Kołowrocki Krzysztof 0000-0002-4836-4976
Gdynia Maritime University, Gdynia, Poland, k.kolowrocki (at) wn.umg.edu.pl

Magryta-Mut Beata 0000-0002-4189-1853
Gdynia Maritime University, Gdynia, Poland, b.magryta (at) wn.umg.edu.pl

Safety of maritime ferry technical system impacted by its operation process

Keywords

critical infrastructure, shipping, maritime ferry, operation process, safety, resilience

Abstract

The chapter focuses on safety examination of a technical system of a maritime ferry that is the component of a shipping critical infrastructure. The model of the critical infrastructure safety without considering outside impacts is applied to determination of the maritime ferry technical system safety indicators. The operation impact model on critical infrastructure safety is created and applied to safety and resilience analysis of this system. The safety and resilience indicators are determined for this system under the assumption that its components' safety functions are piecewise exponential. The comparison of the maritime ferry technical system safety indicators without considering outside impacts with indicators considering its operation impact is done.

1. Introduction

We specify critical infrastructure as a complex system in its operating environment which important features are its internal dependencies inside the system and external dependencies outside the system which in the case of its degradation have a meaningful destructive influence on the health, safety and security, economics and social conditions of large human communities and territory areas [2], [10]. The safety indicators for such a system, that are crucial for its operators, can be obtained by using an original and innovative probabilistic approach to modeling of operation process impact on its safety [6]. At first, we can focus on a simplest pure safety multistate [14]–[16] ageing [11]–[13] model without considering outside impacts and define the critical infrastructure and its subsystems practically useful safety indicators [7]–[9]. This set of safety indicators can be completed by linking the safety pure model with the model of the critical infrastructure operation process [5]–[6]. This way created joint safety model of the critical infrastructure related to its operation process can offer additionally resilience indicators which are measures of the critical infrastructure operation impact on its safety and its resilience to operation [6]. The paper is devoted to modification of this joint safety and operation model and its practical application to safety and resilience examination of the technical system of the maritime ferry, the member of a shipping critical infrastructure [3]–[4].

2. Critical infrastructure safety background

2.1. Critical infrastructure operation process

We consider the critical infrastructure related to the operation process $Z(t)$, $t \in <0, \infty)$, impacted in a various way at its operation states z_k, $k = 1,2,\dots,v$. We assume that the changes of the operation states of the critical infrastructure operation process $Z(t)$ have an influence on the critical infrastructure safety structure and on the safety of the critical infrastructure subsystems [5].
Having the critical infrastructure operation process parameters like the number of operation states, the initial probabilities of operation states, the probabilities of transitions between the operation states and the mean values of conditional sojourn times at the operation states, it is possible to evaluate the critical infrastructure operation process main

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characteristics, the limit transient probabilities p_k, $k = 1,2,\ldots,v$, of the critical infrastructure operation process $Z(t)$ at particular operation states z_k, $k = 1,2,\ldots,v$.

2.2. Modeling safety of ageing system impacted by its operation process

We denote the critical infrastructure conditional lifetime in the safety state subset $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, while its operation process $Z(t)$, $t \in <0,\infty)$, is at the operation state z_k, $k = 1,2,\ldots,v$, by $[T^1(u)]^{(k)}$ and the conditional safety function of the critical infrastructure related to the operation process $Z(t)$, $t \in <0,\infty)$, by the vector

$$[\boldsymbol{S}^1(t,\cdot)]^{(k)} = [[\boldsymbol{S}^1(t,1)]^{(k)}, [\boldsymbol{S}^1(t,2)]^{(k)},\ldots, [\boldsymbol{S}^1(t,z)]^{(k)}], \tag{1}$$

with the coordinates given by

$$[\boldsymbol{S}^1(t,u)]^{(k)} = P([T^1(u)]^{(k)} > t \mid Z(t) = z_k),$$
$$t \in <0,\infty),\ u = 1,2,\ldots,z,\ k = 1,2,\ldots,v. \tag{2}$$

Next, we denote the critical infrastructure related to the operation process $Z(t)$, $t \in <0,\infty)$, unconditional lifetime by $T^1(u)$ and the unconditional safety function of the critical infrastructure related to the operation process $Z(t)$, $t \in <0,\infty)$, by the vector

$$\boldsymbol{S}^1(t,\cdot) = [\boldsymbol{S}^1(t,1), \boldsymbol{S}^1(t,2),\ldots, \boldsymbol{S}^1(t,z)],\ t \in <0,\infty), \tag{3}$$

with the coordinate defined by

$$\boldsymbol{S}^1(t,u) = P(T^1(u) > t) \text{ for } t \in <0,\infty),$$
$$u = 1,2,\ldots,z. \tag{4}$$

In the case when the critical infrastructure operation time θ is large enough, the unconditional safety function of the critical infrastructure related to the operation process $Z(t)$, $t \in <0,\infty)$, defined by (4), is given by [5]

$$\boldsymbol{S}^1(t,u) \cong \sum_{k=1}^{v} p_k [\boldsymbol{S}^1(t,u)]^{(k)} \text{ for } t \in <0,\infty),$$
$$u = 1,2,\ldots,z, \tag{5}$$

where $[\boldsymbol{S}^1(t,u)]^{(k)}$, $t \in <0,\infty)$, $u = 1,2,\ldots,z$, $k = 1,2,\ldots,v$, are the coordinates of critical infrastructure related to the operation process $Z(t)$, $t \in <0,\infty)$, conditional safety function defined by (1)–(2) and p_k, $k = 1,2,\ldots,v$, are the critical infrastructure operation process $Z(t)$, $t \geq 0$, limit transient probabilities at the operation states z_k, $k = 1,2,\ldots,v$.

If r, $r \in \{1,2,\ldots,z\}$, is the critical safety state, then the second safety indicator of the critical infrastructure related to the operation process $Z(t)$, $t \in <0,\infty)$, the risk function

$$\boldsymbol{r}^1(t) = P(s(t) < r \mid s(0) = z) = P(T^1(r) \leq t),$$
$$t \in <0,\infty), \tag{6}$$

is defined as a probability that the critical infrastructure related to the operation process $Z(t)$, $t \in <0,\infty)$, is in the subset of safety states worse than the critical safety state r, $r = 1,2,\ldots,z$, while it was in the best safety state z at the moment $t = 0$ and given by [5]

$$\boldsymbol{r}^1(t) = 1 - \boldsymbol{S}^1(t,r),\ t \in <0,\infty), \tag{7}$$

where $\boldsymbol{S}^1(t,r)$, $t \in <0,\infty)$, is the coordinate of critical infrastructure related to the operation process $Z(t)$ unconditional safety function given by (5) for $u = r$.
The graph of the critical infrastructure risk function $\boldsymbol{r}^1(t)$, $t \in <0,\infty)$, defined by (7), is the safety indicator called the fragility curve [2] of the critical infrastructure related to the operation process $Z(t)$, $t \in <0,\infty)$.
Other useful safety characteristics of the critical infrastructure related to the operation process $Z(t)$ are:

- the mean lifetimes of the critical infrastructure in the safety state subsets $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, given by

$$\boldsymbol{\mu}^1(u) = \int_0^\infty \boldsymbol{S}^1(t,u)dt \cong \sum_{k=1}^{v} p_k [\boldsymbol{\mu}^1(u)]^{(k)},$$
$$u = 1,2,\ldots,z, \tag{8}$$

  where $[\boldsymbol{\mu}^1(u)]^{(k)}$, $u = 1,2,\ldots,z$, $k = 1,2,\ldots,v$, are the mean values of the critical infrastructure conditional lifetimes $[T^1(u)]^{(k)}$ in the safety state subset $\{u,u+1,\ldots,z\}$ at the operation states z_k, $k = 1,2,\ldots,v$, given by

$$[\boldsymbol{\mu}^1(u)]^{(k)} = \int_0^\infty [\boldsymbol{S}^1(t,u)]^{(k)} dt,$$
$$u = 1,2,\ldots,z,\ k = 1,2,\ldots,v; \tag{9}$$

- the mean lifetimes $\bar{\boldsymbol{\mu}}^1(u)$, $u = 1,2,\ldots,z$, of the critical infrastructure in the particular safety states

$$\bar{\boldsymbol{\mu}}^1(u) = \boldsymbol{\mu}^1(u) - \boldsymbol{\mu}^1(u+1),\ u = 1,2,\ldots,z-1,$$
$$\bar{\boldsymbol{\mu}}^1(z) = \boldsymbol{\mu}^1(z); \tag{10}$$

- the mean value $\boldsymbol{\mu}^1(r)$ of the critical infrastructure lifetime $T^1(r)$ up to the exceeding the critical safety state r given by (8), for $u = r$;
- the moment τ^1 of exceeding the acceptable value of critical infrastructure risk function level δ given by

$$\tau^1 = (\boldsymbol{r}^1)^{-1}(\delta),\ t \in <0,\infty), \tag{11}$$

where $(\boldsymbol{r}^1)^{-1}(t)$ is the inverse function of the risk function $\boldsymbol{r}(t)$ given by (7);
- the intensities of ageing of the critical infrastructure (the intensities of the critical infrastructure departure from the safety state subset $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$,)

$$\lambda^1(t,u) = -d\boldsymbol{S}^1(t,u)/\boldsymbol{S}^1(t,u),\ u = 1,2,\ldots,z,$$
$$t \in <0,\infty), \tag{12}$$

where $\boldsymbol{S}^1(t,u)$ for $t \in <0,\infty), u = 1,2,\ldots,z$, are defined by (5).

Moreover, we define resilience indicators

- the coefficients of operation process impact on the critical infrastructure intensities of ageing (intensities of departure from the safety state subset $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$,)

$$\rho^1(t,u) = \lambda^1(t,u)/\lambda^0(t,u),\ t \in <0,\infty),$$
$$u = 1,2,\ldots,z, \tag{13}$$

where and $\lambda^0(t,u)$, $t \in <0,\infty), u = 1,2,\ldots,z$, are the intensities of ageing of the critical infrastructure without of operation process impact, and $\lambda^1(t,u)$, $t \in <0,\infty), u = 1,2,\ldots,z$, are the intensities of ageing of the critical infrastructure with the operation process impact defined by (12);
- the indicator of critical infrastructure resilience to operation process impact defined by

$$\boldsymbol{RI}^1(t) = 1/\rho^1(t,r),\ t \in <0,\infty), \tag{14}$$

where $\boldsymbol{\rho}^1(t,r)$, $t \in <0,\infty)$ is the coefficient of operation process impact on the critical infrastructure intensities od degradation given by (13) for $u = r$.

3. Safety of maritime ferry technical system free of outside impacts

We will examine technical system safety of a selected member of the shipping critical infrastructure. Namely, the maritime ferry technical system safety will by analyzed. We assume, that the maritime ferry is composed of a number of main technical subsystems:

- S_1 – a navigational subsystem,
- S_2 – a propulsion and controlling subsystem,
- S_3 – a loading and unloading subsystem,
- S_4 – a stability control subsystem,
- S_5 – an anchoring and mooring subsystem,

having an essential influence on its safety, further called the ferry technical system.

The subsystems S_1, S_2, S_3, S_4, S_5 are forming a general series safety structure of the ferry technical system shown in *Figure 1*.

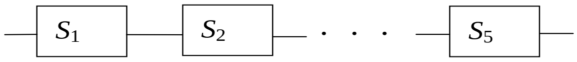

Figure 1. The general structure of the ferry technical system safety

3.1. Maritime ferry technical system safety parameters and indicators

After discussion with experts, taking into account the safety of the operation of the ferry, we distinguish the five safety states (z = 4) of the ferry technical system and its components:

- a safety state 4 – the ferry operation is fully safe,
- a safety state 3 – the ferry operation is less safe and more dangerous because of the possibility of environment pollution,
- a safety state 2 called a critical safety state – the ferry operation is less safe and more dangerous because of the possibility of environment pollution and causing small accidents,
- a safety state 1 – the ferry operation is much less safe and much more dangerous because of the possibility of serious environment pollution and causing extensive accidents,
- a safety state 0 – the ferry technical system is destroyed [4].

Moreover, by the expert opinions, we assume that there are possible the transitions between the components' safety states only from better to worse ones and we assume that the system and its components critical safety state is $r = 2$.

