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Safety optimization of maritime ferry technical system

Keywords

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Abstract

The method that can be used in critical infrastructure safety optimization is shown and adapted to the ferry technical system. The optimal values of the operation process, safety and resilience indicators are determined for the maritime ferry technical system. Practical suggestions on reorganizing the operation process of this member of a shipping critical infrastructure to maximize its lifetime in the safety states not worse than the critical safety state are proposed.

1. Introduction

The critical infrastructure operating at a designated area may be prone to damage and degradation induced by external threats although, it might cause threats to another critical infrastructures [2], [12]. Therefore, the critical infrastructure safety indicators improvement is of high importance in the industrial practice. Hence, there is a need to find the means for searching for the critical infrastructure safety and resilience indicators and their optimal forms and the procedures allowing for changing the system operation process after correlating the values of these indicators with their values before the critical infrastructure operation process optimization in order to improve its safety [12], [16]. In this paper we defined optimal safety function as well as risk function for the maritime ferry technical system. Other optimal, practically significant critical infrastructure safety indicators specified in the paper are its mean lifetime up to the exceeding a critical safety state, the moment when its risk function value exceeds the acceptable safety level, the critical infrastructure intensity of ageing/degradation, the coefficient of operation process impact on critical infrastructure intensities of ageing and the coefficient of critical infrastructure resilience to operation process impact [4]–[7], [13]–[15]. Having the system operation process characteristics and particularly the conditional total mean values at the particular operation states over the fixed time of the system operation it is possible to alter the system operation process through applying the linear programming [3] in order to maximize the ferry technical system mean lifetime in the safety state subset not worse than the critical safety state. In the paper, the model for finding and maximizing the mean value of the system lifetime in the safety state subset not worse than the critical safety state is created and applied to optimization of the maritime ferry technical system operation process. The paper is organized into 8 parts, this Introduction as Section 1, Sections 2–7 and Summary as Section 8. In Section 2, a maritime ferry technical system structure and operation are described. In Section 3, the ferry technical system safety and resilience indicators before optimization are presented. In Section 4, the optimization of the ferry technical system safety is performed. In Section 5, the ferry technical system optimal safety and resilience indicators are fixed. In Section 6, the inventory of the results concerned with the ferry technical system safety and resilience indicators before and after optimization in the form of table is done. The Section 7 is giving the ferry operation new strategy suggested after the performed its safety optimization. In Summary, the evaluation of results achieved is done and the perspective for future research perspective in the field of critical infrastructure safety considered in the paper is given.

2. Ferry technical system structure and operation

We will examine technical safety of a selected member of the shipping critical infrastructure.

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The considered maritime ferry, performing at the Baltic Sea, is described in [11]. Namely, the maritime ferry technical system safety will be analyzed. We consider, that the maritime ferry incorporates a number of main technical subsystems having an crucial impact on its safety, further termed the ferry technical system:

- S_1 – a navigational subsystem,
- S_2 – a propulsion and controlling subsystem,
- S_3 – a loading and unloading subsystem,
- S_4 – a stability control subsystem,
- S_5 – an anchoring and mooring subsystem.

The subsystems $S_1, S_2, S_3, S_4, S_5,$ are forming a general series safety structure of the ferry technical system shown in *Figure 1*.

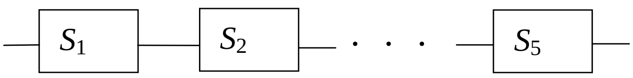

Figure 1. The general structure of the ferry technical system safety

After analyzing the matter with help of experts and taking into consideration the safety of the operation of the ferry, we identify the five safety states of the ferry technical system and its components:

- a safety state 4 – the ferry operation is fully safe,
- a safety state 3 – the ferry operation is less safe and more dangerous because of the possibility of environment pollution,
- a safety state 2 called a critical safety state – the ferry operation is less safe and more dangerous because of the possibility of environment pollution and causing small accidents,
- a safety state 1 – the ferry operation is much less safe and much more dangerous because of the possibility of serious environment pollution and causing extensive accidents,
- a safety state 0 – the ferry technical system is destroyed [9].

Furthermore, by the specialist opinions, we presume that only possible transitions between the components' safety states are those from better to worse. We presume that critical safety state is $r = 2$ for the system and its components.

The maritime ferry operation process $Z(t)$, $t \geq 0$, was identified and specified in [10]. Having regards to the opinions of experts on the varying in time operation process of the pondered maritime ferry system, we identify the eighteen operation states:

- an operation state z_1 – loading at Gdynia Port,
- an operation state z_2 – unmooring operations at Gdynia Port,
- an operation state z_3 – leaving Gdynia Port and navigation to "GD" buoy,
- an operation state z_4 – navigation at restricted waters from "GD" buoy to the end of Traffic Separation Scheme,
- an operation state z_5 – navigation at open waters from the end of Traffic Separation Scheme to "Angoring" buoy,
- an operation state z_6 – navigation at restricted waters from "Angoring" buoy to "Verko" Berth at Karlskrona,
- an operation state z_7 – mooring operations at Karlskrona Port,
- an operation state z_8 – unloading at Karlskrona Port,
- an operation state z_9 – loading at Karlskrona Port,
- an operation state z_{10} – unmooring operations at Karlskrona Port,
- an operation state z_{11} – ferry turning at Karlskrona Port,
- an operation state z_{12} – leaving Karlskrona Port and navigation at restricted waters to "Angoring" buoy,
- an operation state z_{13} – navigation at open waters from "Angoring" buoy to the entering Traffic Separation Scheme,
- an operation state z_{14} – navigation at restricted waters from the entering Traffic Separation Scheme to "GD" buoy,
- an operation state z_{15} – navigation from "GD" buoy to turning area,
- an operation state z_{16} – ferry turning at Gdynia Port,
- an operation state z_{17} – mooring operations at Gdynia Port,
- an operation state z_{18} – unloading at Gdynia Port.

