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Safety analysis of car wheel system impacted by operation process

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Abstract

A practically important approach is proposed for the safety analysis of multistate ageing systems that considers the influence of their operation processes on their safety. The system operation process semi-Markov model is introduced and its characteristics are determined. The system safety function is defined and determined for a multistate ageing complex system impacted by its operation process. As a special case, the safety of a series system is modelled using its components' piecewise exponential safety functions and the results are applied to examine and characterize safety of an exemplary car wheel system.

1. Introduction

The approach to multistate ageing system reliability analysis was introduced in [26]–[31] and widely developed in [15] for reliability analysis of such system impacted by its operation process. Next, the approach was transformed and developed to safety analysis of complex systems [22] and critical infrastructures [23] impacted by their operation processes related to the climate-weather change processes at their operating areas in [15]–[22] and to systems and critical infrastructure networks composed of dependent components and subsystems reliability analysis in [1]–[2]. Those practically important approaches to complex multistate system reliability and safety analysis consider the assumption about system components degradation and changing their safety and reliability parameters impacted by operation conditions [15]. Degradation of multistate system components impacted by their changing operation conditions causes the changes of system reliability and its operation safety. The paper is devoted to joining of the multistate system ageing and its external operation impacts [15]–[22] and to considering them together in system safety analysis and to show the possibility of its real application in practice. The paper is organized into 5 parts, this Introduction as Section 1, Sections 2–4 and Conclusions as Section 5. In Section 2, the multistate ageing system operation process is introduced, its parameters are defined and its characteristics are determined. In Section 3, the safety of multistate ageing system impacted by its operation process is considered. The safety and resilience indicators of the multistate ageing system and its components related to the system operation process impact are proposed. In Section 4, the safety of the car wheel system impacted by its operation process is examined and its safety and resilience indicators are determined.

In conclusions, the results' evaluation and the possibility of their real practical applications are discussed performed and the perspective for future research in the field considered in the paper is suggested.

2. Multistate ageing system operation process

We consider the multistate ageing system impacted by its operation process $Z(t)$, $t \in <0,\infty)$, in various ways depending on the current system state z_b, $b = 1,2,...,v$. We assume that the changes of the operation states of the multistate ageing system operation process $Z(t)$ have an influence on the safety of this system components E_i, $i = 1,2,...,n$, and consequently on the safety of the entire system [12]–[21].

The multistate ageing system operation process is defined by the following parameters that can be identified either statistically using the methods given in [4]–[8], [14]–[15], [24]–[25], or evaluated

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by experts:

- the number of operation states v;
- the operation states $z_1, z_2, \ldots, z_v$;
- the vector $[p_b(0)]_{1\times v}$ of the initial probabilities

$$p_b(0) = P(Z(0) = z_b),\ b = 1,2,...,v, \qquad (1)$$

of the system operation process $Z(t)$, $t \in <0,\infty)$, staying at particular operation states z_b, $b = 1,2,...,v$, at the moment $t = 0$;

- the matrix of probabilities $[p_{bl}]_{v\times v}$ of transition

$$p_{bl},\ b,l = 1,2,...,v,\ p_{bb} = 0,\ b = 1,2,...,v, \qquad (2)$$

of the system operation process $Z(t)$, $t \in <0,\infty)$, between the operation states z_b, $b = 1,2,...,v$, and z_l, $l = 1,2,...,v$;

- the matrix $[M_{bl}]_{v\times v}$ of the mean values of conditional sojourn times

$$M_{bl} = E[\theta_{bl}] = \int_0^\infty t dH_{bl}(t) = \int_0^\infty t h_{bl}(t)dt,$$
$$b,l = 1,2,...,v,\ b \neq l,\ M_{bb} = 0, \qquad (3)$$

of the system operation process $Z(t)$, $t \in <0,\infty)$, conditional sojourn times θ_{bl}, $b,l = 1,2,...,v$, at the operation state z_b, $b = 1,2,...,v$, when the next state is z_l, $l = 1,2,...,v$, where

$$H_{bl}(t) = P(\theta_{bl} < t),\ t \in <0,\infty),$$
$$b,l = 1,2,...,v,\ b \neq l, \qquad (4)$$

are conditional distribution functions of the system operation process $Z(t)$ conditional sojourn times θ_{bl}, $b,l = 1,2,...,v$, $b \neq l$, at the operation states corresponding to conditional density functions

$$h_{bl}(t) = \frac{dH_{bl}(t)}{dt},\ t \in <0,\infty),\ b,l = 1,2,...,v,$$
$$b \neq l. \qquad (5)$$