We assume that the components E_{ij}^{υ}, $\nu = 1,2,3,4,5$, of subsystems (see: Appendix) of the ferry technical system have piecewise exponential safety functions

$$S_{ij}^0(t,\cdot) = [S_{ij}^0(t,1), S_{ij}^0(t,2), S_{ij}^0(t,3), S_{ij}^0(t,4)],$$
$$t \in <0,\infty), \tag{15}$$

with the coordinates

$$S_{ij}^{0}(t,u) = \exp[-\lambda_{ij}^{0}(u)t],\ \lambda_{ij}^{0}(u) \geq 0,$$
$$u = 1,2,3,4. \tag{16}$$

Existing in (16) the intensities $\lambda_{ij}^{0}(u), u$ = 1,2,3,4, of the subsystem components E_{ij} departure from the safety state subsets {1,2,3,4}, {2,3,4}, {3,4}, {4}, calculated on the basis of approximate safety data coming from experts, are as follows [1]:

- for subsystem S_1

$\lambda_{11}^{0}(1)$ = 0.033, $\lambda_{11}^{0}(2)$ = 0.04, $\lambda_{11}^{0}(3)$ = 0.045,
$\lambda_{11}^{0}(4)$ = 0.05;

- for subsystem S_2

$\lambda_{1j}^{0}(1)$ = 0.033, $\lambda_{1j}^{0}(2)$ = 0.04,
$\lambda_{1j}^{0}(3)$ = 0.05, $\lambda_{1j}^{0}(4)$ = 0.055, $j = 1,2,3,4,$
$\lambda_{2j}^{0}(1)$ = 0.066, $\lambda_{2j}^{0}(2)$ = 0.07,
$\lambda_{2j}^{0}(3)$ = 0.075, $\lambda_{2j}^{0}(4)$ = 0.08, $j = 1,2,$
$\lambda_{31}^{0}(1)$ = 0.066, $\lambda_{31}^{0}(2)$ = 0.07,
$\lambda_{31}^{0}(3)$ = 0.075, $\lambda_{31}^{0}(4)$ = 0.08,
$\lambda_{i1}^{0}(1)$ = 0.033, $\lambda_{i1}^{0}(2)$ = 0.04,
$\lambda_{i1}^{0}(3)$ = 0.045, $\lambda_{i1}^{0}(4)$ = 0.05, $i = 4,5,6,7;$

- for subsystem S_3

$\lambda_{11}^{0}(1)$ = 0.02, $\lambda_{11}^{0}(2)$ = 0.03,
$\lambda_{11}^{0}(3)$ = 0.035, $\lambda_{11}^{0}(4)$ = 0.04,
$\lambda_{21}^{0}(1)$ = 0.02, $\lambda_{21}^{0}(2)$ = 0.025,
$\lambda_{21}^{0}(3)$ = 0.03, $\lambda_{21}^{0}(4)$ = 0.04,
$\lambda_{31}^{0}(1)$ = 0.033, $\lambda_{31}^{0}(2)$ = 0.04,
$\lambda_{31}^{0}(3)$ = 0.045, $\lambda_{31}^{0}(4)$ = 0.05,
$\lambda_{41}^{0}(1)$ = 0.033, $\lambda_{41}^{0}(2)$ = 0.04,
$\lambda_{41}^{0}(3)$ = 0.045, $\lambda_{41}^{0}(4)$ = 0.05,
$\lambda_{51}^{0}(1)$ = 0.033, $\lambda_{51}^{0}(2)$ = 0.04,
$\lambda_{51}^{0}(3)$ = 0.045, $\lambda_{51}^{0}(4)$ = 0.05;

- for subsystem S_4

$\lambda_{11}^{0}(1)$ = 0.05, $\lambda_{11}^{0}(2)$ = 0.06,
$\lambda_{11}^{0}(3)$ = 0.065, $\lambda_{11}^{0}(4)$ = 0.07,
$\lambda_{21}^{0}(1)$ = 0.033, $\lambda_{21}^{0}(2)$ = 0.04,
$\lambda_{21}^{0}(3)$ = 0.045, $\lambda_{21}^{0}(4)$ = 0.05;

- for subsystem S_5

$\lambda_{11}^{0}(1)$ = 0.033, $\lambda_{11}^{0}(2)$ = 0.04,
$\lambda_{11}^{0}(3)$ = 0.045, $\lambda_{11}^{0}(4)$ = 0.05,
$\lambda_{21}^{0}(1)$ = 0.033, $\lambda_{21}^{0}(2)$ = 0.04,
$\lambda_{21}^{0}(3)$ = 0.05, $\lambda_{21}^{0}(4)$ = 0.055,
$\lambda_{31}^{0}(1)$ = 0.033, $\lambda_{31}^{0}(2)$ = 0.04,
$\lambda_{31}^{0}(3)$ = 0.05, $\lambda_{31}^{0}(4)$ = 0.06. (17)

Considering (15)–(17) and the safety structures of subsystems presented in the Appendix, the safety function of the series ferry technical system without considering outside impacts, is given by [5]:

$$\boldsymbol{S}^{0}(t,\cdot) = [\ \boldsymbol{S}^{0}(t,1),\ \boldsymbol{S}^{0}(t,2),\ \boldsymbol{S}^{0}(t,3),\ \boldsymbol{S}^{0}(t,4)\ ] \tag{18}$$

where

$\boldsymbol{S}^{0}(t,1)$ = 12 exp[–0.684t] + 8 exp[–0.783t]
–16exp[–0.717t] – 3exp[–0.816t],

$\boldsymbol{S}^{0}(t,2)$ = 12 exp[–0.815t] + 8 exp[–0.925t]
+ 6 exp[–0.895t] – 16 exp[–0.855t]
– 6 exp[–0.885t] – 3 exp[–0.965t],

$\boldsymbol{S}^{0}(t,3)$ = 12 exp[–0.930t] + 8 exp[–1.055t]
+ 6 exp[–1.030t] – 16 exp[–0.980t]
– 6 exp[–1.005t] – 3 exp[–1.105t],

$\boldsymbol{S}^{0}(t,4)$ = 12 exp[–1.035t] + 8 exp[–1.170t]
+ 6 exp[–1.145t] – 16 exp[–1.090t]
– 6 exp[–1.115t] – 3 exp[–1.225],
for $t \in <0,\infty)$. (19)

After he integration of the above safety functions, the expected values of the ferry technical system lifetimes in the safety state subsets {1,2,3,4}, {2,3,4}, {3,4}, {4}, are:

$\mu^{0}(1) \cong$ 1.770, $\mu^{0}(2) \cong$ 1.476, $\mu^{0}(3) \cong$ 1.300,
$\mu^{0}(4) \cong$ 1.164 year, (20)

and further, the mean values of the ferry technical system conditional lifetimes in the particular safety states are:

$$\bar{\mu}^0(1) \cong 0.294,\ \bar{\mu}^0(2) \cong 0.176,\ \bar{\mu}^0(3) \cong 0.136,$$
$$\bar{\mu}^0(4) \cong 1.164 \text{ year}. \quad (21)$$

Hence, the risk function is given by (7) and moment when the system risk function exceeds a permitted level, for instance $\delta = 0.05$, is

$$\tau^0 = (\boldsymbol{r}^0)^{-1}(\delta) \cong 0.077. \quad (22)$$

The ferry technical system critical infrastructure intensities of ageing, after applying (12), are:

$$\lambda^0(1) \cong 0.564972,\ \lambda^0(2) \cong 0.677507,$$
$$\lambda^0(3) \cong 0.769231,\ \lambda^0(4) \cong 0.859107. \quad (23)$$

4. Safety of maritime ferry technical system impacted by operation process

4.1. Parameters and characteristics of maritime ferry technical system operation process

The considered technical system of the maritime ferry is a series system composed of subsystems S_1, S_2, S_3, S_4 and S_5. However, the ferry technical system safety structure and the subsystems and components safety depend on its changing in time operation states. Before indicating this system changing in time safety structures, we define its operation states.

Thus, taking into account the expert opinions concerned with the operation process of the considered ferry technical system, we distinguish the following as its the eighteen operation states:

- an operation state z_1 – loading at Gdynia Port,
- an operation state z_2 – unmooring operations at Gdynia Port,
- an operation state z_3 – leaving Gdynia Port and navigation to “GD” buoy,
- an operation state z_4 – navigation at restricted waters from “GD” buoy to the end of Traffic Separation Scheme,
- an operation state z_5 – navigation at open waters from the end of Traffic Separation Scheme to “Angoring” buoy,
- an operation state z_6 – navigation at restricted waters from “Angoring” buoy to “Verko” Berth at Karlskrona,
- an operation state z_7 – mooring operations at Karlskrona Port,
- an operation state z_8 – unloading at Karlskrona Port,
- an operation state z_9 – loading at Karlskrona Port,
- an operation state z_{10} – unmooring operations at Karlskrona Port,
- an operation state z_{11} – ferry turning at Karlskrona Port,
- an operation state z_{12} – leaving Karlskrona Port and navigation at restricted waters to “Angoring” buoy,
- an operation state z_{13} – navigation at open waters from “Angoring” buoy to the entering Traffic Separation Scheme,
- an operation state z_{14} – navigation at restricted waters from the entering Traffic Separation Scheme to “GD” buoy,
- an operation state z_{15} – navigation from “GD” buoy to turning area,
- an operation state z_{16} – ferry turning at Gdynia Port,
- an operation state z_{17} – mooring operations at Gdynia Port,
- an operation state z_{18} – unloading at Gdynia Port.

To identify the unknown parameters of the ferry technical system operation process the suitable statistical data coming from its real realizations were collected. It was possible to collect these data because of the high frequency of the ferry voyages that result in a large number of its technical system operation process realizations.

The ferry technical system operation process is very regular in the sense that the operation state changes are from the particular state z_k, $k = 1,2,\ldots,17$, to the neighboring state z_{k+1}, $k = 1,2,\ldots,17$, and from z_{18} to z_1 only.

The ferry technical system main operation characteristics, the limit transient probabilities p_k of the operation process $Z(t)$ at the particular operation states z_k, $k = 1,2,\ldots,18$, are [4]:

$$p_1 = 0.038,\ p_2 = 0.002,\ p_3 = 0.026,$$
$$p_4 = 0.036,\ p_5 = 0.363,\ p_6 = 0.026,$$
$$p_7 = 0.005,\ p_8 = 0.016,\ p_9 = 0.037,$$
$$p_{10} = 0.002,\ p_{11} = 0.003,\ p_{12} = 0.016,$$
$$p_{13} = 0.351,\ p_{14} = 0.034,\ p_{15} = 0.024,$$
$$p_{16} = 0.003,\ p_{17} = 0.005,\ p_{18} = 0.013. \quad (24)$$

The influence of the maritime ferry operation states changing on the changes of the ferry technical system safety structure is presented in the Appendix.

4.2. Parameters of maritime ferry operation process impact on its technical system safety

The approximate values of coefficients, $\rho^1{}_{i1}(u)$, u = 1,2,3,4, of the operation process impact on the components E_{ij} of the maritime ferry technical system intensities of ageing, $\lambda^1_{ij}(u)$, u = 1,2,3,4, at the operation states z_k, k = 1,2,…,18, coming from experts are as follows:

- for subsystem S_1

$[\rho^1{}_{i1}(u)]^{(k)}$ = 1.1, for u = 1,2,3,4,
k = 2,3,4,5,6,7,10,11,12,13,14,15,16,
i = 1;

- for subsystem S_2

$[\rho^1{}_{ij}(u)]^{(k)}$ = 1.2, for u = 1,2,3,4,
k = 2,4,6,7,10,14,17, i = 1, j = 1,2,3,4,
$[\rho^1{}_{ij}(u)]^{(k)}$ = 1.15, for u = 1,2,3,4,
k = 3,12,15, i = 1, j = 1,2,3,4,
$[\rho^1{}_{ij}(u)]^{(k)}$ = 1.3, for u = 1,2,3,4, k = 5,13,
i = 1, j = 1,2,3,4,
$[\rho^1{}_{ij}(1)]^{(b)}$ = 1.1, for u = 1,2,3,4, k = 11,16,
i = 1, j = 1,2,3,4,
$[\rho^1{}_{ij}(u)]^{(k)}$ = 1.3, for u = 1,2,3,4,
k = 2,7,10,17, i = 2, j = 1, 2,
$[\rho^1{}_{ij}(u)]^{(k)}$ = 1.1, for u = 1,2,3,4, k = 3,15,
i = 2, j = 1,2,
$[\rho^1{}_{ij}(u)]^{(k)}$ = 1.05, for u = 1,2,3,4, k = 6,
i = 2, j = 1,2,
$[\rho^1{}_{ij}(u)]^{(k)}$ = 1.4, for u = 1,2,3,4, k = 11,16,
i = 2, j = 1, 2,
$[\rho^1{}_{ij}(u)]^{(k)}$ = 1.3, for u = 1,2,3,4,
k = 2,7,10,17, i = 3, j = 1,
$[\rho^1{}_{ij}(u)]^{(k)}$ = 1.1, for u = 1,2,3,4 ,k = 3,15,
i = 3, j = 1,
$[\rho^1{}_{ij}(1)]^{(b)}$ = 1.05, for u = 1,2,3,4 ,k = 6,
i = 3, j = 1,
$[\rho^1{}_{ij}(u)]^{(k)}$ = 1.4, for u = 1,2,3,4 ,k = 11,16,
i = 3, j = 1,
$[\rho^1{}_{ij}(u)]^{(k)}$ = 1.1, for u = 1,2,3,4,
k = 2,7,10,17, i = 4, 5, j = 1,
$[\rho^1{}_{ij}(u)]^{(k)}$ = 1.15, for u = 1,2,3,4 ,k = 3,15,
i = 4,5, j = 1,
$[\rho^1{}_{ij}(u)]^{(k)}$ = 1.2, for u = 1,2,3,4,
k = 4,6,11,12,14,16, i = 4,5, j = 1,
$[\rho^1{}_{ij}(u)]^{(k)}$ = 1.25, for u = 1,2,3,4 ,k = 5,13,
i = 4,5, j = 1,
$[\rho^1{}_{ij}(u)]^{(k)}$ = 1.2, for u = 1,2,3,4,
k = 2,7,10,17, i = 6,7, j = 1,
$[\rho^1{}_{ij}(u)]^{(k)}$ = 1.1, for u = 1,2,3,4,
k = 3 4,11,12,15,16, i = 6,7, j = 1,
$[\rho^1{}_{ij}(u)]^{(k)}$ = 1.05, for u = 1,2,3,4 ,k = 5,13,
i = 6,7, j = 1,
$[\rho^1{}_{ij}(u)]^{(k)}$ = 1.2, for u = 1,2,3,4 ,k = 6,14,
i = 6, j = 1,
$[\rho^1{}_{ij}(u)]^{(k)}$ = 1.1, for u = 1,2,3,4 ,k = 6,14,
i = 7, j = 1;

