**Grabski Franciszek** 0000-0001-5772-4550
*Polish Naval Academy, Gdynia, Poland, f.grabski (at) amw.gdynia.pl*

# Random processes related to accidents on Baltic Sea waters and ports

**Keywords**

nonhomogeneous Poisson process, nonhomogeneous compound Poisson process, safety characteristics

**Abstract**

A crucial role in construction of the models related to accidents on the Baltic Sea water and ports play nonhomogeneous Poisson and nonhomogeneous compound Poisson process. The model of consequences and connected to it model of accidents number on sea and seaports are here presented. Moreover some procedures of the models parameters identification are presented in the chapter. Estimation of model some parameters was made based on data from reports of HELCOM and Interreg project Baltic LINes.

## 1. Introduction

In the paper [7] the models of accidents number in the sea and seaports are presented. A crucial role in construction of the models plays a Poisson process and its extensions especially a nonhomogeneous Poisson process. Moreover some procedures of the model parameters identification are presented in the paper. Estimation of models parameters was made based on available data coming from reports of HELCOM (2014) and Interreg project Baltic LINes (2016–2019). The models allow us to anticipate number of accidents on Baltic Sea water and ports in future. In this chapter a nonhomogeneous compound Poisson process as a model of the accidents consequences is also presented.

## 2. Statistical analysis

Annual report on shipping accidents in the Baltic Sea coming from report we can estimate some parameters in our models. The *Table 1* and *Figure 1* show a total number of ships crossing in the Baltic Sea during 2006–2013. The minimum number of ships crossing amounted 376671 in 2006 and maximum was 430064 in 2008. From 2009 to 2013 the number of ships crossing was in interval 342754 – 391699. Based on the data coming from [10] we have drawn up *Figure 2* that shows a total number of shipping accidents in the Baltic Sea during 2006–2013. According to the reports 149 ship accidents occurred in the Baltic Sea area in 2013, which is the highest recorded number in the last ten years. The number of accidents in the Baltic Sea has shown a slight increase in the last three years. Compared to 2010 the total number of accidents increased by 17% in 2013 (*Figures 3–4*). Over recent years the number of maritime transport accidents increased, most of cargo ships, followed by passenger ships and tankers [9]–[11]. Human error is a major cause of accidents and is primarily related to unintentional action. However, 17% of the accidents occurred after intentional decisions against common rules and plans [9]. The number of collisions with other vessels and contacts to fixed or floating objects has south-western Baltic Sea is the main hotspot for these types of accidents. In the future, the offshore wind power sector will have high spatial requirements, especially when all safety distances are assigned to all components additional space is when ample safety distances are assigned to all components and additional space is reserved for the related service traffic. The expected increase in free traffic will also require more space, which should allow for a greater safety distance to maintain the commercial viability of increasing the safety distance to maintain commercial traffic [9].

Using date form *Table 1* and *Figure 2* we can compute the indicators of shipping accidents intensity in relation to the ships crossing number

$$\alpha = \frac{NSC}{NAC}, \beta = \frac{NAC}{NSC},$$

where *NSC* is a number of ship crossing and *NAC* is a number of accidents.

Safety and Reliability of Systems and Processes, Summer Safety and Reliability Seminar 2020

DOI: 10.26408/srsp-2020-07

*Table 1.* The total number of ships crossing in the Baltic Sea during 2006–2013

| Year | Passeger | Cargo | Tanker | Other | No info | Total |
|---|---|---|---|---|---|---|
| 2006 | 42721 | 226865 | 67458 | 39627 | 0 | 376671 |
| 2007 | 43998 | 237740 | 69281 | 53225 | 8204 | 412448 |
| 2008 | 43060 | 206755 | 60746 | 104814 | 14689 | 430064 |
| 2009 | 37994 | 198427 | 68008 | 61014 | 9234 | 374677 |
| 2010 | 30471 | 181932 | 59409 | 46950 | 23028 | 342754 |
| 2011 | 35398 | 207273 | 64957 | 60123 | 23948 | 391699 |
| 2012 | 33193 | 207056 | 66524 | 54627 | 22948 | 384359 |
| 2013 | 31329 | 182770 | 61193 | 57959 | 17141 | 350392 |

*Table 2.* Indicators of shipping accidents intensity in the Baltic Sea during 2006–2013

| Year | $\alpha$ | $\beta$ |
|---|---|---|
| 2006 | 3275.40 | 0.000305 |
| 2007 | 3495.32 | 0.000286 |
| 2008 | 3130.84 | 0.000320 |
| 2009 | 3258.06 | 0.000306 |
| 2010 | 2698.85 | 0.000370 |
| 2011 | 2739.15 | 0.000365 |
| 2012 | 2596.97 | 0.000385 |
| 2013 | 2351.62 | 0.000425 |