The following multistate ageing system operation process characteristics can be either calculated analytically using the above parameters of the operation process [4]–[8], [14]–[15], [24]–[25] or evaluated approximately by experts:

- the vector $[M_b]_{1\times v}$ of mean values of the system operation process $Z(t)$ unconditional sojourn times θ_b, $b = 1,2,...,v$, at the operation states

$$M_b = E[\theta_b] = \sum_{l=1}^{v} p_{bl} M_{bl},\ b = 1,2,...,v, \qquad (6)$$

where M_{bl}, $b,l = 1,2,...,v$, are defined by (3);

- the vector $[p_b]_{1\times v}$ of limit values of transient probabilities

$$p_b(t) = P(Z(t) = z_b),\ t \in <0,+\infty),$$
$$b = 1,2,...,v, \qquad (7)$$

of the system operation process $Z(t)$ at the particular operation states z_b, $b = 1,2,...,v$, where

$$p_b = \lim_{t\to\infty} p_b(t) = \frac{\pi_b M_b}{\sum_{l=1}^{v} \pi_l M_l},\ b = 1,2,...,v, \quad (8)$$

and M_b, $b = 1,2,...,v$, are given by (6), while the steady probabilities π_b, $b = 1,2,...,v$, of the vector $[\pi_b]_{1\times v}$ satisfy the system of equations [15]

$$\begin{cases} [\pi_b][p_{bl}] = [\pi_b] \\ \sum_{l=1}^{v} \pi_l = 1; \end{cases} \qquad (9)$$

- the vector $[\hat{M}_b]_{1\times v}$ of the mean values of the total sojourn times $\hat{\theta}_b$, $b = 1,2,...,v$,

$$\hat{M}_b = E[\hat{\theta}_b] \cong p_b \theta,\ b = 1,2,...,v, \qquad (10)$$

of the system operation process $Z(t)$, $t \in <0,+\infty)$, at the particular operation states z_b, $b = 1,2,...,v$, during the fixed system opetation time θ, $\theta > 0$, where p_b, $b = 1,2,...,v$, are given by (8).

3. Safety of multistate ageing system impacted by its operation process

We denote by

$$[T_i^2(u)]^{(b)},\ u = 1,2,...,z,\ b = 1,2,...,v,\ i = 1,2,...,n,$$

the multistate ageing system component E_i, $i = 1,2,...,n,$ conditional lifetime in the safety state subset $\{u, u+1,..., z\},$ $u = 1,2,..., z,$ while its operation process $Z(t),$ $t \in < 0,+\infty),$ is at the operation state z_b, $b = 1,2,...,v,$ and its conditional safety function by the vector [15], [17], [19]–[21]

$$[S_i^2(t,\cdot)]^{(b)} = [[S_i^2(t,1)]^{(b)}, [S_i^2(t,2)]^{(b)}, ..., [S_i^2(t,z)]^{(b)}], \quad t \in < 0,\infty), \ b = 1,2,...,v, \ i = 1,2,...,n, \tag{11}$$

with the coordinates

$$[S_i^2(t,u)]^{(b)} = P([T_i^2(u)]^{(b)} > t | Z(t) = z_b), \quad t \in < 0,\infty), \ u = 1,2,..., z, \ b = 1,2,...,v, \ i = 1,2,...,n. \tag{12}$$

The safety function (11) coordinate $[S_i^2(t,u)]^{(b)}$, $t \in < 0,\infty),$ $u = 1,2,..., z,$ $b = 1,2,...,v,$ $i = 1,2,...,n,$ defined by (12), is the conditional probability that the component E_i, $i = 1,2,...,n,$ lifetime $[T_i^2(u)]^{(b)}$, $u = 1,2,..., z,$ $b = 1,2,...,v,$ $i = 1,2,...,n,$ in the safety state subset $\{u, u+1,..., z\},$ is greater than *t*, while the system operation process $Z(t),$ $t \in < 0,+\infty),$ is at the operation state z_b, $b = 1,2,...,v.$