- for subsystem S_3

$[\rho^1{}_{i1}(u)]^{(k)}$ = 1.25, for u = 1,2,3,4,
k = 1,8,9,18, i = 1, 2,
$[\rho^1{}_{i1}(u)]^{(k)}$ = 1.25, for u = 1,2,3,4,
k = 1,18, i = 3;

- for subsystem S_4

$[\rho^1{}_{ij}(u)]^{(k)}$ = 1.1, for u = 1,2,3,4,
k = 1,8,9,18, i = 1, j = 1,
$[\rho^1{}_{ij}(u)]^{(k)}$ = 1.1, for u = 1,2,3,4,
k = 4,6,14, i = 1, j = 1,
$[\rho^1{}_{ij}(u)]^{(k)}$ = 1.25, for u = 1,2,3,4 ,k = 5,13,
i = 1, j = 1,
$[\rho^1{}_{ij}(u)]^{(k)}$ = 1.05, for u = 1,2,3,4 ,k = 12,
i = 1, j = 1;

- for subsystem S_5

$[\rho^1{}_{ij}(u)]^{(k)}$ = 1.1, for u = 1,2,3,4 ,k = 2,10,
i = 1,2,3, j = 1,
$[\rho^1{}_{ij}(u)]^{(k)}$ = 1.35, for u = 1,2,3,4 ,k = 7,17,
i = 1,2,3, j = 1. (25)

In the case, when a component is not used at the operation state z_k, k = 1,2,…,18, the coefficients of the operation process impact on its intensity of ageing is equal to 1.

4.3. Parameters of maritime ferry technical system safety

We assume that the ferry technical system critical infrastructure subsystems S_v, v = 1,2,…,5, are composed of five-state i.e. z = 4, components, $E^{(\upsilon)}_{ij}$, v = 1,2,…,5, having the conditional safety functions given by the vector

$$[S^1_{ij}(t,\cdot)]^{(k)} = [[S^1_{ij}(t,1)]^{(k)}, [S^1_{ij}(t,2)]^{(k)}, [S^1_{ij}(t,3)]^{(k)}, [S^1_{ij}(t,4)]^{(k)}],\ k = 1,2,\ldots,18, \quad (26)$$

with the exponential coordinates

$$[S^1_{ij}(t,u)]^{(k)} = \exp[-[\lambda^1_{ij}(u)]^{(k)} t],\ i = 1,2,\ldots,7,\ j = 1,2,3,4,\ k = 1, 2,\ldots, 18,\ u = 1,2,\ldots,4, \quad (27)$$

where

$$[\lambda^1{}_{ij}(u)]^{(k)} = [\rho^1{}_{ij}(u)]^{(k)} \cdot \lambda^0{}_{ij}(u),\ u = 1,2,3,4,\ i = 1,2,\ldots,7,\ j = 1,2,3,4, \quad (28)$$

and $[\rho^1_{ij}(u)]^{(k)}$, u = 1,2,3,4, i = 1,2,…,7, j = 1,2,3,4, are the coefficients of operation process impact on the intensities of degradation of the ferry technical system components, $E_{ij}^{(\upsilon)}$, v = 1,2,…,5, at the operation states z_k, k = 1,2,…,18, and $\lambda^0_{ij}(u)$, u = 1,2,3,4, i = 1,2,…,7, j = 1,2,3,4,are the intensities of the ferry technical system components without the operation process impact.

Under the assumption (28), considering (25) and (17), it follows that the intensities of the technical system components departure from the safety states subset {1,2,3,4}, {2,3,4}, {3,4}, {4} with operation impact on their safety are:

- for subsystem S_1

$[\lambda^1_{i1}(1)]^{(k)} = 0.0363$, $[\lambda^1_{i1}(1)]^{(k)} = 0.044$,
$[\lambda^1_{i1}(3)]^{(k)} = 0.0495$, $[\lambda^1_{i1}(4)]^{(k)} = 0.055$,
k = 2,3,4,5,6,7,10,11,12,13,14,15,16,
i = 1;

- for subsystem S_2

$[\lambda^1_{ij}(1)]^{(k)} = 0.0396$, $[\lambda^1_{ij}(1)]^{(k)} = 0.048$,
$[\lambda^1_{ij}(3)]^{(k)} = 0.06$, $[\lambda^1_{ij}(4)]^{(k)} = 0.066$,
b = 2,4,6,7,10,14,17, i = 1, j = 1,2,3,4,
$[\lambda^1_{ij}(1)]^{(k)} = 0.03795$, $[\lambda^1_{ij}(1)]^{(k)} = 0.046$,
$[\lambda^1_{ij}(3)]^{(k)} = 0.0575$, $[\lambda^1_{ij}(4)]^{(k)} = 0.06325$,
b = 3,12,15, i = 1, j = 1,2,3,4,
$[\lambda^1_{ij}(1)]^{(k)} = 0.0429$, $[\lambda^1_{ij}(1)]^{(k)} = 0.052$,
$[\lambda^1_{ij}(3)]^{(k)} = 0.065$, $[\lambda^1_{ij}(4)]^{(k)} = 0.0715$,
b = 5,13, i = 1, j = 1,2,3,4,
$[\lambda^1_{ij}(1)]^{(k)} = 0.0363$, $[\lambda^1_{ij}(1)]^{(k)} = 0.044$,
$[\lambda^1_{ij}(3)]^{(k)} = 0.055$, $[\lambda^1_{ij}(4)]^{(k)} = 0.0605$,
b = 11,16, i = 1, j = 1,2,3,4,
$[\lambda^1_{ij}(1)]^{(k)} = 0.0858$, $[\lambda^1_{ij}(1)]^{(k)} = 0.091$,
$[\lambda^1_{ij}(3)]^{(k)} = 0.0975$, $[\lambda^1_{ij}(4)]^{(k)} = 0.104$,
b = 2,7,10,17, i = 2, j = 1,2,
$[\lambda^1_{ij}(1)]^{(k)} = 0.0726$, $[\lambda^1_{ij}(1)]^{(k)} = 0.077$,
$[\lambda^1_{ij}(3)]^{(k)} = 0.0825$, $[\lambda^1_{ij}(4)]^{(k)} = 0.088$,
b = 3,15, i = 2, j = 1,2,
$[\lambda^1_{ij}(1)]^{(k)} = 0.0693$, $[\lambda^1_{ij}(1)]^{(k)} = 0.0735$,
$[\lambda^1_{ij}(3)]^{(k)} = 0.07875$, $[\lambda^1_{ij}(4)]^{(k)} = 0.084$,
b = 6, i = 2, j = 1,2,
$[\lambda^1_{ij}(1)]^{(k)} = 0.0924$, $[\lambda^1_{ij}(1)]^{(k)} = 0.098$,
$[\lambda^1_{ij}(3)]^{(k)} = 0.105$, $[\lambda^1_{ij}(4)]^{(k)} = 0.112$,
b = 11,16, i = 2, j = 1,2,
$[\lambda^1_{ij}(1)]^{(k)} = 0.0858$, $[\lambda^1_{ij}(1)]^{(k)} = 0.091$,
$[\lambda^1_{ij}(3)]^{(k)} = 0.0975$, $[\lambda^1_{ij}(4)]^{(k)} = 0.104$,
b = 2,7,10,17, i = 3, j = 1,
$[\lambda^1_{ij}(1)]^{(k)} = 0.0726$, $[\lambda^1_{ij}(1)]^{(k)} = 0.077$,
$[\lambda^1_{ij}(3)]^{(k)} = 0.0825$, $[\lambda^1_{ij}(4)]^{(k)} = 0.088$,
b = 3,15, i = 3, j = 1,
$[\lambda^1_{ij}(1)]^{(k)} = 0.0693$, $[\lambda^1_{ij}(1)]^{(k)} = 0.0735$,
$[\lambda^1_{ij}(3)]^{(k)} = 0.07875$, $[\lambda^1_{ij}(4)]^{(k)} = 0.084$,
b = 6, i = 3, j = 1,
$[\lambda^1_{ij}(1)]^{(k)} = 0.0924$, $[\lambda^1_{ij}(1)]^{(k)} = 0.098$,
$[\lambda^1_{ij}(3)]^{(k)} = 0.105$, $[\lambda^1_{ij}(4)]^{(k)} = 0.112$,
b = 11,16, i = 3, j = 1,
$[\lambda^1_{ij}(1)]^{(k)} = 0.0363$, $[\lambda^1_{ij}(1)]^{(k)} = 0.044$,
$[\lambda^1_{ij}(3)]^{(k)} = 0.0495$, $[\lambda^1_{ij}(4)]^{(k)} = 0.055$,
b = 2,7,10,17, i = 4,5, j = 1,
$[\lambda^1_{ij}(1)]^{(k)} = 0.03795$, $[\lambda^1_{ij}(1)]^{(k)} = 0.046$,
$[\lambda^1_{ij}(3)]^{(k)} = 0.05175$, $[\lambda^1_{ij}(4)]^{(k)} = 0.0575$,
b = 3,15, i = 4,5, j = 1,
$[\lambda^1_{ij}(1)]^{(k)} = 0.0396$, $[\lambda^1_{ij}(1)]^{(k)} = 0.048$,
$[\lambda^1_{ij}(3)]^{(k)} = 0.054$, $[\lambda^1_{ij}(4)]^{(k)} = 0.06$,
b = 4,6,11,12,14,16, i = 4,5, j = 1,
$[\lambda^1_{ij}(1)]^{(k)} = 0.04125$, $[\lambda^1_{ij}(1)]^{(k)} = 0.05$,
$[\lambda^1_{ij}(3)]^{(k)} = 0.05625$, $[\lambda^1_{ij}(4)]^{(k)} = 0.0625$,
b = 5,13, i = 4,5, j = 1,
$[\lambda^1_{ij}(1)]^{(k)} = 0.0396$, $[\lambda^1_{ij}(1)]^{(k)} = 0.048$,
$[\lambda^1_{ij}(3)]^{(k)} = 0.054$, $[\lambda^1_{ij}(4)]^{(k)} = 0.06$,
b = 2,7,10,17, i = 6,7, j = 1,
$[\lambda^1_{ij}(1)]^{(k)} = 0.0363$, $[\lambda^1_{ij}(1)]^{(k)} = 0.044$,
$[\lambda^1_{ij}(3)]^{(k)} = 0.0495$, $[\lambda^1_{ij}(4)]^{(k)} = 0.055$,
b = 3,4,11,12,15,16, i = 6,7, j = 1,
$[\lambda^1_{ij}(1)]^{(k)} = 0.03465$, $[\lambda^1_{ij}(1)]^{(k)} = 0.042$,
$[\lambda^1_{ij}(3)]^{(k)} = 0.04725$, $[\lambda^1_{ij}(4)]^{(k)} = 0.0525$,
b = 5,13, i = 6,7, j = 1,
$[\lambda^1_{ij}(1)]^{(k)} = 0.0396$, $[\lambda^1_{ij}(1)]^{(k)} = 0.048$,
$[\lambda^1_{ij}(3)]^{(k)} = 0.054$, $[\lambda^1_{ij}(4)]^{(k)} = 0.06$,
b = 6,14, i = 6, j = 1,
$[\lambda^1_{ij}(1)]^{(k)} = 0.0363$, $[\lambda^1_{ij}(1)]^{(k)} = 0.044$,
$[\lambda^1_{ij}(3)]^{(k)} = 0.0495$, $[\lambda^1_{ij}(4)]^{(k)} = 0.055$,
b = 6,14, i = 7, j = 1;

- for subsystem S_3

$[\lambda^1_{i1}(1)]^{(k)} = 0.025$, $[\lambda^1_{i1}(2)]^{(k)} = 0.0375$,
$[\lambda^1_{i1}(3)]^{(k)} = 0.04375$, $[\lambda^1_{i1}(4)]^{(k)} = 0.05$,
k = 1,8,9,18, i = 1,
$[\lambda^1_{i1}(1)]^{(k)} = 0.025$, $[\lambda^1_{i1}(2)]^{(k)} = 0.03125$,
$[\lambda^1_{i1}(3)]^{(k)} = 0.025$, $[\lambda^1_{i1}(4)]^{(k)} = 0.05$,
k = 1,8,9,18, i = 2,
$[\lambda^1_{i1}(1)]^{(k)} = 0.04125$, $[\lambda^1_{i1}(2)]^{(k)} = 0.05$,
$[\lambda^1_{i1}(3)]^{(k)} = 0.05625$, $[\lambda^1_{i1}(4)]^{(k)} = 0.0625$,
k = 1,18, i = 3;