The ferry technical system operation process $Z(t)$ characteristics are the limit values of transients probabilities p_b of the operation process at the particular operation states z_b, $b = 1,2,\ldots,18$ [9]:

$$[p_b]_{1x18} = [0.038, 0.002, 0.026, 0.036, 0.363, 0.026, 0.005, 0.016, 0.037, 0.002, 0.003, 0.016, 0.351, 0.034, 0.024, 0.003, 0.005, 0.013]. \tag{1}$$

3. Ferry technical system safety indicators

The expected values of the analyzed system lifetimes in the safety state subset {2,3,4} at the operation state z_b, $b = 1,2,\ldots,18$, respectively are [8]:

$[\boldsymbol{\mu}(2)]^{(1)} \cong 1.47$, $[\boldsymbol{\mu}(2)]^{(2)} \cong 1.33$, $[\boldsymbol{\mu}(2)]^{(3)} \cong 1.40$,
$[\boldsymbol{\mu}(2)]^{(4)} \cong 1.39$, $[\boldsymbol{\mu}(2)]^{(5)} \cong 1.39$, $[\boldsymbol{\mu}(2)]^{(6)} \cong 1.38$,
$[\boldsymbol{\mu}(2)]^{(7)} \cong 1.28$, $[\boldsymbol{\mu}(2)]^{(8)} \cong 1.44$, $[\boldsymbol{\mu}(2)]^{(9)} \cong 1.44$,
$[\boldsymbol{\mu}(2)]^{(10)} \cong 1.33$, $[\boldsymbol{\mu}(2)]^{(11)} \cong 1.34$, $[\boldsymbol{\mu}(2)]^{(12)} \cong 1.40$,
$[\boldsymbol{\mu}(2)]^{(13)} \cong 1.39$, $[\boldsymbol{\mu}(2)]^{(14)} \cong 1.39$, $[\boldsymbol{\mu}(2)]^{(15)} \cong 1.40$,
$[\boldsymbol{\mu}(2)]^{(16)} \cong 1.34$, $[\boldsymbol{\mu}(2)]^{(17)} \cong 1.28$,
$[\boldsymbol{\mu}(2)]^{(18)} \cong 1.46$ years. (2)

The standard deviation of the considered system lifetimes in the safety state subset {2,3,4} at the operation state z_b, $b = 1,2,\ldots,18$, respectively are [8]:

$[\boldsymbol{\sigma}(2)]^{(1)} \cong 1.45$, $[\boldsymbol{\sigma}(2)]^{(2)} \cong 1.31$, $[\boldsymbol{\sigma}(2)]^{(3)} \cong 1.38$,
$[\boldsymbol{\sigma}(2)]^{(4)} \cong 1.38$, $[\boldsymbol{\sigma}(2)]^{(5)} \cong 1.37$, $[\boldsymbol{\sigma}(2)]^{(6)} \cong 1.37$,
$[\boldsymbol{\sigma}(2)]^{(7)} \cong 1.26$, $[\boldsymbol{\sigma}(2)]^{(8)} \cong 1.42$, $[\boldsymbol{\sigma}(2)]^{(9)} \cong 1.42$,
$[\boldsymbol{\sigma}(2)]^{(10)} \cong 1.31$, $[\boldsymbol{\sigma}(2)]^{(11)} \cong 1.31$, $[\boldsymbol{\sigma}(2)]^{(12)} \cong 1.38$,
$[\boldsymbol{\sigma}(2)]^{(13)} \cong 1.39$, $[\boldsymbol{\sigma}(2)]^{(14)} \cong 1.38$, $[\boldsymbol{\sigma}(2)]^{(15)} \cong 1.37$,
$[\boldsymbol{\sigma}(2)]^{(16)} \cong 1.31$, $[\boldsymbol{\sigma}(2)]^{(17)} \cong 1.26$,
$[\boldsymbol{\sigma}(2)]^{(18)} \cong 1.45$ years. (3)

Thus, considering (1) and (2), the value of the ferry technical system lifetime is

$$\boldsymbol{\mu}(2) = 1.47\cdot p_1 + 1.33\cdot p_2 + 1.40\cdot p_3 + 1.39\cdot p_4 + 1.39\cdot p_5 + 1.38\cdot p_6 + 1.28\cdot p_7 + 1.44\cdot p_8 + 1.44\cdot p_9 + 1.33\cdot p_{10} + 1.34\cdot p_{11} + 1.40\cdot p_{12} + 1.39\cdot p_{13} + 1.39\cdot p_{14} + 1.40\cdot p_{15} + 1.34\cdot p_{16} + 1.28\cdot p_{17} + 1.46\cdot p_{18} \cong 1.39536. \quad (4)$$