Similarly, we denote by

$$[T^2(u)]^{(b)}, \ u = 1,2,..., z, \ b = 1,2,...,v,$$

the multistate ageing system conditional lifetime in the safety state subset $\{u, u+1,..., z\},$ $u = 1,2,..., z,$ while its operation process $Z(t), t \in < 0,\infty),$ is at the operation state z_b, $b = 1,2,...,v,$ and the conditional safety function of the system at this operation state by the vector [8], [15], [19]

$$[\boldsymbol{S}^2(t,\cdot)]^{(b)} = [[\boldsymbol{S}^2(t,1)]^{(b)}, [\boldsymbol{S}^2(t,2)]^{(b)}, ..., [\boldsymbol{S}^2(t,z)]^{(b)}], \tag{13}$$

with the coordinates

$$[\boldsymbol{S}^2(t,u)]^{(b)} = P([T^2(u)]^{(b)} > t | Z(t) = z_b), \quad t \in < 0,\infty), \ u = 1,2,..., z, \ b = 1,2,...,v. \tag{14}$$

The safety function (13) coordinate $[\boldsymbol{S}^2(t,u)]^{(b)}$, $t \in < 0,\infty),$ $u = 1,2,..., z,$ $b = 1,2,...,v,$ defined by (14) is the conditional probability that the aging multistate system impacted by its operation process *Z*(*t*), $t \in < 0,\infty),$ lifetime $[T^2(u)]^{(b)}$, $u = 1,2,..., z,$ in the safety state subset $\{u, u+1,..., z\},$ $u = 1,2,..., z,$ is greater than *t*, while the multistate ageing system operation process *Z*(*t*), $t \in < 0,\infty),$ is at the operation state z_b, $b = 1,2,...,v.$

Next, we denote by

$$T^2(u), \ u = 1,2,..., z,$$

the multistate ageing system impacted by its operation process *Z*(*t*), $t \in < 0,\infty),$ unconditional lifetime in the safety state subset $\{u, u+1,..., z\},$ $u = 1,2,..., z,$ and the unconditional safety function of the multistate ageing system impacted by its operation process *Z*(*t*), $t \in < 0,\infty),$ by the vector

$$\boldsymbol{S}^2(t,\cdot) = [\boldsymbol{S}^2(t,1), \boldsymbol{S}^2(t,2), ..., \boldsymbol{S}^2(t,z)], \quad t \in < 0,\infty), \tag{15}$$

with the coordinates

$$\boldsymbol{S}^2(t,u) = P(T^1(u) > t), \ t \in < 0,\infty), \ u = 1,2,..., z. \tag{16}$$

The safety function (15) coordinate $\boldsymbol{S}^2(t,u)$, $t \in < 0,\infty),$ $u = 1,2,..., z,$ defined by (16) is the unconditional probability that the multistate ageing system impacted by its operation process *Z*(*t*), $t \in < 0,\infty),$ lifetime $T^2(u),$ $u = 1,2,..., z,$ in the safety state subset $\{u, u+1,..., z\},$ $u = 1,2,..., z,$ is greater than *t*, while the multistate ageing system operation process *Z*(*t*), $t \in < 0,\infty),$ is at the operation state z_b, $b = 1,2,...,v.$

In the case when the aging multistate system operation time θ is large enough, the coordinates $\boldsymbol{S}^2(t,u)$, $t \in < 0,\infty),$ $u = 1,2,..., z,$ defined by (16), of the unconditional safety function (15) of the multistate ageing system impacted by its operation process $Z(t), t \in < 0,\infty),$ are given by [15], [21]

$$\boldsymbol{S}^2(t,u) \cong \sum_{b=1}^{v} p_b [\boldsymbol{S}^2(t,u)]^{(b)}, \ t \in < 0,\infty), \quad u = 1,2,..., z, \tag{17}$$

where $[\boldsymbol{S}^2(t,u)]^{(b)}$, $t \in < 0,\infty),$ $u = 1,2,..., z,$ $b = 1,2,...,v,$ are the coordinates of the multistate ageing system impacted by its operation process *Z*(*t*), $t \in < 0,\infty),$ conditional safety functions defined by (13)–(14) and p_b, $b = 1,2,...,v,$ are the aging multistate system operation process $Z(t)$, $t \in <0, \infty)$, limit transient probabilities at the operation states z_b, $b = 1,2,...,\nu$, given by (8).