- for subsystem S_4

$[\lambda^1_{ij}(1)]^{(k)} = 0.055$, $[\lambda^1_{ij}(2)]^{(k)} = 0.066$,
$[\lambda^1_{ij}(3)]^{(k)} = 0.0715$, $[\lambda^1_{ij}(4)]^{(k)} = 0.077$,
k = 1,8,9,18, i = 1, j = 1,
$[\lambda^1_{ij}(1)]^{(k)} = 0.055$, $[\lambda^1_{ij}(2)]^{(k)} = 0.066$,
$[\lambda^1_{ij}(3)]^{(k)} = 0.0715$, $[\lambda^1_{ij}(4)]^{(k)} = 0.077$,
k = 4,6,14, i = 1, j = 1,
$[\lambda^1_{ij}(1)]^{(k)} = 0.0625$, $[\lambda^1_{ij}(2)]^{(k)} = 0.075$,
$[\lambda^1_{ij}(3)]^{(k)} = 0.08125$, $[\lambda^1_{ij}(4)]^{(k)} = 0.0875$,
k = 5,13, i = 1, j = 1,
$[\lambda^1_{ij}(1)]^{(k)} = 0.0525$, $[\lambda^1_{ij}(2)]^{(k)} = 0.063$,
$[\lambda^1_{ij}(3)]^{(k)} = 0.06825$, $[\lambda^1_{ij}(4)]^{(k)} = 0.0735$,
k = 12, i = 1, j = 1;

- for subsystem S_5

$[\lambda^1_{ij}(1)]^{(k)} = 0.0363$, $[\lambda^1_{ij}(2)]^{(k)} = 0.044$,
$[\lambda^1_{ij}(3)]^{(k)} = 0.0495$, $[\lambda^1_{ij}(4)]^{(k)} = 0.055$,
$k = 2{,}10$, $i = 1$, $j = 1$,
$[\lambda^1_{ij}(1)]^{(k)} = 0.0363$, $[\lambda^1_{ij}(2)]^{(k)} = 0.044$,
$[\lambda^1_{ij}(3)]^{(k)} = 0.055$, $[\lambda^1_{ij}(4)]^{(k)} = 0.0605$,
$k = 2{,}10$, $i = 2$, $j = 1$,
$[\lambda^1_{ij}(1)]^{(k)} = 0.0363$, $[\lambda^1_{ij}(2)]^{(k)} = 0.044$,
$[\lambda^1_{ij}(3)]^{(k)} = 0.055$, $[\lambda^1_{ij}(4)]^{(k)} = 0.066$,
$k = 2{,}10$, $i = 3$, $j = 1$,
$[\lambda^1_{ij}(1)]^{(k)} = 0.04455$, $[\lambda^1_{ij}(2)]^{(k)} = 0.054$,
$[\lambda^1_{ij}(3)]^{(k)} = 0.06075$, $[\lambda^1_{ij}(4)]^{(k)} = 0.0675$,
$k = 7{,}17$, $i = 1$, $j = 1$,
$[\lambda^1_{ij}(1)]^{(k)} = 0.04455$, $[\lambda^1_{ij}(2)]^{(k)} = 0.054$,
$[\lambda^1_{ij}(3)]^{(k)} = 0.06075$, $[\lambda^1_{ij}(4)]^{(k)} = 0.07425$,
$k = 7{,}17$, $i = 2$, $j = 1$,
$[\lambda^1_{ij}(1)]^{(k)} = 0.04455$, $[\lambda^1_{ij}(2)]^{(k)} = 0.054$,
$[\lambda^1_{ij}(3)]^{(k)} = 0.06075$, $[\lambda^1_{ij}(4)]^{(k)} = 0.81$,
$k = 7{,}17$, $i = 3$, $j = 1$. (29)

4.4. Safety and resilience characteristics of maritime ferry technical system impacted by its operation process

Considering that the coordinates of the conditional safety functions (26) for the components of the ferry technical system subsystems S_v, $v = 1,2,\ldots,5$, are of the form (27) with the intensities of ageing at the operation states z_k, $k = 1,2,\ldots,18$, given respectively by (29), as the ferry technical system is a five-state ($z = 4$) series system [5], they are given by:

$$[\boldsymbol{S}^1(t,\cdot)]^{(1)} = [\,[\boldsymbol{S}^1(t,1)]^{(1)}, [\boldsymbol{S}^1(t,2)]^{(1)}, [\boldsymbol{S}^1(t,3)]^{(1)}, [\boldsymbol{S}^1(t,4)]^{(1)}\,],$$
$$t \in <0,\infty),$$

where

$[\boldsymbol{S}^1(t,1)]^{(1)} = \exp[-0.37725t](\exp[-0.462t]$
$- 4\exp[-0.429t] + 4\exp[-0.396t] + 4\exp[-0.363t]$
$- 12\exp[-0.33t] + 8\exp[-0.297t]$,

$[\boldsymbol{S}^1(t,2)]^{(1)} = \exp[-0.46475t](\exp[-0.53t]$
$- 4\exp[-0.49t] - 2\exp[-0.46t] + 6\exp[-0.45t]$
$+ 8\exp[-0.42t] - 4\exp[-0.41t] - 12\exp[-0.38t]$
$+ 8\exp[-0.34t]$,

$[\boldsymbol{S}^1(t,3)]^{(1)} = \exp[-0.534t](\exp[-0.605t]$
$- 4\exp[-0.555t] - 2\exp[-0.53t] + 6\exp[-0.505t]$
$+ 8\exp[-0.48t] - 4\exp[-0.455t] - 12\exp[-0.43t]$
$+ 8\exp[-0.38t]$,

$[\boldsymbol{S}^1(t,4)]^{(1)} = \exp[-0.6045t](\exp[-0.66t]$
$- 4\exp[-0.605t] - 2\exp[-0.58t] + 6\exp[-0.55t]$
$+ 8\exp[-0.525t] - 4\exp[-0.495t] - 12\exp[-0.47t]$
$+ 8\exp[-0.415t]$; (30)

$$[\boldsymbol{S}^1(t,\cdot)]^{(2)} = [\,[\boldsymbol{S}^1(t,1)]^{(2)}, [\boldsymbol{S}^1(t,2)]^{(2)}, [\boldsymbol{S}^1(t,3)]^{(2)}, [\boldsymbol{S}^1(t,4)]^{(2)}],$$
$$t \in <0,\infty),$$

where

$[\boldsymbol{S}^1(t,1)]^{(2)} = \exp[-0.3672t](\exp[-0.5676t]$
$- 4\exp[-0.528t] + 6\exp[-0.4884t]$
$- 2\exp[-0.4818t] - 4\exp[-0.4488t]$
$+ 8\exp[-0.4422t] - 12\exp[-0.4026t]$
$+ 8\exp[-0.363t])$,

$[\boldsymbol{S}^1(t,2)]^{(2)} = \exp[-0.461t](\exp[-0.649t]$
$- 4\exp[-0.601t] - 2\exp[-0.558t] + 6\exp[-0.553t]$
$+ 8\exp[-0.51t] - 4\exp[-0.505t] - 12\exp[-0.462t]$
$+ 8\exp[-0.414t])$,

$[\boldsymbol{S}^1(t,3)]^{(2)} = \exp[-0.519t](\exp[-0.7395t]$
$- 4\exp[-0.6795t] - 2\exp[-0.642t]$
$+ 6\exp[-0.6195t] + 8\exp[-0.582t]$
$- 4\exp[-0.5595t] - 12\exp[-0.522t]$
$+ 8\exp[-0.462t])$,

$[\boldsymbol{S}^1(t,4)]^{(2)} = \exp[-0.5865t](\exp[-0.806t]$
$- 4\exp[-0.74t] - 2\exp[-0.702t] + 6\exp[-0.674t]$
$+ 8\exp[-0.636t] - 4\exp[-0.608t] - 12\exp[-0.57t]$
$+ 8\exp[-0.504t])$; (31)

$$[\boldsymbol{S}^1(t,\cdot)]^{(3)} = [\,[\boldsymbol{S}^1(t,1)]^{(3)}, [\boldsymbol{S}^1(t,2)]^{(3)}, [\boldsymbol{S}^1(t,3)]^{(3)}, [\boldsymbol{S}^1(t,4)]^{(3)}],$$
$$t \in <0,\infty),$$

where

$[\boldsymbol{S}^1(t,1)]^{(3)} = \exp[-0.3573t](\exp[-0.5181t]$
$- 4\exp[-0.48015t] - 2\exp[-0.4455t]$
$+ 6\exp[-0.4422t] + 8\exp[-0.40755t]$
$- 4\exp[-0.40425t] - 12\exp[-0.3696t]$
$+ 8\exp[-0.33165t])$,

$[\boldsymbol{S}^1(t,2)]^{(3)} = \exp[-0.449t](\exp[-0.595t]$
$- 4\exp[-0.549t] - 2\exp[-0.518t] + 6\exp[-0.503t]$
$+ 8\exp[-0.472t] - 4\exp[-0.457t] - 12\exp[-0.426t]$
$+ 8\exp[-0.38t])$,

$[\boldsymbol{S}^1(t,3)]^{(3)} = \exp[-0.5045t](\exp[-0.68t]$
$- 4\exp[-0.6225t] - 2\exp[-0.5975t]$
$+ 6\exp[-0.565t] + 8\exp[-0.54t] - 4\exp[-0.5075t]$
$- 12\exp[-0.4825t] + 8\exp[-0.425t])$,

$[\boldsymbol{S}^1(t,4)]^{(3)} = \exp[-0.57t](\exp[-0.742t]$
$- 4\exp[-0.67875t] - 2\exp[-0.654t]$
$+ 6\exp[-0.6155t] + 8\exp[-0.59075t]$
$- 4\exp[-0.55225t] - 12\exp[-0.5275t]$
$+ 8\exp[-0.46425t])$; (32)

$$[\boldsymbol{S}^1(t,\cdot)]^{(4)} = [\,[\boldsymbol{S}^1(t,1)]^{(4)}, [\boldsymbol{S}^1(t,2)]^{(4)}, [\boldsymbol{S}^1(t,3)]^{(4)}, [\boldsymbol{S}^1(t,4)]^{(4)}\,],\quad t \in <0,\infty),$$

where

$[\boldsymbol{S}^1(t,1)]^{(4)} = \exp[-0.3606t](\exp[-0.50821t] - 4\exp[-0.4686t] - 2\exp[-0.4422t] + 6\exp[-0.429t] + 8\exp[-0.4026t] - 4\exp[-0.3894t] - 12\exp[-0.363t] + 8\exp[-0.3234t])$,

$[\boldsymbol{S}^1(t,2)]^{(4)} = \exp[-0.452t](\exp[-0.586t] - 4\exp[-0.538t] - 2\exp[-0.516t] + 6\exp[-0.49t] + 8\exp[-0.468t] - 4\exp[-0.442t] - 12\exp[-0.42t] + 8\exp[-0.372t])$,

$[\boldsymbol{S}^1(t,3)]^{(4)} = \exp[-0.509t](\exp[-0.672t] - 4\exp[-0.612t] - 2\exp[-0.597t] + 6\exp[-0.552t] + 8\exp[-0.537t] - 4\exp[-0.492t] - 12\exp[-0.477t] + 8\exp[-0.417t])$,

$[\boldsymbol{S}^1(t,4)]^{(4)} = \exp[-0.575t](\exp[-0.734t] - 4\exp[-0.668t] - 2\exp[-0.654t] + 6\exp[-0.602t] + 8\exp[-0.588t] - 4\exp[-0.536t] - 12\exp[-0.522t] + 8\exp[-0.456t])$; (33)

$$[\boldsymbol{S}^1(t,\cdot)]^{(5)} = [\,[\boldsymbol{S}^1(t,1)]^{(5)}, [\boldsymbol{S}^1(t,2)]^{(5)}, [\boldsymbol{S}^1(t,3)]^{(5)}, [\boldsymbol{S}^1(t,4)]^{(5)}\,],\quad t \in <0,\infty),$$

where

$[\boldsymbol{S}^1(t,1)]^{(5)} = \exp[-0.3606t](\exp[-0.5241t] - 4\exp[-0.4785t] - 2\exp[-0.4554t] + 6\exp[-0.4356t] + 8\exp[-0.4125t] - 4\exp[-0.3927t] - 12\exp[-0.3696t] + 8\exp[-0.3267t])$,

$[\boldsymbol{S}^1(t,2)]^{(5)} = \exp[-0.453t](\exp[-0.602t] - 4\exp[-0.55t] - 2\exp[-0.532t] + 6\exp[-0.498t] + 8\exp[-0.48t] - 4\exp[-0.446t] - 12\exp[-0.428t] + 8\exp[-0.376t])$,

$[\boldsymbol{S}^1(t,3)]^{(5)} = \exp[-0.509t](\exp[-0.692t] - 4\exp[-0.627t] - 2\exp[-0.617t] + 6\exp[-0.562t] + 8\exp[-0.552t] - 4\exp[-0.497t] - 12\exp[-0.487t] + 8\exp[-0.422t])$,