Moreover applying (1), the corresponding unconditional safety function of the ferry technical system takes the form

$$\boldsymbol{S}(t,2) = [\boldsymbol{S}(t,2)]^{(1)}\cdot 0.038 + [\boldsymbol{S}(t,2)]^{(2)}\cdot 0.002 + [\boldsymbol{S}(t,2)]^{(3)}\cdot 0.026 + [\boldsymbol{S}(t,2)]^{(4)}\cdot 0.036 + [\boldsymbol{S}(t,2)]^{(5)}\cdot 0.363 + [\boldsymbol{S}(t,2)]^{(6)}\cdot 0.026 + [\boldsymbol{S}(t,2)]^{(7)}\cdot 0.005 + [\boldsymbol{S}(t,2)]^{(8)}\cdot 0.016 + [\boldsymbol{S}(t,2)]^{(9)}\cdot 0.037 + [\boldsymbol{S}(t,2)]^{(10)}\cdot 0.002 + [\boldsymbol{S}(t,2)]^{(11)}\cdot 0.003 + [\boldsymbol{S}(t,2)]^{(12)}\cdot 0.0016 + [\boldsymbol{S}(t,2)]^{(13)}\cdot 0.351 + [\boldsymbol{S}(t,2)]^{(14)}\cdot 0.034 + [\boldsymbol{S}(t,2)]^{(15)}\cdot 0.024 + [\boldsymbol{S}(t,2)]^{(16)}\cdot 0.003 + [\boldsymbol{S}(t,2)]^{(17)}\cdot 0.005 + [\boldsymbol{S}(t,2)]^{(18)}\cdot 0.013,$$
$$t \in \langle 0,+\infty), \quad (5)$$

where $[\boldsymbol{S}(t,2)]^{(b)}$, $b = 1,2,\ldots,18$, are determined in [8].
Further, considering (4)–(5), the corresponding standard deviations of the analyzed system unconditional lifetime in the state subset is

$$\boldsymbol{\sigma}(2) \cong 1.38317 \text{ years.} \quad (6)$$

As the ferry technical system critical safety state is $r = 2$, then its system risk function is [8]

$$\boldsymbol{r}(t) \cong 1 - \boldsymbol{S}(t,2),\ t \in \langle 0,+\infty), \quad (7)$$

where $\boldsymbol{S}(t,2)$ is given by (5).
Hence, and considering (7), the moment when the system risk function exceeds a permitted level, for instance $\delta = 0.05$, is

$$\boldsymbol{\tau} = \boldsymbol{r}^{-1}(\delta) \cong 0.0727 \text{ year.} \quad (8)$$

By (4) the analyzed system mean value intensity of ageing is

$$\boldsymbol{\lambda}(t,2) = \frac{1}{\boldsymbol{\mu}(2)} = \frac{1}{1.39536} \cong 0.716661. \quad (9)$$

Considering (9) and the values of the ferry technical system without operation impact intensity of ageing $\lambda^0(t)$, determined in [9], the coefficient of the operation process effect on the ferry technical system intensity of ageing is

$$\boldsymbol{\rho}(t,2) = \frac{\lambda\ (t)}{\lambda^0(t)} = \frac{0.716661}{0.677507} \cong 1.05779. \quad (10)$$

Finally, the ferry technical system resilience indicator, i.e. the coefficient of the ferry technical system resilience to operation process impact, is

$$\boldsymbol{RI}(t) = \frac{1}{\boldsymbol{\rho}(t,2)} = \frac{1}{1.05779} \cong 0.945 = 94.5\%. \quad (11)$$

4. Ferry technical system safety optimization

The considered system critical state is $r = 2$, considering the conditional mean values determined by (2), the objective function defined in [8] takes the form

$$\boldsymbol{\mu}(2) = 1.47\cdot p_1 + 1.33\cdot p_2 + 1.40\cdot p_3 + 1.39\cdot p_4 + 1.39\cdot p_5 + 1.38\cdot p_6 + 1.28\cdot p_7 + 1.44\cdot p_8 + 1.44\cdot p_9 + 1.33\cdot p_{10} + 1.34\cdot p_{11} + 1.40\cdot p_{12} + 1.39\cdot p_{13} + 1.39\cdot p_{14} + 1.40\cdot p_{15} + 1.34\cdot p_{16} + 1.28\cdot p_{17} + 1.46\cdot p_{18}, \quad (12)$$

where transient probabilities p_b, $b = 1,2,\ldots,18$, are given by (1).
The approximate values of the lower $\breve{p}_b$ as well as upper $\hat{p}_b$ bounds of the unknown transient probabilities p_b, $b = 1,2,\ldots,18$, accordingly are [9]:

$\breve{p}_1 = 0.0006$, $\breve{p}_2 = 0.001$, $\breve{p}_3 = 0.018$, $\breve{p}_4 = 0.027$,
$\breve{p}_5 = 0.286$, $\breve{p}_6 = 0.018$, $\breve{p}_7 = 0.002$, $\breve{p}_8 = 0.001$,
$\breve{p}_9 = 0.001$, $\breve{p}_{10} = 0.001$, $\breve{p}_{11} = 0.002$, $\breve{p}_{12} = 0.013$,
$\breve{p}_{13} = 0.286$, $\breve{p}_{14} = 0.025$, $\breve{p}_{15} = 0.018$, $\breve{p}_{16} = 0.002$,
$\breve{p}_{17} = 0.002$, $\breve{p}_{18} = 0.001$,

$$\widehat{p}_1 = 0.056,\ \widehat{p}_2 = 0.002,\ \widehat{p}_3 = 0.027,\ \widehat{p}_4 = 0.056,$$
$$\widehat{p}_5 = 0.780,\ \widehat{p}_6 = 0.024,\ \widehat{p}_7 = 0.018,\ \widehat{p}_8 = 0.018,$$
$$\widehat{p}_9 = 0.056,\ \widehat{p}_{10} = 0.003,\ \widehat{p}_{11} = 0.004,\ \widehat{p}_{12} = 0.024,$$
$$\widehat{p}_{13} = 0.780,\ \widehat{p}_{14} = 0.043,\ \widehat{p}_{15} = 0.024,$$
$$\widehat{p}_{16} = 0.004,\ \widehat{p}_{17} = 0.007,\ \widehat{p}_{18} = 0.018. \quad (13)$$

Thus, we assume bound constraints as follows

$$0.0006 \le p_1 \le 0.056,\quad 0.001 \le p_2 \le 0.002,$$
$$0.018 \le p_3 \le 0.027,\quad 0.027 \le p_4 \le 0.056,$$
$$0.286 \le p_5 \le 0.780,\quad 0.018 \le p_6 \le 0.024,$$
$$0.002 \le p_7 \le 0.018,\quad 0.001 \le p_8 \le 0.018,$$
$$0.001 \le p_9 \le 0.056,\quad 0.001 \le p_{10} \le 0.003,$$
$$0.002 \le p_{11} \le 0.004,\quad 0.013 \le p_{12} \le 0.024,$$
$$0.286 \le p_{13} \le 0.780,\quad 0.025 \le p_{14} \le 0.043,$$
$$0.018 \le p_{15} \le 0.024,\quad 0.002 \le p_{16} \le 0.004,$$
$$0.002 \le p_{17} \le 0.007,\quad 0.001 \le p_{18} \le 0.018,$$
$$\sum_{b=1}^{18} p_b = 1. \quad (14)$$

At this moment, before we get optimal values $\dot{p}_b$ of the transient probabilities p_b, b = 1,2,…,18, that maximize the objective function, we establish the system conditional lifetime mean values $[\boldsymbol{\mu}(2)]^{(b)}$, b = 1,2,…,18, in the safety state subset {2,3,4}, in non-increasing order

$$1.47 \ge 1.46 \ge 1.44 \ge 1.44 \ge 1.40 \ge 1.40 \ge 1.40$$
$$\ge 1.39 \ge 1.39 \ge 1.39 \ge 1.39 \ge 1.38 \ge 1.34 \ge 1.34$$
$$\ge 1.33 \ge 1.33 \ge 1.28 \ge 1.28,$$

i.e.

$$[\boldsymbol{\mu}(2)]^{(1)} \ge [\boldsymbol{\mu}(2)]^{(18)} \ge [\boldsymbol{\mu}(2)]^{(8)} \ge [\boldsymbol{\mu}(2)]^{(9)} \ge [\boldsymbol{\mu}(2)]^{(3)}$$
$$\ge [\boldsymbol{\mu}(2)]^{(12)} \ge [\boldsymbol{\mu}(2)]^{(15)} \ge [\boldsymbol{\mu}(2)]^{(4)} \ge [\boldsymbol{\mu}(2)]^{(5)}$$
$$\ge [\boldsymbol{\mu}(2)]^{(13)} \ge [\boldsymbol{\mu}(2)]^{(14)} \ge [\boldsymbol{\mu}(2)]^{(6)} \ge [\boldsymbol{\mu}(2)]^{(11)}$$
$$\ge [\boldsymbol{\mu}(2)]^{(16)} \ge [\boldsymbol{\mu}(2)]^{(2)} \ge [\boldsymbol{\mu}(2)]^{(10)} \ge [\boldsymbol{\mu}(2)]^{(7)}$$
$$\ge [\boldsymbol{\mu}(2)]^{(17)}. \quad (15)$$