4. Application

4.1. Car wheel system operation process parameters

We consider the car wheel system impacted by its operation process. On the basis of the statistical data and expert opinions, it is possible to fix and to evaluate the unknown basic parameters of the car wheel system operation process $Z(t)$. Thus, we arbitrarily assume that the number of operation states of this process is $\nu = 5$ and we define the operation states as follows:

- the operation state z_1 – the system is not operating,
- the operation state z_2 – the system is operating on the good-quality road surface in good hydro-meteorological conditions,
- the operation state z_3 – the system is operating on the bad-quality road surface in good hydro-meteorological conditions,
- the operation state z_4 – the system is operating on the good-quality road surface in bad hydro-meteorological conditions,
- the operation state z_5 – the system is operating on the bad-quality road surface in bad hydro-meteorological conditions.

Next, the arbitrarily assumed basic parameters of the car wheel system operation process $Z(t)$ are as follows:

- the vector of the initial probabilities of the car wheel system operation process $Z(t)$ staying at particular operation states z_b, b = 1,2,3,4,5, at the moment $t = 0$

$$[p(0)]_{1x5} = [0.40, 0.24, 0.16, 0.15, 0.05]; \quad (18)$$

- the matrix of probabilities of the car wheel system operation process transitions from the operation state z_b, b = 1,2,3,4,5, into the operation state z_l, $l = 1,2,3,4,5$,

$$[p_{bl}]_{5x5} = \begin{bmatrix} 0 & 0.4 & 0.3 & 0.2 & 0.1 \\ 0.3 & 0 & 0.4 & 0.2 & 0.1 \\ 0.3 & 0.4 & 0 & 0.2 & 0.1 \\ 0.2 & 0.1 & 0.4 & 0 & 0.3 \\ 0.5 & 0.1 & 0.2 & 0.2 & 0 \end{bmatrix}; \quad (19)$$

- the matrix of mean values $M_{bl} = E[\theta_{bl}]$, $b,l = 1,2,3,4,5$, of the car wheel system operation process conditional sojourn times θ_{bl}, $b,l = 1,2,3,4,5$, at the particular operation states

$$[M_{bl}]_{5x5} = \begin{bmatrix} 0 & 10 & 12 & 12 & 14 \\ 10 & 0 & 8 & 8 & 6 \\ 8 & 10 & 0 & 10 & 8 \\ 8 & 4 & 6 & 0 & 4 \\ 6 & 2 & 4 & 6 & 0 \end{bmatrix}. \quad (20)$$

This way, the car wheel system operation process is arbitrarily defined and we may predict its main characteristics.

4.2. Car wheel system operation process characteristics

Applying (6), (19) and (20), we have:

- the mean values of the car wheel system operation process unconditional sojourn times θ_b, $b = 1,2,3,4,5$, at the particular operation states:

$$M_1 = E[\theta_1] = p_{12}M_{12} + p_{13}M_{13} + p_{14}M_{14} + p_{15}M_{15} = 0.4 \cdot 10 + 0.3 \cdot 12 + 0.2 \cdot 12 + 0.1 \cdot 14 = 11.4,$$

$$M_2 = E[\theta_2] = p_{21}M_{21} + p_{23}M_{23} + p_{24}M_{24} + p_{25}M_{25} = 0.3 \cdot 10 + 0.4 \cdot 8 + 0.2 \cdot 8 + 0.1 \cdot 6 = 8.4,$$