$[\boldsymbol{S}^1(t,4)]^{(5)} = \exp[-0.575t](\exp[-0.756t] - 4\exp[-0.6845t] - 2\exp[-0.676t] + 6\exp[-0.613t] + 8\exp[-0.6045t] - 4\exp[-0.5415t] - 12\exp[-0.533t] + 8\exp[-0.4615t])$; (34)

$$[\boldsymbol{S}^1(t,\cdot)]^{(6)} = [\,[\boldsymbol{S}^1(t,1)]^{(6)}, [\boldsymbol{S}^1(t,2)]^{(6)}, [\boldsymbol{S}^1(t,3)]^{(6)}, [\boldsymbol{S}^1(t,4)]^{(6)}\,],\quad t \in <0,\infty),$$

where

$[\boldsymbol{S}^1(t,1)]^{(6)} = \exp[-0.3606t](\exp[-0.5214t] - 4\exp[-0.4818t] - 2\exp[-0.4521t] + 6\exp[-0.4422t] + 8\exp[-0.4125t] - 4\exp[-0.4026t] - 12\exp[-0.3729t] + 8\exp[-0.3333t])$,

$[\boldsymbol{S}^1(t,2)]^{(6)} = \exp[-0.453t](\exp[-0.5005t] - 4\exp[-0.5525t] - 2\exp[-0.527t] + 6\exp[-0.5045t] + 8\exp[-0.479t] - 4\exp[-0.4565t] - 12\exp[-0.431t] + 8\exp[-0.383t])$,

$[\boldsymbol{S}^1(t,3)]^{(6)} = \exp[-0.509t](\exp[-0.68775t] - 4\exp[-0.62775t] - 2\exp[-0.609t] + 6\exp[-0.56775t] + 8\exp[-0.549t] - 4\exp[-0.50775t] - 12\exp[-0.489t] + 8\exp[-0.429t])$,

$[\boldsymbol{S}^1(t,4)]^{(6)} = \exp[-0.575t](\exp[-0.751t] - 4\exp[-0.685t] - 2\exp[-0.667t] + 6\exp[-0.619t] + 8\exp[-0.601t] - 4\exp[-0.553t] - 12\exp[-0.535t] + 8\exp[-0.469t])$; (35)

$$[\boldsymbol{S}^1(t,\cdot)]^{(7)} = [\,[\boldsymbol{S}^1(t,1)]^{(7)}, [\boldsymbol{S}^1(t,2)]^{(7)}, [\boldsymbol{S}^1(t,3)]^{(7)}, [\boldsymbol{S}^1(t,4)]^{(7)}\,],\quad t \in <0,\infty),$$

where

$[\boldsymbol{S}^1(t,1)]^{(7)} = \exp[-0.39195t](\exp[-0.5676t] - 4\exp[-0.528t] + 6\exp[-0.4884t] - 2\exp[-0.4818t] - 4\exp[-0.4488t] + 8\exp[-0.4422t] - 12\exp[-0.4026t] + 8\exp[-0.363t])$,

$[\boldsymbol{S}^1(t,2)]^{(7)} = \exp[-0.491t](\exp[-0.649t] - 4\exp[-0.601t] - 2\exp[-0.558t] + 6\exp[-0.553t] + 8\exp[-0.51t] - 4\exp[-0.505t] - 12\exp[-0.462t] + 8\exp[-0.414t])$,

$[\boldsymbol{S}^1(t,3)]^{(7)} = \exp[-0.54175t](\exp[-0.7395t] - 4\exp[-0.6795t] - 2\exp[-0.642t] + 6\exp[-0.6195t] + 8\exp[-0.582t] - 4\exp[-0.5595t] - 12\exp[-0.522t] + 8\exp[-0.462t])$,

$[\boldsymbol{S}^1(t,4)]^{(7)} = \exp[-0.621t](\exp[-0.806t] - 4\exp[-0.74t] - 2\exp[-0.702t] + 6\exp[-0.674t]$

$+ 8\exp[-0.636t] - 4\exp[-0.608t] - 12\exp[-0.57t]$
$+ 8\exp[-0.504t]);$ (36)

$$[\boldsymbol{S}^1(t,\cdot)]^{(8)} = [[\boldsymbol{S}^1(t,1)]^{(8)}, [\boldsymbol{S}^1(t,2)]^{(8)}, [\boldsymbol{S}^1(t,3)]^{(8)}, [\boldsymbol{S}^1(t,4)]^{(8)}],$$
$t \in <0,\infty),$

where

$[\boldsymbol{S}^1(t,1)]^{(8)} = \exp[-0.369t](\exp[-0.462t]$
$- 4\exp[-0.429t] + 4\exp[-0.396t] + 4\exp[-0.363t]$
$- 12\exp[-0.33t] + 8\exp[-0.297t],$

$[\boldsymbol{S}^1(t,2)]^{(8)} = \exp[-0.45475t](\exp[-0.53t]$
$- 4\exp[-0.49t] - 2\exp[-0.46t] + 6\exp[-0.45t]$
$+ 8\exp[-0.42t] - 4\exp[-0.41t] - 12\exp[-0.38t]$
$+ 8\exp[-0.34t],$

$[\boldsymbol{S}^1(t,3)]^{(8)} = \exp[-0.52275t](\exp[-0.605t]$
$- 4\exp[-0.555t] - 2\exp[-0.53t] + 6\exp[-0.505t]$
$+ 8\exp[-0.48t] - 4\exp[-0.455t] - 12\exp[-0.43t]$
$+ 8\exp[-0.38t],$

$[\boldsymbol{S}^1(t,4)]^{(8)} = \exp[-0.592t](\exp[-0.66t]$
$- 4\exp[-0.605t] - 2\exp[-0.58t] + 6\exp[-0.55t]$
$+ 8\exp[-0.525t] - 4\exp[-0.495t] - 12\exp[-0.47t]$
$+ 8\exp[-0.415t];$ (37)

$$[\boldsymbol{S}^1(t,\cdot)]^{(9)} = [[\boldsymbol{S}^1(t,1)]^{(9)}, [\boldsymbol{S}^1(t,2)]^{(9)}, [\boldsymbol{S}^1(t,3)]^{(9)}, [\boldsymbol{S}^1(t,4)]^{(9)}],$$
$t \in <0,\infty),$

where

$[\boldsymbol{S}^1(t,1)]^{(9)} = \exp[-0.369t](\exp[-0.462t]$
$- 4\exp[-0.429t] + 4\exp[-0.396t] + 4\exp[-0.363t]$
$- 12\exp[-0.33t] + 8\exp[-0.297t],$

$[\boldsymbol{S}^1(t,2)]^{(9)} = \exp[-0.45475t](\exp[-0.53t]$
$- 4\exp[-0.49t] - 2\exp[-0.46t] + 6\exp[-0.45t]$
$+ 8\exp[-0.42t] - 4\exp[-0.41t] - 12\exp[-0.38t]$
$+ 8\exp[-0.34t],$

$[\boldsymbol{S}^1(t,3)]^{(9)} = \exp[-0.52275t](\exp[-0.605t]$
$- 4\exp[-0.555t] - 2\exp[-0.53t] + 6\exp[-0.505t]$
$+ 8\exp[-0.48t] - 4\exp[-0.455t] - 12\exp[-0.43t]$
$+ 8\exp[-0.38t],$

$[\boldsymbol{S}^1(t,4)]^{(9)} = \exp[-0.592t](\exp[-0.66t]$
$- 4\exp[-0.605t] - 2\exp[-0.58t] + 6\exp[-0.55t]$
$+ 8\exp[-0.525t] - 4\exp[-0.495t] - 12\exp[-0.47t]$
$+ 8\exp[-0.415t];$ (38)

$$[\boldsymbol{S}^1(t,\cdot)]^{(10)} = [[\boldsymbol{S}^1(t,1)]^{(10)}, [\boldsymbol{S}^1(t,2)]^{(10)}, [\boldsymbol{S}^1(t,3)]^{(10)}, [\boldsymbol{S}^1(t,4)]^{(10)}],$$
$t \in <0,\infty),$

where

$[\boldsymbol{S}^1(t,1)]^{(10)} = \exp[-0.3672t](\exp[-0.5676t]$
$- 4\exp[-0.528t] + 6\exp[-0.4884t]$
$- 2\exp[-0.4818t] - 4\exp[-0.4488t]$
$+ 8\exp[-0.4422t] - 12\exp[-0.4026t]$
$+ 8\exp[-0.363t]),$

$[\boldsymbol{S}^1(t,2)]^{(10)} = \exp[-0.461t](\exp[-0.649t]$
$- 4\exp[-0.601t] - 2\exp[-0.558t] + 6\exp[-0.553t]$
$+ 8\exp[-0.51t] - 4\exp[-0.505t] - 12\exp[-0.462t]$
$+ 8\exp[-0.414t]),$

$[\boldsymbol{S}^1(t,3)]^{(10)} = \exp[-0.519t](\exp[-0.7395t]$
$- 4\exp[-0.6795t] - 2\exp[-0.642t]$
$+ 6\exp[-0.6195t] + 8\exp[-0.582t]$
$- 4\exp[-0.5595t] - 12\exp[-0.522t]$
$+ 8\exp[-0.462t]),$

$[\boldsymbol{S}^1(t,4)]^{(10)} = \exp[-0.5865t](\exp[-0.806t]$
$- 4\exp[-0.74t] - 2\exp[-0.702t] + 6\exp[-0.674t]$
$+ 8\exp[-0.636t] - 4\exp[-0.608t] - 12\exp[-0.57t]$
$+ 8\exp[-0.504t]);$ (39)

$$[\boldsymbol{S}^1(t,\cdot)]^{(11)} = [[\boldsymbol{S}^1(t,1)]^{(11)}, [\boldsymbol{S}^1(t,2)]^{(11)}, [\boldsymbol{S}^1(t,3)]^{(11)}, [\boldsymbol{S}^1(t,4)]^{(11)}],$$
$t \in <0,\infty),$

where

$[\boldsymbol{S}^1(t,1)]^{(11)} = \exp[-0.3573t](\exp[-0.5742t]$
$- 4\exp[-0.5379t] + 6\exp[-0.5016t]$
$- 2\exp[-0.4818t] - 4\exp[-0.4653t]$
$+ 8\exp[-0.4455t] - 12\exp[-0.4092t]$
$+ 8\exp[-0.3729t]),$

$[\boldsymbol{S}^1(t,2)]^{(11)} = \exp[-0.449t](\exp[-0.654t]$
$- 4\exp[-0.61t] + 6\exp[-0.566t] - 2\exp[-0.556t]$
$- 4\exp[-0.522t] + 8\exp[-0.512t] - 12\exp[-0.468t]$
$+ 8\exp[-0.424t]),$

$[\boldsymbol{S}^1(t,3)]^{(11)} = \exp[-0.5045t](\exp[-0.742t]$
$- 4\exp[-0.687t] - 2\exp[-0.637t] + 6\exp[-0.632t]$
$+ 8\exp[-0.582t] - 4\exp[-0.577t] - 12\exp[-0.527t]$
$+ 8\exp[-0.472t]),$

$[\boldsymbol{S}^1(t,4)]^{(11)} = \exp[-0.57t](\exp[-0.808t]$
$- 4\exp[-0.7475t] - 2\exp[-0.696t] + 6\exp[-0.687t]$
$+ 8\exp[-0.6355t] - 4\exp[-0.6265t]$
$- 12\exp[-0.575t] + 8\exp[-0.5145t]);$ (40)

$$[\boldsymbol{S}^1(t,\cdot)]^{(12)} = [\,[\boldsymbol{S}^1(t,1)]^{(12)}, [\boldsymbol{S}^1(t,2)]^{(12)}, [\boldsymbol{S}^1(t,3)]^{(12)}, [\boldsymbol{S}^1(t,4)]^{(12)}\,],$$
$$t \in <0,\infty),$$

where

$[\boldsymbol{S}^1(t,1)]^{(12)} = \exp[-0.35855t](\exp[-0.5016t] - 4\exp[-0.46365t] - 2\exp[-0.4356t] + 6\exp[-0.4257t] + 8\exp[-0.39765t] - 4\exp[-0.38775t] - 12\exp[-0.3597t] + 8\exp[-0.32175t])$,

$[\boldsymbol{S}^1(t,2)]^{(12)} = \exp[-0.451t](\exp[-0.578t] - 4\exp[-0.532t] - 2\exp[-0.508t] + 6\exp[-0.486t] + 8\exp[-0.462t] - 4\exp[-0.44t] - 12\exp[-0.416t] + 8\exp[-0.37t])$,

$[\boldsymbol{S}^1(t,3)]^{(12)} = \exp[-0.50675t](\exp[-0.662t] - 4\exp[-0.6045t] - 2\exp[-0.587t] + 6\exp[-0.547t] + 8\exp[-0.5295t] - 4\exp[-0.4895t] - 12\exp[-0.472t] + 8\exp[-0.4145t])$,

$[\boldsymbol{S}^1(t,4)]^{(12)} = \exp[-0.5725t](\exp[-0.723t] - 4\exp[-0.65975t] - 2\exp[-0.643t] + 6\exp[-0.5965t] + 8\exp[-0.57975t] - 4\exp[-0.53325t] - 12\exp[-0.5165t] + 8\exp[-0.45325t])$; (41)

$$[\boldsymbol{S}^1(t,\cdot)]^{(13)} = [\,[\boldsymbol{S}^1(t,1)]^{(13)}, [\boldsymbol{S}^1(t,2)]^{(13)}, [\boldsymbol{S}^1(t,3)]^{(13)}, [\boldsymbol{S}^1(t,4)]^{(13)}\,],$$
$$t \in <0,\infty),$$

where

$[\boldsymbol{S}^1(t,1)]^{(13)} = \exp[-0.3606t](\exp[-0.5241t] - 4\exp[-0.4785t] - 2\exp[-0.4554t] + 6\exp[-0.4356t] + 8\exp[-0.4125t] - 4\exp[-0.3927t] - 12\exp[-0.3696t] + 8\exp[-0.3267t])$,