Further, we substitute

$$x_1 = p_1,\ x_2 = p_{18},\ x_3 = p_8,\ x_4 = p_9,\ x_5 = p_3$$
$$x_6 = p_{12},\ x_7 = p_{15},\ x_8 = p_4,\ x_9 = p_5,\ x_{10} = p_{13},$$
$$x_{11} = p_{14},\ x_{12} = p_6,\ x_{13} = p_{11},\ x_{14} = p_{16},$$
$$x_{15} = p_2,\ x_{16} = p_{10},\ x_{17} = p_7,\ x_{18} = p_{17}, \quad (16)$$

and

$$\breve{x}_1 = \breve{p}_1 = 0.0006,\ \breve{x}_2 = \breve{p}_{18} = 0.001,$$
$$\breve{x}_3 = \breve{p}_8 = 0.001,\ \breve{x}_4 = \breve{p}_9 = 0.001,$$
$$\breve{x}_5 = \breve{p}_3 = 0.018,\ \breve{x}_6 = \breve{p}_{12} = 0.013,$$
$$\breve{x}_7 = \breve{p}_{15} = 0.018,\ \breve{x}_8 = \breve{p}_4 = 0.027,$$
$$\breve{x}_9 = \breve{p}_5 = 0.286,\ \breve{x}_{10} = \breve{p}_{13} = 0.286,$$
$$\breve{x}_{11} = \breve{p}_{14} = 0.025,\ \breve{x}_{12} = \breve{p}_6 = 0.018,$$
$$\breve{x}_{13} = \breve{p}_{11} = 0.002,\ \breve{x}_{14} = \breve{p}_{16} = 0.002,$$
$$\breve{x}_{15} = \breve{p}_2 = 0.001,\ \breve{x}_{16} = \breve{p}_{10} = 0.001,$$
$$\breve{x}_{17} = \breve{p}_7 = 0.002,\ \breve{x}_{18} = \breve{p}_{17} = 0.002,$$

$$\widehat{x}_1 = \widehat{p}_1 = 0.056,\ \widehat{x}_2 = \widehat{p}_{18} = 0.018,$$
$$\widehat{x}_3 = \widehat{p}_8 = 0.018,\ \widehat{x}_4 = \widehat{p}_9 = 0.056,$$
$$\widehat{x}_5 = \widehat{p}_3 = 0.027,\ \widehat{x}_6 = \widehat{p}_{12} = 0.024,$$
$$\widehat{x}_7 = \widehat{p}_{15} = 0.024,\ \widehat{x}_8 = \widehat{p}_4 = 0.056,$$
$$\widehat{x}_9 = \widehat{p}_5 = 0.78,\ \widehat{x}_{10} = \widehat{p}_{13} = 0.78,$$
$$\widehat{x}_{11} = \widehat{p}_{14} = 0.043,\ \widehat{x}_{12} = \widehat{p}_6 = 0.024,$$
$$\widehat{x}_{13} = \widehat{p}_{11} = 0.004,\ \widehat{x}_{14} = \widehat{p}_{16} = 0.004,$$
$$\widehat{x}_{15} = \widehat{p}_2 = 0.002,\ \widehat{x}_{16} = \widehat{p}_{10} = 0.003,$$
$$\widehat{x}_{17} = \widehat{p}_7 = 0.018,\ \widehat{x}_{18} = \widehat{p}_{17} = 0.007, \quad (17)$$

and we maximize with respect to x_i, i = 1,2,…,18, the linear form (12) that according to (16)–(17) takes the form

$$\boldsymbol{\mu}(2) = 1.47\cdot x_1 + 1.46\cdot x_2 + 1.44\cdot x_3 + 1.44\cdot x_4$$
$$+ 1.44\cdot x_5 + 1.40\cdot x_6 + 1.40\cdot x_7 + 1.39\cdot x_8 + 1.39\cdot x_9$$
$$+ 1.39\cdot x_{10} + 1.39\cdot x_{11} + 1.38\cdot x_{12} + 1.34\cdot x_{13}$$
$$+ 1.34\cdot x_{14}\cdot + 1.33\cdot x_{15} + 1.33\cdot x_{16} + 1.28\cdot x_{17}$$
$$+ 1.28\cdot x_{18}, \quad (18)$$

with the following bound constraints

$$0.0006 \le x_1 \le 0.056,\quad 0.001 \le x_2 \le 0.018,$$
$$0.001 \le x_3 \le 0.018,\quad 0.001 \le x_4 \le 0.056,$$
$$0.018 \le x_5 \le 0.027,\quad 0.013 \le x_6 \le 0.024,$$
$$0.018 \le x_7 \le 0.024,\quad 0.027 \le x_8 \le 0.056,$$
$$0.286 \le x_9 \le 0.78,\quad 0.286 \le x_{10} \le 0.78,$$
$$0.025 \le x_{11} \le 0.043,\quad 0.018 \le x_{12} \le 0.024,$$
$$0.002 \le x_{13} \le 0.004,\quad 0.002 \le x_{14} \le 0.004,$$
$$0.001 \le x_{15} \le 0.002,\quad 0.001 \le x_{16} \le 0.003,$$
$$0.002 \le x_{17} \le 0.018,\quad 0.002 \le x_{18} \le 0.007,$$
$$\sum_{i=1}^{18} x_i = 1. \quad (19)$$