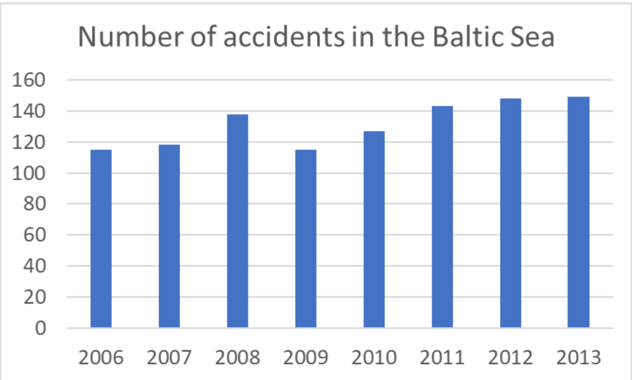


*Figure 1.* Number of accidents in the Baltic Sea during 2006–2013 [10]

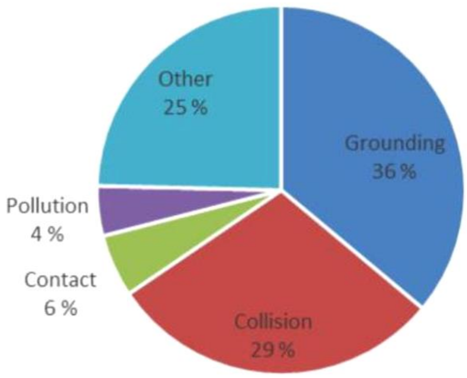


*Figure 3.* Types of accidents in the 2012 [10]

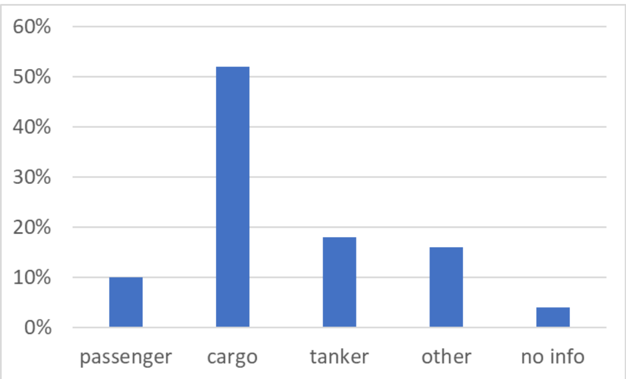


*Figure 2.* Different types of cruises in ships crossing in the Baltic Sea during 2006–2013

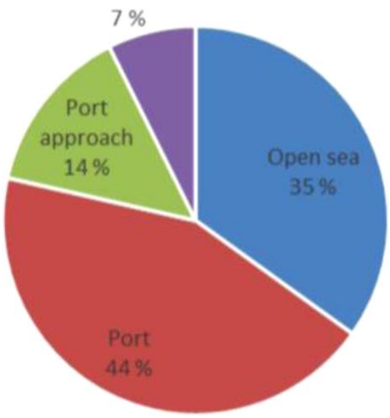


*Figure 4.* Location of accidents in the Baltic Sea in the Baltic Sea during 2004–2013 [11]

## 3. Nonhomogeneous Poisson process

We will begin with a reminder of the concept of nonhomogeneous Poisson's process.
We suppose

$$\tau_0 = \vartheta_0 = 0,\ \tau_n = \sum_{i=0}^{n} \vartheta_{\mathrm{i}} \tag{1}$$

where $\vartheta_1, \vartheta_2, \dots, \vartheta_n$ are positive independent and idendical distributed random variables. Let

$$\tau_\infty = \lim_{n\to\infty} \tau_n = \sup\{\tau_n:\ n \in \mathbb{N}_0\}. \tag{2}$$

A stochastic process $\{N(t): t \geq 0\}$ defined by the formula

$$N(t) = \sup\{n \in \mathbb{N}_0: \tau_n \leq t\} \tag{3}$$

is called a *counting process* corresponding to a random sequence $\{\tau_n : \in \mathbb{N}_0\}$.
Let $\{N(t): t \geq 0\}$ be a stochastic process taking values on $S = \{0,1,2,\dots\}$, value of which represents the number of events in a time interval $[0, t]$.
A counting process $\{N(t): t \geq 0\}$ is said to be nonhomogeneous Poisson process (NPP) with an intensity function $\lambda(\,t) \geq 0, t \geq 0$, if

1) $P(N(0) = 0) = 1;$ (4)