$[\boldsymbol{S}^1(t,2)]^{(13)} = \exp[-0.453t](\exp[-0.602t] - 4\exp[-0.55t] - 2\exp[-0.532t] + 6\exp[-0.498t] + 8\exp[-0.48t] - 4\exp[-0.446t] - 12\exp[-0.428t] + 8\exp[-0.376t])$,

$[\boldsymbol{S}^1(t,3)]^{(13)} = \exp[-0.509t](\exp[-0.692t] - 4\exp[-0.627t] - 2\exp[-0.617t] + 6\exp[-0.562t] + 8\exp[-0.552t] - 4\exp[-0.497t] - 12\exp[-0.487t] + 8\exp[-0.422t])$,

$[\boldsymbol{S}^1(t,4)]^{(13)} = \exp[-0.575t](\exp[-0.756t] - 4\exp[-0.6845t] - 2\exp[-0.676t] + 6\exp[-0.613t] + 8\exp[-0.6045t] - 4\exp[-0.5415t] - 12\exp[-0.533t] + 8\exp[-0.4615t])$; (42)

$$[\boldsymbol{S}^1(t,\cdot)]^{(14)} = [\,[\boldsymbol{S}^1(t,1)]^{(14)}, [\boldsymbol{S}^1(t,2)]^{(14)}, [\boldsymbol{S}^1(t,3)]^{(14)}, [\boldsymbol{S}^1(t,4)]^{(14)}\,],$$
$$t \in <0,\infty),$$

where

$[\boldsymbol{S}^1(t,1)]^{(14)} = \exp[-0.3606t](\exp[-0.5115t] - 4\exp[-0.4719t] - 2\exp[-0.4455t] + 6\exp[-0.4323t] + 8\exp[-0.4059t] - 4\exp[-0.3927t] - 12\exp[-0.3663t] + 8\exp[-0.3267t])$,

$[\boldsymbol{S}^1(t,2)]^{(14)} = \exp[-0.453t](\exp[-0.59t] - 4\exp[-0.542t] - 2\exp[-0.52t] + 6\exp[-0.494t] + 8\exp[-0.472t] - 4\exp[-0.446t] - 12\exp[-0.424t] + 8\exp[-0.376t])$,

$[\boldsymbol{S}^1(t,3)]^{(14)} = \exp[-0.509t](\exp[-0.6765t] - 4\exp[-0.6165t] - 2\exp[-0.6015t] + 6\exp[-0.5565t] + 8\exp[-0.5415t] - 4\exp[-0.4965t] - 12\exp[-0.4815t] + 8\exp[-0.4215t])$,

$[\boldsymbol{S}^1(t,4)]^{(14)} = \exp[-0.575t](\exp[-0.739t] - 4\exp[-0.673t] - 2\exp[-0.659t] + 6\exp[-0.607t] + 8\exp[-0.593t] - 4\exp[-0.541t] - 12\exp[-0.527t] + 8\exp[-0.461t])$; (43)

$$[\boldsymbol{S}^1(t,\cdot)]^{(15)} = [\,[\boldsymbol{S}^1(t,1)]^{(15)}, [\boldsymbol{S}^1(t,2)]^{(15)}, [\boldsymbol{S}^1(t,3)]^{(15)}, [\boldsymbol{S}^1(t,4)]^{(15)}\,],$$
$$t \in <0,\infty),$$

where

$[\boldsymbol{S}^1(t,1)]^{(15)} = \exp[-0.3573t](\exp[-0.5181t] - 4\exp[-0.48015t] - 2\exp[-0.4455t] + 6\exp[-0.4422t] + 8\exp[-0.40755t] - 4\exp[-0.40425t] - 12\exp[-0.3696t] + 8\exp[-0.33165t])$,

$[\boldsymbol{S}^1(t,2)]^{(15)} = \exp[-0.449t](\exp[-0.595t] - 4\exp[-0.549t] - 2\exp[-0.518t] + 6\exp[-0.503t] + 8\exp[-0.472t] - 4\exp[-0.457t] - 12\exp[-0.426t] + 8\exp[-0.38t])$,

$[\boldsymbol{S}^1(t,3)]^{(15)} = \exp[-0.5045t](\exp[-0.68t] - 4\exp[-0.6225t] - 2\exp[-0.5975t] + 6\exp[-0.565t] + 8\exp[-0.54t] - 4\exp[-0.5075t] - 12\exp[-0.4825t] + 8\exp[-0.425t])$,

$[\boldsymbol{S}^1(t,4)]^{(15)} = \exp[-0.57t](\exp[-0.742t] - 4\exp[-0.67875t] - 2\exp[-0.654t] + 6\exp[-0.6155t] + 8\exp[-0.59075t] - 4\exp[-0.55225t] - 12\exp[-0.5275t] + 8\exp[-0.46425t])$; (44)

$$[\boldsymbol{S}^1(t,\cdot)]^{(16)} = [[\boldsymbol{S}^1(t,1)]^{(16)}, [\boldsymbol{S}^1(t,2)]^{(16)}, [\boldsymbol{S}^1(t,3)]^{(16)}, [\boldsymbol{S}^1(t,4)]^{(16)}],$$
$$t \in <0,\infty),$$

where

$[\boldsymbol{S}^1(t,1)]^{(16)} = \exp[-0.3573t](\exp[-0.5742t] - 4\exp[-0.5379t] + 6\exp[-0.5016t] - 2\exp[-0.4818t] - 4\exp[-0.4653t] + 8\exp[-0.4455t] - 12\exp[-0.4092t] + 8\exp[-0.3729t])$,

$[\boldsymbol{S}^1(t,2)]^{(16)} = \exp[-0.449t](\exp[-0.654t] - 4\exp[-0.61t] + 6\exp[-0.566t] - 2\exp[-0.556t] - 4\exp[-0.522t] + 8\exp[-0.512t] - 12\exp[-0.468t] + 8\exp[-0.424t])$,

$[\boldsymbol{S}^1(t,3)]^{(16)} = \exp[-0.5045t](\exp[-0.742t] - 4\exp[-0.687t] - 2\exp[-0.637t] + 6\exp[-0.632t] + 8\exp[-0.582t] - 4\exp[-0.577t] - 12\exp[-0.527t] + 8\exp[-0.472t])$,

$[\boldsymbol{S}^1(t,4)]^{(16)} = \exp[-0.57t](\exp[-0.808t] - 4\exp[-0.7475t] - 2\exp[-0.696t] + 6\exp[-0.687t] + 8\exp[-0.6355t] - 4\exp[-0.6265t] - 12\exp[-0.575t] + 8\exp[-0.5145t])$; (45)

$$[\boldsymbol{S}^1(t,\cdot)]^{(17)} = [[\boldsymbol{S}^1(t,1)]^{(17)}, [\boldsymbol{S}^1(t,2)]^{(17)}, [\boldsymbol{S}^1(t,3)]^{(17)}, [\boldsymbol{S}^1(t,4)]^{(17)}],$$
$$t \in <0,\infty),$$

where

$[\boldsymbol{S}^1(t,1)]^{(17)} = \exp[-0.39195t](\exp[-0.5676t] - 4\exp[-0.528t] + 6\exp[-0.4884t] - 2\exp[-0.4818t] - 4\exp[-0.4488t] + 8\exp[-0.4422t] - 12\exp[-0.4026t] + 8\exp[-0.363t])$,

$[\boldsymbol{S}^1(t,2)]^{(17)} = \exp[-0.491t](\exp[-0.649t] - 4\exp[-0.601t] - 2\exp[-0.558t] + 6\exp[-0.553t] + 8\exp[-0.51t] - 4\exp[-0.505t] - 12\exp[-0.462t] + 8\exp[-0.414t])$,

$[\boldsymbol{S}^1(t,3)]^{(17)} = \exp[-0.54175t](\exp[-0.7395t] - 4\exp[-0.6795t] - 2\exp[-0.642t] + 6\exp[-0.6195t] + 8\exp[-0.582t] - 4\exp[-0.5595t] - 12\exp[-0.522t] + 8\exp[-0.462t])$,

$[\boldsymbol{S}^1(t,4)]^{(17)} = \exp[-0.621t](\exp[-0.806t] - 4\exp[-0.74t] - 2\exp[-0.702t] + 6\exp[-0.674t] + 8\exp[-0.636t] - 4\exp[-0.608t] - 12\exp[-0.57t] + 8\exp[-0.504t])$; (46)

$$[\boldsymbol{S}^1(t,\cdot)]^{(18)} = [[\boldsymbol{S}^1(t,1)]^{(18)}, [\boldsymbol{S}^1(t,2)]^{(18)}, [\boldsymbol{S}^1(t,3)]^{(18)}, [\boldsymbol{S}^1(t,4)]^{(18)}],$$
$$t \in <0,\infty),$$

where

$[\boldsymbol{S}^1(t,1)]^{(18)} = \exp[-0.37725t](\exp[-0.462t] - 4\exp[-0.429t] + 4\exp[-0.396t] + 4\exp[-0.363t] - 12\exp[-0.33t] + 8\exp[-0.297t]$,

$[\boldsymbol{S}^1(t,2)]^{(18)} = \exp[-0.46475t](\exp[-0.53t] - 4\exp[-0.49t] - 2\exp[-0.46t] + 6\exp[-0.45t] + 8\exp[-0.42t] - 4\exp[-0.41t] - 12\exp[-0.38t] + 8\exp[-0.34t]$,

$[\boldsymbol{S}^1(t,3)]^{(18)} = \exp[-0.534t](\exp[-0.605t] - 4\exp[-0.555t] - 2\exp[-0.53t] + 6\exp[-0.505t] + 8\exp[-0.48t] - 4\exp[-0.455t] - 12\exp[-0.43t] + 8\exp[-0.38t]$,

$[\boldsymbol{S}^1(t,4)]^{(18)} = \exp[-0.6045t](\exp[-0.66t] - 4\exp[-0.605t] - 2\exp[-0.58t] + 6\exp[-0.55t] + 8\exp[-0.525t] - 4\exp[-0.495t] - 12\exp[-0.47t] + 8\exp[-0.415t]$. (47)

Hence, the expected values of the ferry technical system lifetimes in the safety state subsets {1,2,3,4}, {2,3,4}, {3,4}, {4} at the operation state z_k, $k = 1,2,\ldots,18$ respectively are:

$[\mu^1(1)]^{(1)} \cong 1.70$, $[\mu^1(2)]^{(1)} \cong 1.42$,
$[\mu^1(3)]^{(1)} \cong 1.23$, $[\mu^1(4)]^{(1)} \cong 1.12$ years,
$[\mu^1(1)]^{(2)} \cong 1.61$, $[\mu^1(2)]^{(2)} \cong 1.33$,
$[\mu^1(3)]^{(2)} \cong 1.19$, $[\mu^1(4)]^{(2)} \cong 1.07$ years,
$[\mu^1(1)]^{(3)} \cong 1.68$, $[\mu^1(2)]^{(3)} \cong 1.39$,
$[\mu^1(3)]^{(3)} \cong 1.25$, $[\mu^1(4)]^{(3)} \cong 1.12$ years,
$[\mu^1(1)]^{(4)} \cong 1.70$, $[\mu^1(2)]^{(4)} \cong 1.39$,
$[\mu^1(3)]^{(4)} \cong 1.25$, $[\mu^1(4)]^{(4)} \cong 1.12$ years,
$[\mu^1(1)]^{(5)} \cong 1.70$, $[\mu^1(2)]^{(5)} \cong 1.39$,
$[\mu^1(3)]^{(5)} \cong 1.25$, $[\mu^1(4)]^{(5)} \cong 1.12$ years,
$[\mu^1(1)]^{(6)} \cong 1.67$, $[\mu^1(2)]^{(6)} \cong 1.38$,
$[\mu^1(3)]^{(6)} \cong 1.23$, $[\mu^1(4)]^{(6)} \cong 1.10$ years,
$[\mu^1(1)]^{(7)} \cong 1.55$, $[\mu^1(2)]^{(7)} \cong 1.28$,
$[\mu^1(3)]^{(7)} \cong 1.16$, $[\mu^1(4)]^{(7)} \cong 1.03$ years,
$[\mu^1(1)]^{(8)} \cong 1.73$, $[\mu^1(2)]^{(8)} \cong 1.44$,
$[\mu^1(3)]^{(8)} \cong 1.27$, $[\mu^1(4)]^{(8)} \cong 1.13$ years,
$[\mu^1(1)]^{(9)} \cong 1.73$, $[\mu^1(2)]^{(9)} \cong 1.44$,
$[\mu^1(3)]^{(9)} \cong 1.27$, $[\mu^1(4)]^{(9)} \cong 1.13$ years,
$[\mu^1(1)]^{(10)} \cong 1.61$, $[\mu^1(2)]^{(10)} \cong 1.33$,
$[\mu^1(3)]^{(10)} \cong 1.19$, $[\mu^1(4)]^{(10)} \cong 1.07$ years,