We calculate

$$\breve{x} = \sum_{i=1}^{18} \breve{x}_i = 0.7046,$$
$$\hat{y} = 1 - \breve{x} = 1 - 0.7046 = 0.2954, \quad (20)$$

and we find

$$\breve{x}^0 = 0,\ \widehat{x}^0 = 0,\ \widehat{x}^0 - \breve{x}^0 = 0,$$

$$\breve{x}^1 = 0.0006,\ \hat{x}^1 = 0.056,\ \hat{x}^1 - \breve{x}^1 = 0.0554,$$
$$\breve{x}^2 = 0.0016,\ \hat{x}^2 = 0.074,\ \hat{x}^2 - \breve{x}^2 = 0.0724,$$
$$\breve{x}^3 = 0.0026,\ \hat{x}^3 = 0.092,\ \hat{x}^3 - \breve{x}^3 = 0.08994,$$
$$\breve{x}^4 = 0.0036,\ \hat{x}^4 = 0.148,\ \hat{x}^4 - \breve{x}^4 = 0.1444,$$
$$\breve{x}^5 = 0.0216,\ \hat{x}^5 = 0.175,\ \hat{x}^5 - \breve{x}^5 = 0.1534,$$
$$\breve{x}^6 = 0.0346,\ \hat{x}^6 = 0.199,\ \hat{x}^6 - \breve{x}^6 = 0.1644,$$
$$\breve{x}^7 = 0.0526,\ \hat{x}^7 = 0.223,\ \hat{x}^7 - \breve{x}^7 = 0.1704,$$
$$\breve{x}^8 = 0.0796,\ \hat{x}^8 = 0.279,\ \hat{x}^8 - \breve{x}^8 = 0.1994,$$
$$\breve{x}^9 = 0.3656,\ \hat{x}^9 = 1.059,\ \hat{x}^9 - \breve{x}^9 = 0.6934,$$
$$\breve{x}^{10} = 0.6516,\ \hat{x}^{10} = 1.839,\ \hat{x}^{10} - \breve{x}^{10} = 1.1874,$$
$$\breve{x}^{11} = 0.6766,\ \hat{x}^{11} = 1.882,\ \hat{x}^{11} - \breve{x}^{11} = 1.2054,$$
$$\breve{x}^{12} = 0.6946,\ \hat{x}^{12} = 1.906,\ \hat{x}^{12} - \breve{x}^{12} = 1.2114,$$
$$\breve{x}^{13} = 0.6966,\ \hat{x}^{13} = 1.91,\ \hat{x}^{13} - \breve{x}^{13} = 1.2134,$$
$$\breve{x}^{14} = 0.6986,\ \hat{x}^{14} = 1.914,\ \hat{x}^{14} - \breve{x}^{14} = 1.2154,$$
$$\breve{x}^{15} = 0.6996,\ \hat{x}^{15} = 1.916,\ \hat{x}^{15} - \breve{x}^{15} = 1.2164,$$
$$\breve{x}^{16} = 0.7006,\ \hat{x}^{16} = 1.919,\ \hat{x}^{16} - \breve{x}^{16} = 1.2184,$$
$$\breve{x}^{17} = 0.7026,\ \hat{x}^{17} = 1.937,\ \hat{x}^{17} - \breve{x}^{17} = 1.2344,$$
$$\breve{x}^{18} = 0.7046,\ \hat{x}^{18} = 1.944,\ \hat{x}^{18} - \breve{x}^{18} = 1.2394. \quad (21)$$

From the above

$$\hat{x}^I - \breve{x}^I < 0.2954, \quad (22)$$

it follows that the largest value $I \in \{0,1,\ldots,18\}$ such that this inequality holds is

$I = 8.$

Consequently, we fix the optimal solution that maximize linear function (18). Namely, we have

$$\dot{x}_1 = \hat{x}_1 = 0.056,\quad \dot{x}_2 = \hat{x}_2 = 0.018,$$
$$\dot{x}_3 = \hat{x}_3 = 0.018,\quad \dot{x}_4 = \hat{x}_4 = 0.056,$$
$$\dot{x}_5 = \hat{x}_5 = 0.027,\quad \dot{x}_6 = \hat{x}_6 = 0.024,$$
$$\dot{x}_7 = \hat{x}_7 = 0.024,\quad \dot{x}_8 = \hat{x}_8 = 0.056,$$
$$\dot{x}_9 = \hat{y} - \hat{x}^8 + \breve{x}^8 + \breve{x}_9 = 0.2954 - 0.279$$
$$+ 0.0796 + 0.286 = 0.382,$$
$$\dot{x}_{10} = \breve{x}_{10} = 0.286,\quad \dot{x}_{11} = \breve{x}_{11} = 0.025,$$
$$\dot{x}_{12} = \breve{x}_{12} = 0.018,\quad \dot{x}_{13} = \breve{x}_{13} = 0.002,$$
$$\dot{x}_{14} = \breve{x}_{14} = 0.002,\quad \dot{x}_{15} = \breve{x}_{15} = 0.001,$$
$$\dot{x}_{16} = \breve{x}_{16} = 0.001,\quad \dot{x}_{17} = \breve{x}_{17} = 0.002,$$
$$\dot{x}_{18} = \breve{x}_{18} = 0.002. \quad (23)$$