2) the process $\{N(t): t \geq 0\}$ is the stochastic process with independent increments, the right continuous and piecewise constant trajectories;

3) $P(N(t+h) - N(t) = k) =$
$$= \frac{\left(\int_t^{t+h} \lambda(x)dx\right)^k}{k\,!} e^{-\int_t^{t+h} \lambda(x)dx}. \tag{5}$$

From this definition it follows that the one dimensional distribution of NPP is given by the rule

$$P(N(t) = k) = \frac{\left(\int_0^t \lambda(x)dx\right)^k}{k\,!} e^{-\int_0^t \lambda(x)dx}, \tag{6}$$

where $k = 0,1,2, \dots$ .
The expectation and variance of NPP are the functions

$$\Lambda(t) = E[N(t)] = \int_0^t \lambda(x)dx\,, t \geq 0, \tag{7}$$

$$V(\mathrm{t}) = V[N(t)] = \int_0^t \lambda(x)dx\,, t \geq 0. \tag{8}$$

The corresponding standard deviation is

$$D(\mathrm{t}) = \sqrt{V[N(t)]} = \sqrt{\int_0^t \lambda(x)dx}\,, t \geq 0. \tag{9}$$

The expected value of the increment $N(t+h) - N(t)$ is

$$\Delta(t;h) = E\big(N(t+h) - N(t)\big)$$

$$= \int_t^{t+h} \lambda(x)dx,\ t,h \geq 0. \tag{10}$$

The corresponding standard deviation is

$$\sigma(t;h) = \sqrt{\int_t^{t+h} \lambda(x)dx},\ t,h \geq 0. \tag{11}$$

An nonhomogeneous Poisson process with $\lambda(\,t) = \lambda, t \geq 0$ for each $t \geq 0$, is a regular Poisson process. The increments of an nonhomogeneous Poisson process are independent, but not necessarily stationary. A nonhomogeneous Poisson process is a Markov process.

## 4. Compound Poisson process

Let $\{N(t): t \geq 0\}$ be a Poisson proces with intensity $\lambda > 0$ and $X_1, X_2, \dots$ be sequence of independent and identically distributed (i.i.d.) random variables independent of $\{N(t): t \geq 0\}$. A stochastic process

$$X(t) = X_1 + X_2 + \dots + X_{N(t)}, t \geq 0, \tag{12}$$

is called a compound Poisson process (CPP).
The probability discrete distribution function of $\{N(t): t \geq 0\}$ at $k$ is

$$p(k;t) = P(N(t) = k) = \frac{(\lambda\, t)^k}{k\,!} e^{-\lambda\, t},\ k = 0,1,2, \dots .$$

We quote a well-known result.

If $E(X_1^2) < \infty$, then

1) $E[X(t)] = \lambda\, t\, E(X_1), t \geq 0,$ (13)

2) $V[X(t)] = \lambda\, t\, E(X_1^2), t \geq 0.$ (14)

The concepts and facts can be generalized. We assume now that $\{N(t): t \geq 0\}$ is a nonhomogeneous Poisson process (NPP) with an intensity function $\lambda(\,t), t \geq 0$, such that $\lambda(\,t) \geq 0$, for $t \geq 0$, and $X_1, X_2, \dots$, is a sequence of the independent and identically distributed (i.i.d.) random variables independent of $\{N(t): t \geq 0\}$. A stochastic process $\{X(t): t \geq 0\}$ determines by the formula

$$X(t) = X_1 + X_2 + \dots + X_{N(t)}, t \geq 0, \tag{15}$$

is said to be a nonhomogeneous compound Poisson process (NCPP).

*Proposition 1*
If $\{N(t): t \geq 0\}$ is a nonhomogeneous Poisson process (NPP) with an intensity function $\lambda(t), t \geq 0$, such that $\lambda(t) \geq 0$ for $t \geq 0$, then cumulative distribution function (CDF) of the nonhomogeneous compound Poisson process is given by the rule:

$$G(x,t) = I_{[0,\infty)}(x) e^{-\Lambda(t)} + \sum_{k=1}^{\infty} p(k;t) F_X^{(k)}(x), \quad (16)$$

where $F_X^{(k)}(x)$ denotes the *k*–fold convolution of CDF of the random variables $X_i$, *i* = 1,2,…, and

$$p(k;t) = \frac{(\Lambda(t))^k}{k!} e^{-\Lambda(t)}, t \geq 0, \ k = 0,1,\ldots, \quad (17)$$

where

$$\Lambda(t) = E[N(t)] = \int_0^t \lambda(x) dx. \quad (18)$$

*Proof*: Using total probability low, we obtain cumulative distribution function (CDF) of NCPP.

