{11\,10} = 62.8304$	20.904
exponential	$C_{16\,15}$	$x_{16\,15} = 0.28$ $\alpha_{16\,15} = 0.2591$	4.14
double trapezium	C_{62}	$x_{62} = 0$ $y_{62} = 356.82$ $z_6 = 69.9324$ $w_{62} = 0.0002$ $q_{62} = 0.0161$	163.471
	$C_{6\,10}$	$x_{6\,10} = 0$ $y_{6\,10} = 544.32$ $z_{6\,10} = 34.0985$ $w_{6\,10} = 0.0001$ $q_{6\,10} = 0.0336$	217.497
	$C_{6\,11}$	$x_{6\,11} = 0$ $y_{6\,11} = 315.8$ $z_{6\,11} = 17.8138$ $w_{6\,11} = 0.0002$ $q_{6\,11} = 0.0492$	127.826
	C_{76}	$x_{76} = 0.71$ $y_{76} = 14.28$ $z_{76} = 2.3299$ $w_{76} = 0.0035$ $q_{76} = 0.6981$	6.799
	$C_{7\,11}$	$x_{7\,11} = 0.69$ $y_{7\,11} = 14.33$ $z_{7\,11} = 2.2437$ $w_{7\,11} = 0.0037$ $q_{7\,11} = 0.7458$	6.817
	$C_{10\,6}$	$x_{10\,6} = 0.66$ $y_{10\,6} = 10.32$ $z_{10\,6} = 2.4231$ $w_{10\,6} = 0.008$ $q_{10\,6} = 0.6609$	5.606
	$C_{10\,11}$	$x_{10\,11} = 0$ $y_{10\,11} = 43.68$ $z_{10\,11} = 2.0721$ $w_{10\,11} = 0.0017$ $q_{10\,11} = 0.2954$	17.954
	$C_{11\,15}$	$x_{11\,15} = 0$ $y_{11\,15} = 388$ $z_{11\,15} = 28.8754$ $w_{11\,15} = 0.0001$ $q_{11\,15} = 0.0546$	151.504
	$C_{15\,11}$	$x_{15\,11} = 0$ $y_{15\,11} = 70$ $z_{15\,11} = 5.8173$ $w_{15\,11} = 0.0004$ $q_{15\,11} = 0.1964$	26.906
	$C_{15\,16}$	$x_{15\,16} = 0$ $y_{15\,16} = 59.4$ $z_{15\,16} = 8.9167$ $w_{15\,16} = 0.0019$ $q_{15\,16} = 0.0714$	28.359

For the distributions identified in this section, by application either the general formulae for the mean value given by (2.12) or the particular formulae (2.13)–(2.19) in (Kołowrocki & Soszyńska-Budny, 2011), the mean values $M_{bl} = E[C_{bl}]$, $b,l = 1,2,...,16$, $b \neq l$, of the climate-weather change process conditional sojourn times at the particular climate-weather states related to the land Baltic seaside area can be determined (Table 13).

Because of the lack of sufficient numbers of realizations of the climate-weather change process conditional sojourn times at the climate-weather states, it is not possible to identify statistically their distributions. In those cases when the number of the conditional sojourn times realizations is non-zero it is possible to find the approximate empirical values of the mean values $M_{bl} = E[C_{bl}]$ of the conditional sojourn times at the particular climate-weather states that are given in Table 14.

Table 14. Distribution functions of conditional sojourn times when their realizations are less than 30

Distribution	Sojourn Time	Parameters, intervals	Mean
empirical	C_{12}	{0, 2/8, 6/8, 7/8, 1}; {(-∞, 1⟩, (1, 2⟩, (2, 6⟩, (6, 10⟩, (10, +∞)}	3.25
	C_{21}	{0, 2/11, 3/11, 5/11, 6/11, 7/11, 8/11, 9/11, 10/11, 1}; {(-∞, 1⟩, (1, 4⟩, (4, 16⟩, (16, 22⟩, (22, 28⟩, (28, 40⟩, (40, 41⟩, (41, 47⟩, (47, 51⟩, (51, +∞)}	24.273
	$C_{7\,10}$	{0, 1}; {(-∞, 5⟩, (5, +∞)}	5
	$C_{10\,7}$	{0, 1/2, 1}; {(-∞, 2⟩, (2, 4⟩, (4, +∞)}	3
	$C_{11\,12}$	{0, 1}; {(-∞, 7⟩, (7, +∞)}	7
	$C_{16\,11}$	{0, 1/2, 1}; {(-∞, 1⟩, (1, 2⟩, (2, +∞)}	1.5

As there are no realizations of the rest climate-weather change process conditional sojourn times at the climate-weather states then it is impossible to estimate their empirical conditional mean values.

4. Conclusion

The proposed statistical methods of identification of the unknown parameters of the climate-weather change processes' models allow us to identify their parameters, to determine their characteristics and next to apply them practically in evaluation, prediction and optimization of reliability, availability and safety of real complex critical infrastructures impacted by the climate-weather change processes (Bogalecka, 2020; Dąbrowska, 2020; Kołowrocki, 2021; Torbicki, 2018).

Acknowledgment

The chapter presents the results developed in the scope of the research project WN/PZ/04 "Safety of critical infrastructure transport networks", granted by Gdynia Maritime University in 2021.

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