Eventually, after making the substitution inverse to (16), we get the optimal transient probabilities

$$\dot{p}_1 = \dot{x}_1 = 0.056,\quad \dot{p}_{18} = \dot{x}_2 = 0.018,$$
$$\dot{p}_8 = \dot{x}_3 = 0.018,\quad \dot{p}_9 = \dot{x}_4 = 0.056,$$
$$\dot{p}_3 = \dot{x}_5 = 0.027,\quad \dot{p}_{12} = \dot{x}_6 = 0.024,$$
$$\dot{p}_{15} = \dot{x}_7 = 0.024,\quad \dot{p}_4 = \dot{x}_8 = 0.056,$$
$$\dot{p}_5 = \dot{x}_9 = 0.382,\quad \dot{p}_{13} = \dot{x}_{10} = 0.286,$$
$$\dot{p}_{14} = \dot{x}_{11} = 0.025,\quad \dot{p}_6 = \dot{x}_{12} = 0.018,$$
$$\dot{p}_{11} = \dot{x}_{13} = 0.002,\quad \dot{p}_{16} = \dot{x}_{14} = 0.002,$$
$$\dot{p}_2 = \dot{x}_{15} = 0.001,\quad \dot{p}_{10} = \dot{x}_{16} = 0.001,$$
$$\dot{p}_7 = \dot{x}_{17} = 0.002,\quad \dot{p}_{17} = \dot{x}_{18} = 0.002, \quad (24)$$

that maximize the ferry technical system mean lifetime $\mu(2)$, expressed by the linear form (12).

5. Ferry technical system optimal safety indicators

Thus, considering (12) and (24), the optimal value of the ferry technical system lifetime is

$$\dot{\boldsymbol{\mu}}(2) = 1.47 \cdot \dot{p}_1 + 1.33 \cdot \dot{p}_2 + 1.40 \cdot \dot{p}_3 + 1.39 \cdot \dot{p}_4$$
$$+ 1.39 \cdot \dot{p}_5 + 1.38 \cdot \dot{p}_6 + 1.28 \cdot \dot{p}_7 + 1.44 \cdot \dot{p}_8$$
$$+ 1.44 \cdot \dot{p}_9 + 1.33 \cdot \dot{p}_{10} + 1.34 \cdot \dot{p}_{11} + 1.40 \cdot \dot{p}_{12}$$
$$+ 1.39 \cdot \dot{p}_{13} + 1.39 \cdot \dot{p}_{14} + 1.40 \cdot \dot{p}_{15} + 1.34 \cdot \dot{p}_{16}$$
$$+ 1.28 \cdot \dot{p}_{17} + 1.46 \cdot \dot{p}_{18} \cong 1.39925 \text{ years}. \quad (25)$$

Moreover, the corresponding optimal unconditional safety function of the ferry technical system takes the form

$$\dot{\boldsymbol{S}}(t,2) = [\boldsymbol{S}(t,2)]^{(1)} \cdot 0.056 + [\boldsymbol{S}(t,2)]^{(2)} \cdot 0.001$$
$$+ [\boldsymbol{S}(t,2)]^{(3)} \cdot 0.027 + [\boldsymbol{S}(t,2)]^{(4)} \cdot 0.056$$
$$+ [\boldsymbol{S}(t,2)]^{(5)} \cdot 0.382 + [\boldsymbol{S}(t,2)]^{(6)} \cdot 0.018$$
$$+ [\boldsymbol{S}(t,2)]^{(7)} \cdot 0.002 + [\boldsymbol{S}(t,2)]^{(8)} \cdot 0.018$$
$$+ [\boldsymbol{S}(t,2)]^{(9)} \cdot 0.056 + [\boldsymbol{S}(t,2)]^{(10)} \cdot 0.001$$
$$+ [\boldsymbol{S}(t,2)]^{(11)} \cdot 0.002 + [\boldsymbol{S}(t,2)]^{(12)} \cdot 0.024$$
$$+ [\boldsymbol{S}(t,2)]^{(13)} \cdot 0.286 + [\boldsymbol{S}(t,2)]^{(14)} \cdot 0.025$$
$$+ [\boldsymbol{S}(t,2)]^{(15)} \cdot 0.024 + [\boldsymbol{S}(t,2)]^{(16)} \cdot 0.002$$
$$+ [\boldsymbol{S}(t,2)]^{(17)} \cdot 0.002 + [\boldsymbol{S}(t,2)]^{(18)} \cdot 0.018,$$
$$t \in \langle 0, +\infty), \quad (26)$$

where $[\boldsymbol{S}(t,2)]^{(b)}$, $b = 1,2,\ldots,18$, are determined in [8]. Moreover, considering (25) and (26), the corresponding optimal standard deviations of the ferry technical system unconditional lifetime in the state subset is [8]

$$\dot{\boldsymbol{\sigma}}(2) \cong 1.38586 \text{ years}. \quad (27)$$

As the ferry technical system critical safety state is $r = 2$, then considering (26), its optimal system risk function, is given by

$$\dot{\boldsymbol{r}}(t) \cong 1 - \dot{\boldsymbol{S}}(t,2),\ t \in <0,+\infty). \tag{28}$$

Hence, and considering (28) the moment when the optimal system risk function exceeds a permitted level, for instance $\delta = 0.05$, is

$$\dot{\boldsymbol{\tau}} = \dot{\boldsymbol{r}}^{-1}(\delta) \cong 0.0729 \text{ year}. \tag{29}$$

By (25) the ferry technical system mean value of the optimal intensity of ageing is

$$\dot{\boldsymbol{\lambda}}(t,2) = \frac{1}{\dot{\boldsymbol{\mu}}(2)} = \frac{1}{1.39925} \cong 0.714669. \tag{30}$$