$$\begin{aligned}
&G(x,t) = P(X(t) \leq x) \\
&= P\left(X_1 + X_2 + \cdots + X_{N(t)} \leq x\right) \\
&= \sum_{k=0}^{\infty} P\left(X_1 + \cdots + X_{N(t)} \leq x | N(t) = k\right) \\
&\cdot P(N(t) = k) = \sum_{k=0}^{\infty} p(k;t) F_X^{(k)}(x) \\
&= I_{[0,\infty)}(x) e^{-\Lambda(t)} + \sum_{k=1}^{\infty} p(k;t) F_X^{(k)}(x),
\end{aligned}$$

where $F_X^{(k)}(x)$ denotes the *k*–fold convolution of CDF of the random variables $X_i$, *i* = 1,2,…, and

$$p(k;t) = \frac{(\Lambda(t))^k}{k!} e^{-\Lambda(t)}, t \geq 0, k = 0,1,\ldots,$$

where

$$\Lambda(t) = E[N(t)] = \int_0^t \lambda(x) dx.$$

*Conclusion 1*
If the random variables $X_i$, *i* =1,2,…, are absolutely continuous with density function $f_X(\cdot)$, then the density of NCPP is given by the rule

$$g(x,t) = \sum_{k=1}^{\infty} p(k;t) f_X^{(k)}(x), x \neq 0, \ t > 0, \quad (19)$$

where $f_X^{(k)}(x)$ denotes *k*–fold convolution of the density function $f_X(x)$.

*Conclusion 2*
If the random variables $X_i$, *i* = 1,2,…, are discrete distributed with density function $p_X(x) = P(X = x), x \in S$, then the density of NCPP is given by the rule:

$$g(x,t) = \sum_{k=1}^{\infty} p(k;t) p_X^{(k)}(x), x \neq 0, t > 0, \quad (20)$$

where $p_X^{(k)}(x)$ denotes *k*–fold convolution of the discrete density function $p_X(x)$.

*Proposition 2*
Let $\{X(t): t \geq 0\}$ be a compound nonhomogeneous Poisson process (NCPP).
If $E(X_1^2) < \infty$, then

1) $E[X(t)] = \Lambda(t) \, E(X_1), t \geq 0,$ (21)

2) $V[X(t)] = \Lambda(t) E(X_1^2), t \geq 0.$ (22)

*Proof*: Applying the property of conditional expectation

$$E[X(t)] = E[E(X(t)|N(t))]$$

we have

$$\begin{aligned}
&E[E(X(t)|N(t))] \\
&= E\left(E(X_1 + X_2 + \cdots + X_{N(t)}) \middle| N(t)\right) \\
&= \sum_{n=0}^{\infty} E\left(X_1 + X_2 + \cdots + X_{N(t)} \middle| N(t) = n\right) \\
&\cdot P(N(t) = n) \\
&= \sum_{n=0}^{\infty} E(X_1 + X_2 + \cdots + X_n) \, P(N(t) = n) \\
&= \sum_{n=0}^{\infty} E(X_1) \, n \, P(N(t) = n) = E(X_1) E\left(N(t)\right) \\
&= \Lambda(t) \, E(X_1).
\end{aligned}$$

Using a formula

$$V[X(t)] = E[V(X(t)|N(t))] + V[E(X(t)|N(t))]$$

we get

$$\begin{aligned}
&E[V(X(t)|N(t))] \\
&= E\left(V(X_1 + X_2 + \cdots + X_{N(t)}) \middle| N(t)\right) \\
&= \sum_{n=0}^{\infty} V\left(X_1 + X_2 + \cdots + X_{N(t)} \middle| N(t) = n\right) \\
&\cdot P(N(t) = n) \\
&= \sum_{n=0}^{\infty} V(X_1 + X_2 + \cdots + X_n) \, P(N(t) = n) \\
&= \sum_{n=0}^{\infty} V(X_1) \, n \, P(N(t) = n) = \\
&V(X_1) E\left(N(t)\right) = V(X_1) \Lambda(t),
\end{aligned}$$

and

$$\begin{aligned}
&V\left[E(X(t)|N(t))\right] \\
&= V\left(E(X_1 + X_2 + \cdots + X_{N(t)}) \middle| N(t)\right) \\
&= V(E(X_1) N(t)) = (E(X_1))^2 V\left(N(t)\right) \\
&= (E(X_1))^2 \Lambda(t).
\end{aligned}$$

Therefore

$$V[X(t)] = V(X_1)[\Lambda(t) + (E(X_1))^2] = \Lambda(t)[E(X_1^2) - (E(X_1))^2 + (E(X_1))^2] = \Lambda(t)\, E(X_1^2).$$

*Proposition 2*

Let $\{X(t+h) - X(t): t \geq 0\}$ be an increament of compound nonhomogeneous Poisson process sit (CNPP).