$[\mu^1(1)]^{(11)} \cong 1.61,\ [\mu^1(2)]^{(11)} \cong 1.33,$
$[\mu^1(3)]^{(11)} \cong 1.20,\ [\mu^1(4)]^{(11)} \cong 1.07$ years,
$[\mu^1(1)]^{(12)} \cong 1.70,\ [\mu^1(2)]^{(12)} \cong 1.40,$
$[\mu^1(3)]^{(12)} \cong 1.25,\ [\mu^1(4)]^{(12)} \cong 1.12$ years,
$[\mu^1(1)]^{(13)} \cong 1.70,\ [\mu^1(2)]^{(13)} \cong 1.39,$
$[\mu^1(3)]^{(13)} \cong 1.25,\ [\mu^1(4)]^{(13)} \cong 1.12$ years,
$[\mu^1(1)]^{(14)} \cong 1.69,\ [\mu^1(2)]^{(14)} \cong 1.39,$
$[\mu^1(3)]^{(14)} \cong 1.24,\ [\mu^1(4)]^{(14)} \cong 1.11$ years,
$[\mu^1(1)]^{(15)} \cong 1.68,\ [\mu^1(2)]^{(15)} \cong 1.39,$
$[\mu^1(3)]^{(15)} \cong 1.25,\ [\mu^1(4)]^{(15)} \cong 1.12$ years,
$[\mu^1(1)]^{(16)} \cong 1.61,\ [\mu^1(2)]^{(16)} \cong 1.33,$
$[\mu^1(3)]^{(16)} \cong 1.20,\ [\mu^1(4)]^{(16)} \cong 1.07$ years,
$[\mu^1(1)]^{(17)} \cong 1.55,\ [\mu^1(2)]^{(17)} \cong 1.28,$
$[\mu^1(3)]^{(17)} \cong 1.16,\ [\mu^1(4)]^{(17)} \cong 1.03$ years,
$[\mu^1(1)]^{(18)} \cong 1.70,\ [\mu^1(2)]^{(18)} \cong 1.42,$
$[\mu^1(3)]^{(18)} \cong 1.23, [\mu^1(4)]^{(18)} \cong 1.12$ years. (48)

From the results (24) and (30)–(47), applying (2), the ferry technical system unconditional safety function is given by

$$\boldsymbol{S}^1(t,\cdot) = [\ \boldsymbol{S}^1(t,1),\ \boldsymbol{S}^1(t,2),\ \boldsymbol{S}^1(t,3),\ \boldsymbol{S}^1(t,4)\,],$$

where

$$\begin{aligned}\boldsymbol{S}^1(t,1) &= 0.038\cdot[\boldsymbol{S}^1(t,1)]^{(1)} + 0.002\cdot[\boldsymbol{S}^1(t,1)]^{(2)}\\
&+ 0.026\cdot[\boldsymbol{S}^1(t,1)]^{(3)} + 0.036\cdot[\boldsymbol{S}^1(t,1)]^{(4)}\\
&+ 0.363\cdot[\boldsymbol{S}^1(t,1)]^{(5)} + 0.026\cdot[\boldsymbol{S}^1(t,1)]^{(6)}\\
&+ 0.005\cdot[\boldsymbol{S}^1(t,1)]^{(7)} + 0.016\cdot[\boldsymbol{S}^1(t,1)]^{(8)}\\
&+ 0.037\cdot[\boldsymbol{S}^1(t,1)]^{(9)} + 0.002\cdot[\boldsymbol{S}^1(t,1)]^{(10)}\\
&+ 0.003\cdot[\boldsymbol{S}^1(t,1)]^{(11)} + 0.016\cdot[\boldsymbol{S}^1(t,1)]^{(12)}\\
&+ 0.351\cdot[\boldsymbol{S}^1(t,1)]^{(13)} + 0.034\cdot[\boldsymbol{S}^1(t,1)]^{(14)}\\
&+ 0.024\cdot[\boldsymbol{S}^1(t,1)]^{(15)} + 0.003\cdot[\boldsymbol{S}^1(t,1)]^{(16)}\\
&+ 0.005\cdot[\boldsymbol{S}^1(t,1)]^{(17)} + 0.013\cdot[\boldsymbol{S}^1(t,1)]^{(18)},\end{aligned}$$

$$\begin{aligned}\boldsymbol{S}^1(t,2) &= 0.038\cdot[\boldsymbol{S}^1(t,2)]^{(1)} + 0.002\cdot[\boldsymbol{S}^1(t,2)]^{(2)}\\
&+ 0.026\cdot[\boldsymbol{S}^1(t,2)]^{(3)} + 0.036\cdot[\boldsymbol{S}^1(t,2)]^{(4)}\\
&+ 0.363\cdot[\boldsymbol{S}^1(t,2)]^{(5)} + 0.026\cdot[\boldsymbol{S}^1(t,2)]^{(6)}\\
&+ 0.005\cdot[\boldsymbol{S}^1(t,2)]^{(7)} + 0.016\cdot[\boldsymbol{S}^1(t,2)]^{(8)}\\
&+ 0.037\cdot[\boldsymbol{S}^1(t,2)]^{(9)} + 0.002\cdot[\boldsymbol{S}^1(t,2)]^{(10)}\\
&+ 0.003\cdot[\boldsymbol{S}^1(t,2)]^{(11)} + 0.016\cdot[\boldsymbol{S}^1(t,2)]^{(12)}\\
&+ 0.351\cdot[\boldsymbol{S}^1(t,2)]^{(13)} + 0.034\cdot[\boldsymbol{S}^1(t,2)]^{(14)}\\
&+ 0.024\cdot[\boldsymbol{S}^1(t,2)]^{(15)} + 0.003\cdot[\boldsymbol{S}^1(t,2)]^{(16)}\\
&+ 0.005\cdot[\boldsymbol{S}^1(t,2)]^{(17)} + 0.013\cdot[\boldsymbol{S}^1(t,2)]^{(18)},\end{aligned}$$

$$\begin{aligned}\boldsymbol{S}^1(t,3) &= 0.038\cdot[\boldsymbol{S}^1(t,3)]^{(1)} + 0.002\cdot[\boldsymbol{S}^1(t,3)]^{(2)}\\
&+ 0.026\cdot[\boldsymbol{S}^1(t,3)]^{(3)} + 0.036\cdot[\boldsymbol{S}^1(t,3)]^{(4)}\\
&+ 0.363\cdot[\boldsymbol{S}^1(t,3)]^{(5)} + 0.026\cdot[\boldsymbol{S}^1(t,3)]^{(6)}\\
&+ 0.005\cdot[\boldsymbol{S}^1(t,3)]^{(7)} + 0.016\cdot[\boldsymbol{S}^1(t,3)]^{(8)}\\
&+ 0.037\cdot[\boldsymbol{S}^1(t,3)]^{(9)} + 0.002\cdot[\boldsymbol{S}^1(t,3)]^{(10)}\\
&+ 0.003\cdot[\boldsymbol{S}^1(t,3)]^{(11)} + 0.016\cdot[\boldsymbol{S}^1(t,3)]^{(12)}\\
&+ 0.351\cdot[\boldsymbol{S}^1(t,3)]^{(13)} + 0.034\cdot[\boldsymbol{S}^1(t,3)]^{(14)}\\
&+ 0.024\cdot[\boldsymbol{S}^1(t,3)]^{(15)} + 0.003\cdot[\boldsymbol{S}^1(t,3)]^{(16)}\\
&+ 0.005\cdot[\boldsymbol{S}^1(t,3)]^{(17)} + 0.013\cdot[\boldsymbol{S}^1(t,3)]^{(18)},\end{aligned}$$

$$\begin{aligned}\boldsymbol{S}^1(t,4) &= 0.038\cdot[\boldsymbol{S}^1(t,4)]^{(1)} + 0.002\cdot[\boldsymbol{S}^1(t,4)]^{(2)}\\
&+ 0.026\cdot[\boldsymbol{S}^1(t,4)]^{(3)} + 0.036\cdot[\boldsymbol{S}^1(t,4)]^{(4)}\\
&+ 0.363\cdot[\boldsymbol{S}^1(t,4)]^{(5)} + 0.026\cdot[\boldsymbol{S}^1(t,4)]^{(6)}\\
&+ 0.005\cdot[\boldsymbol{S}^1(t,4)]^{(7)} + 0.016\cdot[\boldsymbol{S}^1(t,4)]^{(8)}\\
&+ 0.037\cdot[\boldsymbol{S}^1(t,4)]^{(9)} + 0.002\cdot[\boldsymbol{S}^1(t,4)]^{(10)}\\
&+ 0.003\cdot[\boldsymbol{S}^1(t,4)]^{(11)} + 0.016\cdot[\boldsymbol{S}^1(t,4)]^{(12)}\\
&+ 0.351\cdot[\boldsymbol{S}^1(t,4)]^{(13)} + 0.034\cdot[\boldsymbol{S}^1(t,4)]^{(14)}\\
&+ 0.024\cdot[\boldsymbol{S}^1(t,4)]^{(15)} + 0.003\cdot[\boldsymbol{S}^1(t,4)]^{(16)}\\
&+ 0.005\cdot[\boldsymbol{S}^1(t,4)]^{(17)} + 0.013\cdot[\boldsymbol{S}^1(t,4)]^{(18)}, \qquad (49)\end{aligned}$$

where $[\boldsymbol{S}^1(t,u)]^{(k)}$, $u = 1,2,3,4$, $k = 1,2,\ldots,18$ are given by (30)–(47).

Considering (24) and (48), the mean values of the ferry technical system unconditional lifetimes in the safety state subsets, {1,2,3,4}, {2,3,4}, {3,4}, {4} respectively are:

$$\begin{aligned}\boldsymbol{\mu}(1) \cong\ & 0.038\cdot 1.70 + 0.002\cdot 1.61 + 0.026\cdot 1.68\\
&+ 0.036\cdot 1.70 + 0.363\cdot 1.70 + 0.026\cdot 1.67\\
&+ 0.005\cdot 1.55 + 0.016\cdot 1.73 + 0.037\cdot 1.73\\
&+ 0.002\cdot 1.61 + 0.003\cdot 1.61 + 0.016\cdot 1.70\\
&+ 0.351\cdot 1.70 + 0.034\cdot 1.69 + 0.024\cdot 1.68\\
&+ 0.003\cdot 1.61 + 0.005\cdot 1.55 + 0.013\cdot 1.70\\
&\cong 1.70 \text{ years,}\end{aligned}$$

$$\begin{aligned}\boldsymbol{\mu}(2) \cong\ & 0.038\cdot 1.42 + 0.002\cdot 1.33 + 0.026\cdot 1.39\\
&+ 0.036\cdot 1.39 + 0.363\cdot 1.39 + 0.026\cdot 1.38\\
&+ 0.005\cdot 1.28 + 0.016\cdot 1.44 + 0.037\cdot 1.44\\
&+ 0.002\cdot 1.33 + 0.003\cdot 1.33 + 0.016\cdot 1.40\\
&+ 0.351\cdot 1.39 + 0.034\cdot 1.39 + 0.024\cdot 1.39\\
&+ 0.003\cdot 1.33 + 0.005\cdot 1.28 + 0.013\cdot 1.42\\
&\cong 1.39 \text{ years,}\end{aligned}$$

$$\begin{aligned}\boldsymbol{\mu}(3) \cong\ & 0.038\cdot 1.23 + 0.002\cdot 1.19 + 0.026\cdot 1.25\\
&+ 0.036\cdot 1.25 + 0.363\cdot 1.25 + 0.026\cdot 1.23\\
&+ 0.005\cdot 1.16 + 0.016\cdot 1.27 + 0.037\cdot 1.27\\
&+ 0.002\cdot 1.19 + 0.003\cdot 1.20 + 0.016\cdot 1.25\\
&+ 0.351\cdot 1.25 + 0.034\cdot 1.24 + 0.024\cdot 1.25\\
&+ 0.003\cdot 1.20 + 0.005\cdot 1.16 + 0.013\cdot 1.23\\
&\cong 1.25 \text{ years,}\end{aligned}$$

$$\boldsymbol{\mu}(4) \cong 0.038 \cdot 1.12 + 0.002 \cdot 1.07 + 0.026 \cdot 1.12 + 0.036 \cdot 1.12 + 0.363 \cdot 1.12 + 0.026 \cdot 1.10 + 0.005 \cdot 1.03 + 0.016 \cdot 1.13 + 0.037 \cdot 1.13 + 0.002 \cdot 1.07 + 0.003 \cdot 1.07 + 0.016 \cdot 1.12 + 0.351 \cdot 1.12 + 0.034 \cdot 1.11 + 0.024 \cdot 1.12 + 0.003 \cdot 1.07 + 0.005 \cdot 1.03 + 0.013 \cdot 1.12 \cong 1.12 \text{ years.} \quad (50)$$

Further, considering (10), the mean values of the ferry technical system lifetimes in the particular safety states are:

$$\bar{\boldsymbol{\mu}}(1) = \boldsymbol{\mu}(1) - \boldsymbol{\mu}(2) = 0.31 \text{ year},$$
$$\bar{\boldsymbol{\mu}}(2) = \boldsymbol{\mu}(2) - \boldsymbol{\mu}(3) = 0.14 \text{ year},$$
$$\bar{\boldsymbol{\mu}}(3) = \boldsymbol{\mu}(3) - \boldsymbol{\mu}(4) = 0.13 \text{ year},$$
$$\bar{\boldsymbol{\mu}}(4) = \boldsymbol{\mu}(4) = 1.12 \text{ years.} \quad (51)$$