$$M_3 = E[\theta_3] = p_{31}M_{31} + p_{32}M_{32} + p_{34}M_{34} + p_{35}M_{35} = 0.3 \cdot 8 + 0.4 \cdot 10 + 0.2 \cdot 10 + 0.1 \cdot 8 = 9.2,$$

$$M_4 = E[\theta_4] = p_{41}M_{41} + p_{42}M_{42} + p_{43}M_{43} + p_{45}M_{45} = 0.2 \cdot 8 + 0.1 \cdot 4 + 0.4 \cdot 6 + 0.3 \cdot 4 = 5.6,$$

$$M_5 = E[\theta_5] = p_{51}M_{51} + p_{52}M_{52} + p_{53}M_{53} + p_{54}M_{54} = 0.5 \cdot 6 + 0.1 \cdot 2 + 0.2 \cdot 4 + 0.2 \cdot 6 = 5.2; \quad (21)$$

- the limit values of transient probabilities of the car wheel system operation process at the particular operation states z_b, $b = 1,2,3,4,5$:

$$p_1 \cong 0.272, \quad p_2 \cong 0.261, \quad p_3 \cong 0.280, \quad p_4 \cong 0.112, \quad p_5 \cong 0.075, \tag{22}$$

obtained after considering (19) in the system of equations (9) that takes the form

$$\begin{cases} [\pi_1, \pi_2, \pi_3, \pi_4, \pi_5][p_{bl}]_{5x5} = [\pi_1, \pi_2, \pi_3, \pi_4, \pi_5] \\ \pi_1 + \pi_2 + \pi_3 + \pi_4 + \pi_5 = 1, \end{cases}$$

equivalent to the system of equations

$$\begin{cases} 0.3\pi_2 + 0.3\pi_3 + 0.2\pi_4 + 0.5\pi_5 = \pi_1 \\ 0.4\pi_1 + 0.4\pi_3 + 0.1\pi_4 + 0.1\pi_5 = \pi_2 \\ 0.3\pi_1 + 0.4\pi_2 + 0.4\pi_4 + 0.2\pi_5 = \pi_3 \\ 0.2\pi_1 + 0.2\pi_2 + 0.2\pi_3 + 0.2\pi_5 = \pi_4 \\ 0.1\pi_1 + 0.1\pi_2 + 0.1\pi_3 + 0.3\pi_4 = \pi_5 \\ \pi_1 + \pi_2 + \pi_3 + \pi_4 + \pi_5 = 1, \end{cases}$$

resulting in its following solution

$$\pi_1 \cong 0.199, \quad \pi_2 \cong 0.259, \quad \pi_3 \cong 0.254, \quad \pi_4 \cong 0.167, \quad \pi_5 \cong 0.121, \tag{23}$$

and next, after applying (8) to (21) and (23), resulting in determining the limit values p_b, $b = 1,2,3,4,5$, of the car wheel system operation process transient probabilities $p_b(t)$, $t \in <0, \infty)$, $b = 1,2,3,4,5$, at the operation states z_b, $b = 1,2,3,4,5$, that are given by (22);

- the expected values of the total sojourn times $\hat{\theta}_b$, $b = 1,2,3,4,5$, of the car wheel system operation process at the particular operation states z_b, $b = 1,2,3,4,5$, during the fixed operation time $\theta = 1$ year = 365 days, determined according to (10) and (22):

$$\hat{M}_1 \cong E[\hat{\theta}_1] = 0.272 \cdot 1 \text{ year} = 99.3 \text{ days},$$
$$\hat{M}_2 \cong E[\hat{\theta}_2] = 0.261 \cdot 1 \text{ year} = 95.3 \text{ days},$$
$$\hat{M}_3 \cong E[\hat{\theta}_3] = 0.280 \cdot 1 \text{ year} = 102.2 \text{ days},$$
$$\hat{M}_4 \cong E[\hat{\theta}_4] = 0.112 \cdot 1 \text{ year} = 40.9 \text{ days},$$
$$\hat{M}_5 \cong E[\hat{\theta}_5] = 0.075 \cdot 1 \text{ year} = 267.3 \text{ days}. \tag{24}$$