If $E(X_1^2) < \infty$, then

$$E[X(t+h) - X(t)] = \Delta(t;h)\, E(X_1), \quad (23)$$

$$V[X(t+h) - X(t)] = \Delta(t;h)\, E(X_1^2), \quad (24)$$

$$\Delta(t;h) = \int_t^{t+h} \lambda(x)dx. \quad (25)$$

## 5. Models of accidents number in the Baltic Sea and seaports

We will quote information from the paper [7], which is necessary for further consideration.

From statistical anlysis in chapter 2 it follows that a stochastic process $\{N(t);\ t \geq 0\}$ representing the number of accidents in the Baltic Sea in a time interval $[0, t]$ of this process is given by (4)–(6) while its one dimensional distribution is determined by (6). We can use practically these rules if will know the intensity function $\lambda(t) > 0$. To define this function, we utilize information presented above in statistical analysis.

Dividing the number of accidents in each year, that are shown in *Figure 1*, by 365 or 366 we get the intensity in units of [1 / day ]. The results are shown in *Table 3*. *Figure 7* shows the empirical intensity of accidents in the Baltic Sea and seaports.

As a parameters of the models we will approximate the empirical intensity by a linear regression function $y = ax + b$ that satisfied condition

$$S(a,b) = \sum_{i=1}^{n} [y_i - (ax_i + b)]^2 \to \min.$$

Recall that parameters $a$ and $b$ are given by the rules

$$a = \frac{\mu_{11}}{\mu_{20}}, \quad b = m_{01} - am_{10} \quad (26)$$

or

$$a = r\frac{s_Y}{s_X}, \quad b = \bar{y} - a\bar{x}, \quad (27)$$

where

$$\bar{x} = m_{10} = \frac{1}{n}\sum_{i=1}^{n} x_i,$$

$$\bar{y} = m_{01} = \frac{1}{n}\sum_{i=1}^{n} y_i,$$

$$m_{11} = \frac{1}{n}\sum_{i=1}^{n} x_i y_i,$$

$$\mu_{11} = m_{11} - m_{10}\, m_{01},$$

$$r = \frac{\mu_{11}}{\sqrt{\mu_{20}\,\mu_{02}}}.$$

*Table 3*. The empirical intensity of accidents in the Baltic Sea and seaports

| Year | Interval | Center of interval | Number of accidents | Intensity [1/day] |
|---|---|---|---|---|
| 2004 | [0, 366) | 183 | 133 | 0.36338 |
| 2005 | [366,731) | 731.5 | 146 | 0.40000 |
| 2006 | [731, 1096) | 913.5 | 115 | 0.31506 |
| 2007 | [1096, 1461) | 1278.5 | 118 | 0.32328 |
| 2008 | [1461, 1827) | 1644 | 138 | 0.37704 |
| 2009 | [1827, 2192) | 2009.5 | 115 | 0.31506 |
| 2010 | [2192, 2557) | 2374.5 | 127 | 0.34794 |
| 2011 | [2557, 2922) | 2374.5 | 143 | 0.39178 |
| 2012 | [2922, 3288) | 3105 | 148 | 0.40437 |
| 2013 | [3288, 3653) | 3470.5 | 149 | 0.40821 |

Applaying the rules (26)–(27) for the data from *Table 3* and using Excel we obtain the linear intensity of accidents

$$\lambda(x) = 0.0000147564\,x + 0.3379257233, \quad x \geq 0. \quad (28)$$

From (7), we have

$$\Lambda(t) = \int_0^t (0.0000147564\,x + 0.3379257233)dx.$$

Hence we obtain

$$\Lambda(t) = 0.0000073782t^2 + 0.3379257233t, \quad t \geq 0. \quad (29)$$

From (6) and (7) we obtain one dimensional distribution of NPP

$$P(N(t)=k)=\frac{(\Lambda(t))^k}{k!}e^{-\Lambda(t)},\ k=0,1,2,\dots, \quad (30)$$

where $\Lambda(t)$ is given by (29).
Finnally we can say that the model of the accident number in the Baltic Sea and seaport is the nonhomogeneous Poisson process with the parameter $\Lambda(t),\ \ t\geq 0$ determines by (29).