Since the critical safety state is $r = 2$, then the ferry technical system risk function, according to (7), is given by

$$\boldsymbol{r}^1(t) = 1 - \boldsymbol{S}^1(t,2), \quad (52)$$

where $\boldsymbol{S}^1(t,2)$ is given by (49).
From (32) according to (11) the moment when the ferry technical system risk function exceeds a permitted level, for instance $\delta = 0.05$ is

$$\tau^1 = (\boldsymbol{r}^1)^{-1}(\delta) \cong 0.073 \text{ year.} \quad (53)$$

Applying (12), the ferry technical system intensities of ageing are:

$$\lambda^1(1) \cong 0.58824, \ \lambda^1(2) \cong 0.71942, \ \lambda^1(3) \cong 0.8,$$
$$\lambda^1(4) \cong 0.89286. \quad (54)$$

Considering (23) and (54) and applying (13), the coefficients of impact on the ferry technical system intensities of ageing, are:

$$\rho^1(t,1) \cong 1.041184, \ \rho^1(t,2) \cong 1.061864,$$
$$\rho^1(t,3) \cong 1.04, \ \rho^1(t,4) \cong 1.039288. \quad (55)$$

Finally, by (14) and (55), the ferry technical system resilience indicator of the ferry technical system critical infrastructure to the operation process impact is

$$\boldsymbol{RI}^1(t) = 1/\rho^1(t,2) \cong 0.9417 = 94.17\%.$$

5. Conclusion

The safety models of critical infrastructure without considering outside impacts and with considering operation process influence on its safety are applied to the safety and resilience examination of a maritime ferry technical system. The comparison of the maritime ferry technical system safety indicators without considering outside impacts with indicators considering impact of its operation process proves an influence of the operation process on its safety. The results justify the improvement of the accuracy of the system safety analysis and evaluation through considering the influence of the system operation process states changing on changing the system safety structures and the system components safety parameters and consequently on changing the entire system safety. Thus, the proposed models are appropriate for safety analysis of a wide class of real complex systems and critical infrastructures changing during the operation their safety structures and their components and subsystems safety parameters.

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Appendix: maritime ferry technical system safety structure changing

The influence of the changing defined in the paper operation states on the changes of the ferry technical system safety structure is as follows. At the operation states z_1 and z_{18}, the ferry technical system is composed of two subsystems S_3 and S_4 forming a series structure shown in *Figure A.1*. At the operation states z_2, z_7, z_{10} and z_{17}, the ferry technical system is composed of three subsystems S_1, S_2 and S_5 forming a series structure shown in *Figure A.2*. At the operation states z_3, z_{11}, z_{15} and z_{16}, the ferry technical system is composed of two subsystems S_1 and S_2 forming a series structure shown in *Figure A.3*. At the operation states z_4, z_5, z_{12}, z_{13} and z_{14}, the ferry technical system is composed of three subsystems S_1, S_2 and S_4 forming a series structure shown in *Figure A.4*. At the operation state z_6, the ferry technical system is composed of three subsystems S_1, S_2, and S_4 forming a series structure shown in *Figure A.5*. At the operation state z_8, and z_9, the ferry technical system is composed of two subsystems S_3 and S_4 forming a series structure shown in *Figure A.6*.

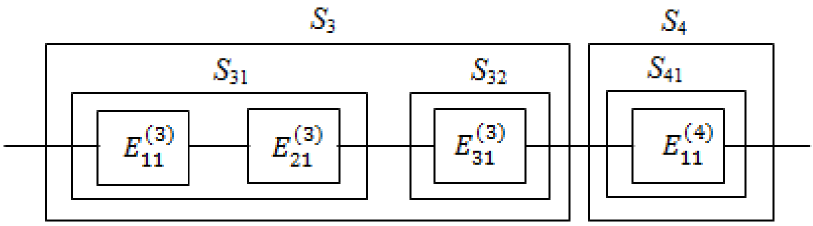

Figure A.1. The scheme of the ferry technical system structure at the operation states z_1 and z_{18}

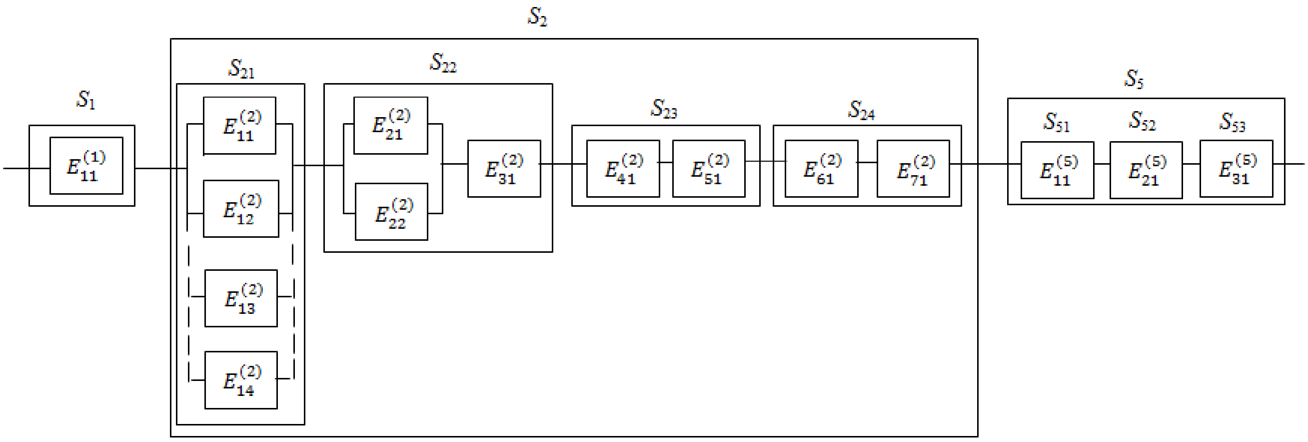

Figure A.2. The scheme of the ferry technical system structure at the operation states z_2, z_7, z_{10} and z_{17}

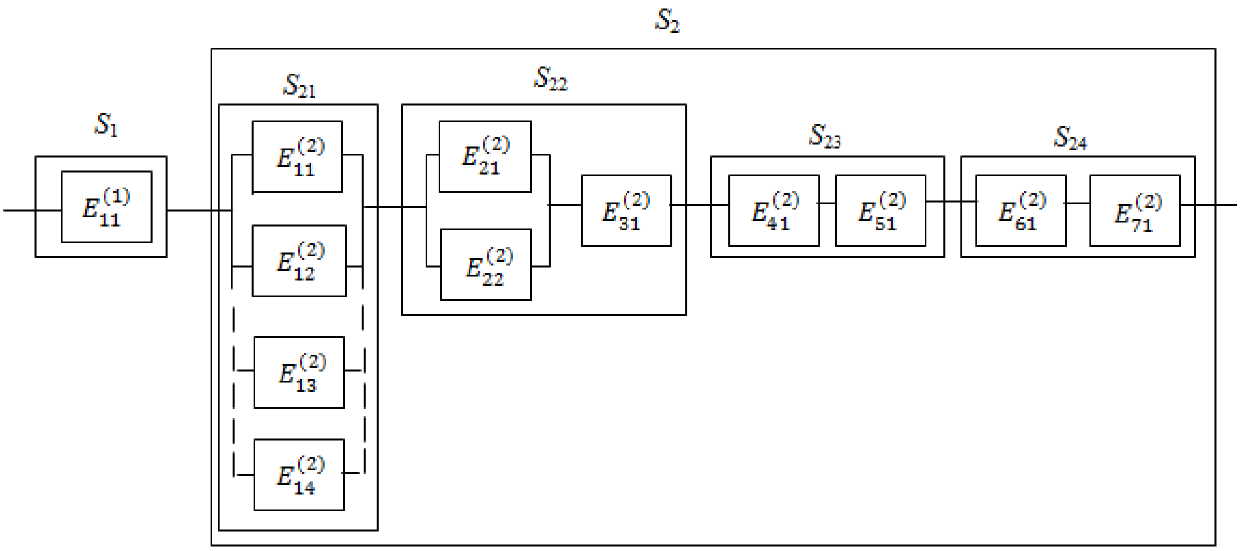

Figure A.3. The scheme of the ferry technical system structure at the operation states z_3, z_{11}, z_{15} and z_{16}

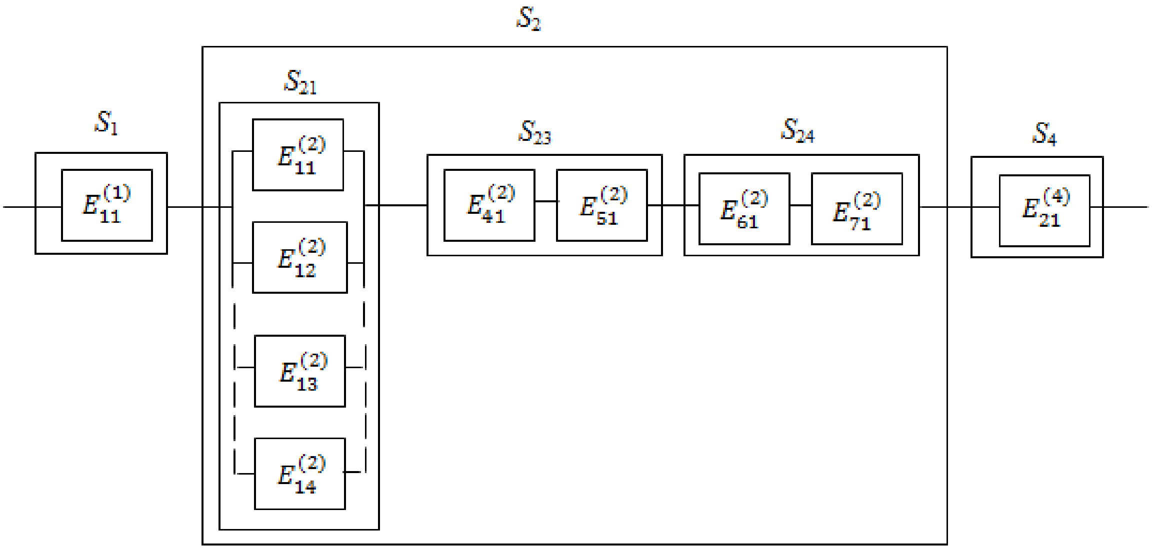

Figure A.4. The scheme of the ferry technical system structure at the operation states z_4, z_5, z_{12}, z_{13} and z_{14}

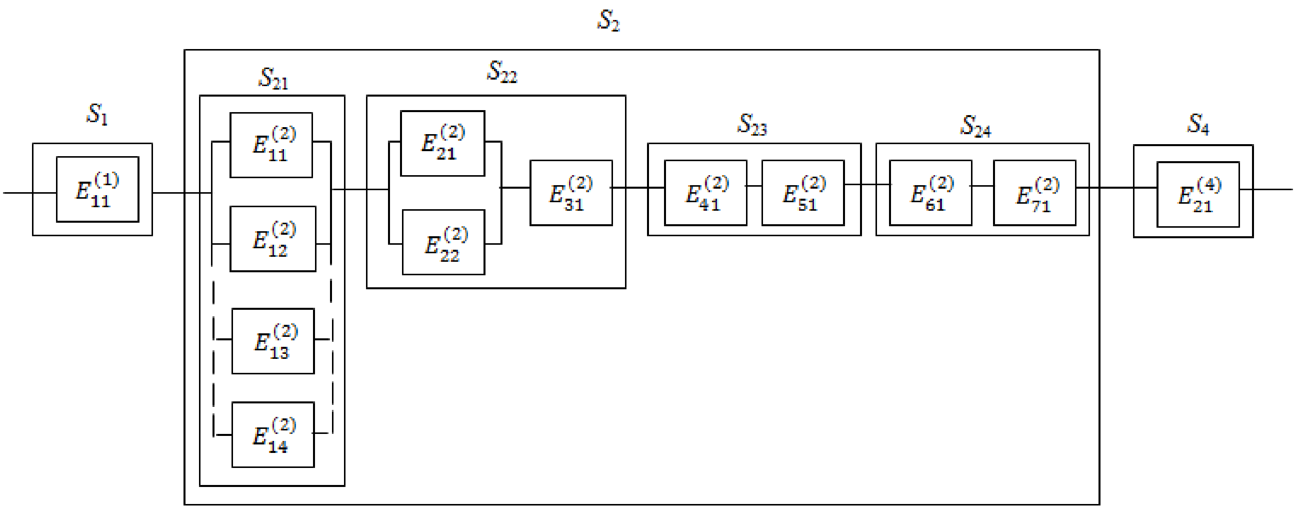

Figure A.5. The scheme of the ferry technical system structure at the operation state z_6

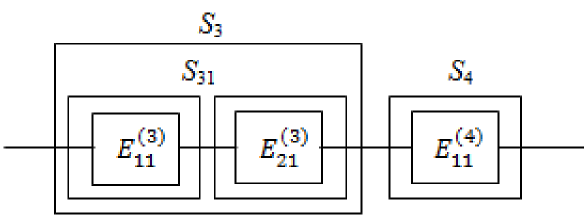

Figure A.6. The scheme of the ferry technical system structure at the operation states z_8 and z_9

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