4.3. Multistate ageing car wheel system impacted by its operation process safety parameters

We assume that the operation impact on the intensities of ageing $[\lambda_i^2(u)]^{(b)}$, $u = 1,2,3,4$, $b = 1,2,3,4,5$, $i = 1,2,3,4$, of the car wheel system components E_i, $i = 1,2,3,4$, at the operation process $Z(t)$ states z_b, $b = 1,2,3,4,5$, is expressed by

$$[\lambda_i^2(u)]^{(b)} = [\rho_i^2(u)]^{(b)} \cdot \lambda_i^0(u), \; u = 1,2,3,4, \; b = 1,2,3,4,5, \; i = 1,2,3,4, \tag{25}$$

where $\lambda_i^0(u)$, $u = 1,2,3,4$, $i = 1,2,3,4$, are the intensities of ageing of the car wheel system components E_i, $i = 1,2,3,4$, without operation process impact and $[\rho_i^2(u)]^{(b)}$, $u = 1,2,3,4$, $b = 1,2,3,4,5$, $i = 1,2,3,4$, are the coefficients of operation process impact on the car wheel system components E_i, $i = 1,2,3,4$, intensities of ageing at the operation states z_b, $b = 1,2,3,4,5$.

Further, we arbitrarily assume the following coefficients $[\rho_i^2(u)]^{(b)}$, $u = 1,2,3,4$, $b = 1,2,3,4,5$, i = 1,2,3,4, of the operation process impact on the car wheel system components E_i, i = 1,2,3,4, intensities of ageing at the operation states z_b, $b = 1,2,3,4,5$, in the safety state subsets $\{u, u+1, \ldots, 4\}$, $u = 1,2,3,4$:

$$[\rho_i^2(u)]^{(1)} = 1.00, \quad [\rho_i^2(u)]^{(2)} = 1.04, \quad [\rho_i^2(u)]^{(3)} = 1.08, \quad [\rho_i^2(u)]^{(4)} = 1.06, \quad [\rho_i^2(u)]^{(5)} = 1.10, \quad u = 1,2,3,4, \; i = 1,2,3,4. \tag{26}$$

After that, from (25) and (26), considering the intensities of ageing of the car wheel system components E_i, $i = 1,2,3,4$, without operation process impact $\lambda_i^0(u)$, $u = 1,2,3,4$, $i = 1,2,3,4$, given by (18) in [30], it follows that the intensities of the components departure from the safety states subset

$\{u, u+1, ..., 4\}$, $u = 1,2,3,4$, of the car wheel system impacted by its operation process at the particular operation states are:

- at the operation state z_1

$$[\lambda_i^2(1)]^{(1)} = [\rho_i^2(1)]^{(1)} \cdot \lambda_i^0(1) = 1.00 \cdot 0.0125 = 0.0125,$$

$$[\lambda_i^2(2)]^{(1)} = [\rho_i^2(2)]^{(1)} \cdot \lambda_i^0(2) = 1.00 \cdot 0.0200 = 0.0200,$$

$$[\lambda_i^2(3)]^{(1)} = [\rho_i^2(3)]^{(1)} \cdot \lambda_i^0(3) = 1.00 \cdot 0.0250 = 0.0250,$$

$$[\lambda_i^2(4)]^{(1)} = [\rho_i^2(4)]^{(1)} \cdot \lambda_i^0(4) = 1.00 \cdot 0.03125 = 0.03125, \ i = 1,2,3,4;$$

- at the operation state z_2

$$[\lambda_i^2(1)]^{(2)} = [\rho_i^2(1)]^{(2)} \cdot \lambda_i^0(1) = 1.04 \cdot 0.0125 = 0.0130,$$

$$[\lambda_i^2(2)]^{(2)} = [\rho_i^2(2)]^{(2)} \cdot \lambda_i^0(2) = 1.04 \cdot 0.0200 = 0.0208,$$

$$[\lambda_i^2(3)]^{(2)} = [\rho_i^2(3)]^{(2)} \cdot \lambda_i^0(3) = 1.04 \cdot 0.0250 = 0.0260,$$

$$[\lambda_i^2(4)]^{(2)} = [\rho_i^2(4)]^{(2)} \cdot \lambda_i^0(4) = 1.04 \cdot 0.03125 = 0.0325, \ i = 1,2,3,4;$$