*Example 1* [6]
The predicted number of injured person and ships in accidents in the Baltic Sea and ports from June 1, 2017 to August 31, 2017 we will get in a similar way. (*Tables 4–5*, *Figures 5–7*) In this case

$\boldsymbol{\mu} = 0.224.$

The expected value and standard deviation of injured people number at considered period are

$$ENI = 34.45 \cdot 0.224 = 7.7168,$$
$$DNI = \sqrt{34.45}\cdot(0.224+0.2242) = 3.0733.$$

*Table* 4. Distribution of injured person number

| $x$ | $g(x)$ |
|---|---|
| 0 | 0.00099 |
| 1 | 0.00613 |
| 2 | 0.01959 |
| 3 | 0.04317 |
| 4 | 0.07358 |
| 5 | 0.10332 |
| 6 | **0.12431** |
| 7 | **0.13164** |
| 8 | **0.12507** |
| 9 | 0.1082 |
| 10 | 0.0862 |
| 11 | 0.0638 |
| 12 | 0.04426 |
| 13 | 0.02891 |
| 14 | 0.0179 |
| 15 | 0.01053 |
| 16 | 0.00592 |
| 17 | 0.00319 |
| 18 | 0.0016 |
| 19 | 0.0008 |

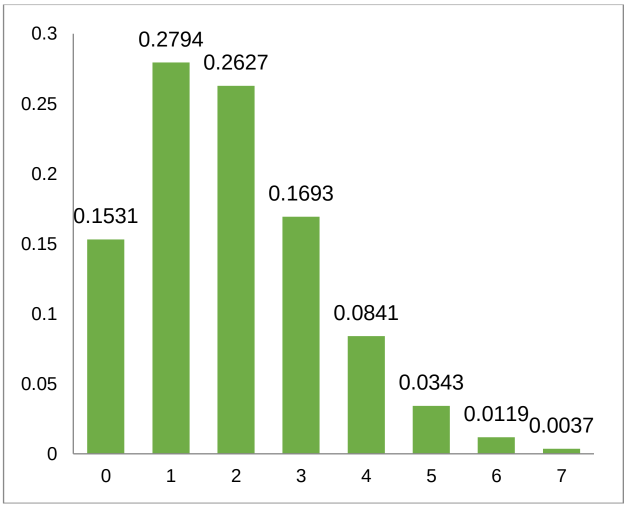


*Figure 5.* Distribution of injured person number in a single accident

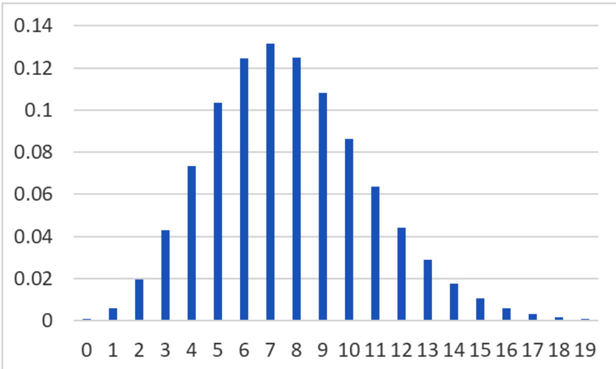


*Figure 6.* Distribution of injured person number

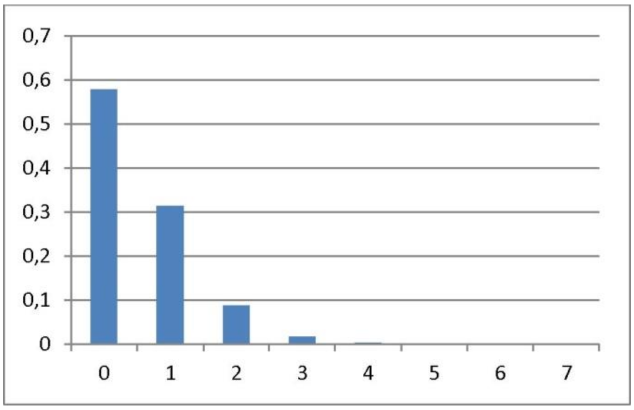


*Figure 7.* Distribution of the ships lost number in a single accident

*Table 5.* Distribution of ships lost number

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $g(x)$ | 0.57879 | 0.31396 | 0.08766 | 0.01677 |
| $x$ | 4 | 5 | 6 | 7 |
| $g(x)$ | 0.00247 | 0.00029 | 0.00003 | 0.0000003 |

## 6. Random parameter in Poisson model

The expected number of accidents often depends on changing randomly external conditions. Thus it can be assumed that the parameter $\lambda$ is a random variable.