Considering (30) and the values of the analyzed system without operation impact intensity of ageing $\lambda^0(t)$, determined in [9], the optimal coefficient of the operation process impact on the ferry technical system intensity of ageing is

$$\dot{\boldsymbol{\rho}}(t,2) = \frac{\lambda\ (t)}{\lambda^0(t)} = \frac{0.714669}{0.677507} \cong 1.05485. \tag{31}$$

Finally, the ferry technical system optimal resilience indicator, i.e. the optimal coefficient of the ferry technical system resilience to operation process impact, is

$$\dot{\boldsymbol{RI}}(t) = \frac{1}{\dot{\boldsymbol{\rho}}(t,2)} = \frac{1}{1.055} \cong 0.948 = 94.8\%. \tag{32}$$

6. Inventory of results

In *Table 1* given below, to compare the optimal results of safety indicators given by (25), (27), (29)–(32) with their values before optimization given by (4), (6), (8)–(11) are presented.

7. Ferry operation new strategy

To maximize the ferry technical system mean lifetime, we can modify its operation process. Namely, cconsidering (24) and presuming the system operation time θ = 1 year = 365 days and using the approximate formula from [9]

$$\hat{M}_b = p_b \cdot \theta,\ \ b = 1,2,...,18,$$

we get the approximate mean values of total sojourn times of the ferry technical system at the particular

Table 1. The values of safety indicators before optimization with their values after optimization

Indicators	Before optimization	After optimization
The mean value of the ferry technical system lifetime in the safety state subset {2, 3, 4}	1.395 years	1.399 years
The standard deviation in the safety state subset {2, 3, 4}	1.383 years	1.386 years
The moment when the system risk function exceeds a permitted level	0.0727 years	0.0729 years
The ferry technical system intensity of ageing	0.717	0.715
The coefficient of the operation process impact on the ferry technical system intensity of ageing	1.058	1.055
The coefficient of the ferry technical system resilience to operation process impact	94.5%	94.8%

operation states before the optimization:

$$\hat{M}_1 = 13.87,\ \hat{M}_2 = 0.73,\ \hat{M}_3 = 9.49,$$
$$\hat{M}_4 = 13.14,\ \hat{M}_5 = 132{,}495,\ \hat{M}_6 = 9.49,$$
$$\hat{M}_7 = 1.825,\ \hat{M}_8 = 5.84,\ \hat{M}_9 = 13.505,$$
$$\hat{M}_{10} = 0.73,\ \hat{M}_{11} = 1.095,\ \hat{M}_{12} = 5.84,$$
$$\hat{M}_{13} = 128.115,\ \hat{M}_{14} = 12.41,\ \hat{M}_{15} = 8.76,$$
$$\hat{M}_{16} = 1.095,\ \hat{M}_{17} = 1.825,\ \hat{M}_{18} = 4.745, \qquad (33)$$

and considering and using the approximate formula from [9]

$$\dot{\hat{M}}_b = p_b \cdot \theta,\ b = 1,2,...,18,$$

we get the optimal mean values of the total sojourn times of the ferry technical system at the particular operation states after the optimization:

$$\dot{\hat{M}}_1 = 20.44,\ \dot{\hat{M}}_2 = 0.365,\ \dot{\hat{M}}_3 = 9.855,$$
$$\dot{\hat{M}}_4 = 20.44,\ \dot{\hat{M}}_5 = 139.43,\ \dot{\hat{M}}_6 = 6.57,$$
$$\dot{\hat{M}}_7 = 0.73,\ \dot{\hat{M}}_8 = 6.57,\ \dot{\hat{M}}_9 = 20.44,$$
$$\dot{\hat{M}}_{10} = 0.365,\ \dot{\hat{M}}_{11} = 0.73,\ \dot{\hat{M}}_{12} = 8.76,$$
$$\dot{\hat{M}}_{13} = 104.39,\ \dot{\hat{M}}_{14} = 9.125,\ \dot{\hat{M}}_{15} = 8.76,$$
$$\dot{\hat{M}}_{16} = 0.73,\ \dot{\hat{M}}_{17} = 0.73,\ \dot{\hat{M}}_{18} = 6.57. \qquad (34)$$

Further, having the above values the easiest way of modification of the ferry technical system operation process is to change it through the reorganizing this process in the way that depends on substitutioning approximately (nearing to in its real operation) the total sojourn times $\hat{M}_b$ of the system at the particular operation states before the optimization determined in (33) by their optimal values $\dot{\hat{M}}_b$ after the optimization given in (34).

8. Conclusion

The optimization procedure applied to safety and resilience optimization of the ferry technical system influenced by its operation process gives practically important possibility of its safety improvement through its new operation strategy. The proposed optimization procedure can be used in operation and safety optimization of members of various real critical infrastructures. Further research can be related to other impacts, for instance to climate-weather factors [17], and resolving the issues of critical infrastructure safety optimization and discovering optimal values of safety and resilience indicators of system impact by climate-weather conditions. These developments can also benefit the mitigation of critical infrastructure accident circumstances [1] and to improve critical infrastructure resilience to operation and climate-weather conditions [17].

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