We assume that this random variable has a gamma distribution with a density

$$f(u) = \begin{cases} \frac{\alpha^\nu}{\Gamma(\nu)} u^{\nu-1} e^{-\alpha\,\nu} & \\ 0 & \text{for } u \le 0, \end{cases} \quad (31)$$

where $\alpha > 0$, $\nu > 1$.

Suppose that a condition distribution of the accidents number given $\lambda$ has a Poisson distribution

$$P(N(t) = k|\,\lambda) = \frac{(\lambda\, t)^k}{k!} e^{-\lambda\, t}, \quad k = 0,1,2,\dots . \quad (32)$$

From the total probability low we obtain unconditional one-dimensional distribution of the process

$$P(N(t) = k) = \int_0^\infty \frac{(u\, t)^k e^{-u\, t}}{k!} \frac{\alpha^\nu}{\Gamma(\nu)} u^{\nu-1} e^{-\alpha\,\nu} du. \quad (33)$$

For $k \ge 0$ we obtain

$$P(N(t) = 0) = P(\vartheta_1 > t) = \left(\frac{\alpha}{\alpha+t}\right)^\nu. \quad (34)$$

Finally for $k$ = 1,2 , … we have [2]

$$P(N(t) = k) = \frac{\nu(\nu+1)\dots(\nu+k-1)}{k!} \left(\frac{\alpha}{\alpha+t}\right)^k \left(\frac{\alpha}{\alpha+t}\right)^\nu \quad (35)$$

where $k$ = 1,2 ,…, $\nu > 1$, $\lambda > 0$.

The random variable $T = \vartheta_1$ denotes a lifetime of an object. The function

$$R(t) = P(T > t) = 1 - P(T \ge t) = \left(\frac{\alpha}{\alpha+t}\right)^\nu \quad (36)$$

is called a survival function.

An expected value of the random variable $T$ is

$$E(T) = \int_0^\infty \nu \left(\frac{\alpha}{\alpha+t}\right)^\nu dt. \quad (37)$$

The second moment is

$$2\int_0^\infty t \left(\frac{\alpha}{\alpha+t}\right)^\nu dt = \frac{2a^2}{(\nu-1)(\nu-2)}. \quad (38)$$

The variance is

$$V(T) = \frac{\alpha^2 \nu}{(\nu-1)^2(\nu-2)}. \quad (39)$$

It should be mentioned that the variance there exists if $\nu > 2$. The standard deviation is

$$\sigma(T) = \sqrt{V(T)}. \quad (40)$$

## 7. Procedure of parameters identification

Notice that values mentioned in Section 6 depend on the two parameters: both $\alpha$ and $\nu$. There is natural question, how to determine these parameters. One method of estimating the unknown parameters is the so called the method of moments. In this method the unknown parameters are replaced by their statistical estimates derived from the results of observation. In this case, the expected value is replaced by the average of the sample and the second moment of the random variable is replaced by the second moment of the sample. Solving the corresponding system of equations we obtain the unknown parameters of the distribution. An estimate of the expectation $E(T)$ is mean:

$$\bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n} \quad (41)$$

and the estimate of is second moment from the sample

$$\overline{x^2} = \frac{1}{n} \sum_{k=1}^n \overline{x_k^2}. \quad (42)$$

An empirical variance takes the form

$$\boldsymbol{s^2} = \overline{x^2} - (\overline{x})^2. \quad (43)$$

A standard deviation is

$$\boldsymbol{s} = \sqrt{\overline{x^2} - (\overline{x})^2}. \quad (44)$$

Then, the parameters $\alpha$ and $\nu$ can be evaluated by:

$$\alpha = \frac{2(\overline{x})^3}{(\overline{x})^2 + \boldsymbol{s^2}}, \quad (45)$$

$$\nu = \frac{2(\overline{x})^2}{(\overline{x})^2 - \boldsymbol{s^2}}. \quad (46)$$

## 8. Conclusion

The random processes theory delivers concepts and theorems that enable to construct stochastic models concerning accidents. The counting processes and processes with independent increments are the most appropriate for modelling number of the accidents number in the Baltic Sea waters and ports in specified period of time. A crucial role in the models construction plays a nonhomogeneous

Poisson process and nonhomogeneous compound Poisson process. Based on the nonhomogeneous Poisson process the models of accidents number in the Baltic Sea Waters and Seaports have been constructed. Moreover, some procedures of the model parameters identification are presented in the paper. Estimation of model parameters was made based on data from reports of HELCOM (2014) and Interreg project Baltic LINes (2016).

The nonhomogeneous compound Poisson process as a model of the accidents consequences is also presented in this paper. Theoretical results are applied for anticipation the number of fatalities, number injured people and number of lost ships in accidents at the Baltic Sea waters and ports in specified period of time. The expected number of accidents often depends on changing randomly external conditions. Thus it can be assumed that the parameter $\lambda$ is a random variable. In the paper is assumed that this random variable has Weibull distribution.

## Acknowledgement

I would like to thank Professor Lauri Ojala for pointing to the reports Baltic LINes (2016) [13] and HELCOM (2014) [14]–[15], from which the data allowed me to estimate the models parameters.

I want to thank Professor Krzysztof Kołowrocki for mobilization to write this chapter.

## References

[1] Bernaciak, K. 2005. Multicriterial optimization of semi-Markov processes with discount. *Advances in Safety and Reliability*, 171–178.

[2] Boussemart, M., Bicard, T. & Limnios, N. 2001. Markov decision processes with aconstraint on the average asymptotic failure rate. *Methodology and Computing in Applied Probability* 3(2), 199–214.

[3] Beutler, F. & Ross, K. 1986. Time-average optimal constrained semi-Markov decision processes. *Advances in Applied Probability* 18, 341–359.

[4] Boussemart, M & Limnios, N. 2004. Markov decision processes with aconstraint on the average asymptotic failure rate. *Communication in Statistics – Theory and Methods* 33(7), 1689–1714.

[5] Feinberg, E. 1994. Constrained semi-Markov decision processes with average rewards. *ZOR – Mathematical Methods of Operations Research* 39, 257–288.

[6] Gertsbakh, I. B. 1969. *Models of Preventive Service.* Sovetskoe radio. Moscow (in Russian).

[7] Grabski, F. 2009. *Application of the Functions of Random Arguments in Reliability, Safety and Logistic Problems*. Wydawnictwo Komunikacji i Łączności, Warszawa (in Polish: *Funkcje o losowych argumentach w zagadnieniach niezawodności, bezpieczeństwa i logistyki*), 344.

[8] Grabski, F. 2015. *Semi-Markov Processes: Applications in Systems Reliability and Maintenance*. Elsevier, Amsterdam – Boston – Heidelberd – London – New York – Oxford – Paris – San Diego – San Francisco – Sidney – Tokyo.

[9] Grabski, F. 2018. Optimization problem of reliability subject to constraint of availability in semi-Markov models of operation. *AIP Conference Proceedings.* Data Analysis and Applicaions 4: financial data analysis and methods 6; Big Data, Artificial Intelligence and Mata Analysis SET. Editors: Makrides A., Karagrigoriou A., Skidas C. H. London, ISTE LTD 2020, 103–118.

[10] Grabski, F. 2017. Nonhomogeneous Poisson process application to modelling accidents number at Baltic waters and ports. *Journal of Polish Safety and Reliability Association, Summer Safety and Reliability Seminars* 8(1), 39–46.

[11] Limnios, N. & Oprisan, G. 2001. *Semi-Markov Processes and Reliability*, Boston, Birkhauser.

[12] Shiryayev, A. N. 1984. *Probability*. Springer–Verlag, New York, Berlin, Heidelberg, Tokyo.

[13] Baltic LINes. 2016. *Shipping in the Baltic Sea – Past, Present and Future Developments Relevant for Maritime Spatial Planning*. Project Report I, 35.

[14] HELCOM. 2014. *Annual Report on Shipping Accidents in the Baltic Sea Area During 2012*, 43.

[15] HELCOM. 2014. *Annual Report on Shipping Accidents in the Baltic Sea in 2013*.

[16] Howard, R. A. 1960. *Dynamic Programing and Markov Processes*. Cambridge, Massachusetts, MIT Press.

[17] Howard, R. A. 1964. Research of semi-markovian decision structures. *Journal of Operations Research Society* 6, 163–199.

[18] Howard, R. A. 1971. Dynamic probabilistic system. *Semi-Markow and Decision Processes* 2. New York – London – Sydney – Toronto, Wiley.

[19] Jewell, W. S. 1963. Markov-renewal programming. *Operation Research* 11, 938–971.

[20] Korolyuk, V. S. & Turbin, A. F. 1976. *Semi-Markov Processes and their Applications* (in Russian), Kiev, Naukova Dumka.

[21] Limnios, N & Oprisan, G. 2001. *Semi-Markov Processes and Reliability*. Boston, Birkhauser.

[22] Mine, H. & Osaki, S. 1970. *Markovian Decision Processes*. New York, AEPCI.
[23] Silvestrov, D. C. 1980. *Semi-Markov Processes with a Discrete State Space* (in Russian), Moscow, Sovetskoe Radio.