- at the operation state z_3

$$[\lambda_i^2(1)]^{(3)} = [\rho_i^2(1)]^{(3)} \cdot \lambda_i^0(1) = 1.08 \cdot 0.0125 = 0.0135,$$

$$[\lambda_i^2(2)]^{(3)} = [\rho_i^2(2)]^{(3)} \cdot \lambda_i^0(2) = 1.08 \cdot 0.0200 = 0.0216,$$

$$[\lambda_i^2(3)]^{(3)} = [\rho_i^2(3)]^{(3)} \cdot \lambda_i^0(3) = 1.08 \cdot 0.0250 = 0.0270,$$

$$[\lambda_i^2(4)]^{(3)} = [\rho_i^2(4)]^{(3)} \cdot \lambda_i^0(4) = 1.08 \cdot 0.03125 = 0.03375, \ i = 1,2,3,4;$$

- at the operation state z_4

$$[\lambda_i^2(1)]^{(4)} = [\rho_i^2(1)]^{(4)} \cdot \lambda_i^0(1) = 1.06 \cdot 0.0125 = 0.01325,$$

$$[\lambda_i^2(2)]^{(4)} = [\rho_i^2(2)]^{(4)} \cdot \lambda_i^0(2) = 1.06 \cdot 0.0200 = 0.0212,$$

$$[\lambda_i^2(3)]^{(4)} = [\rho_i^2(3)]^{(4)} \cdot \lambda_i^0(3) = 1.06 \cdot 0.0250 = 0.0265,$$

$$[\lambda_i^2(4)]^{(4)} = [\rho_i^2(4)]^{(4)} \cdot \lambda_i^0(4) = 1.06 \cdot 0.03125 = 0.033125, \ i = 1,2,3,4;$$

- at the operation state z_5

$$[\lambda_i^2(1)]^{(5)} = [\rho_i^2(1)]^{(5)} \cdot \lambda_i^0(1) = 1.10 \cdot 0.0125 = 0.01375,$$

$$[\lambda_i^2(2)]^{(5)} = [\rho_i^2(2)]^{(5)} \cdot \lambda_i^0(2) = 1.10 \cdot 0.0200 = 0.0220,$$

$$[\lambda_i^2(3)]^{(5)} = [\rho_i^2(3)]^{(5)} \cdot \lambda_i^0(3) = 1.10 \cdot 0.0250 = 0.0275,$$

$$[\lambda_i^2(4)]^{(5)} = [\rho_i^2(4)]^{(5)} \cdot \lambda_i^0(4) = 1.10 \cdot 0.03125 = 0.034375, \ i = 1,2,3,4. \quad (27)$$

Considering that the conditional safety functions (11)–(12) of the car wheel system components E_i, $i = 1,2,3,4$, are piecewise exponential, i.e. they are given by the vectors

$$[S_i^2(t,\cdot)]^{(b)} = [[S_i^2(t,1)]^{(b)}, [S_i^2(t,2)]^{(b)}, [S_i^2(t,3)]^{(b)}, [S_i^2(t,4)]^{(b)}], \ t \in <0, \infty), \ i = 1,2,3,4; \ b = 1,2,3,4,5,$$

with the coordinates

$$[S_i^2(t,u)]^{(b)} = \exp[-[\lambda_i^2(u)]^{(b)} t], \ t \in <0, \infty), \ u = 1,2,3,4, \ i = 1,2,3,4, \ b = 1,2,3,4,5,$$

and the intensities of ageing $[\lambda_i^1(u)]^{(b)}$, $t \in <0, \infty)$, $u = 1,2,3,4$, $i = 1,2,3,4$, $b = 1,2,3,4,5$, at the operation states z_b, $b = 1,2,3,4,5$, are given respectively by (27), we get their coordinates following final forms:

- at the operation state z_1

$$[S_i^2(t,1)]^{(1)} = \exp[-0.0125t],$$
$$[S_i^2(t,2)]^{(1)} = \exp[-0.0200t],$$

$[S_i^2(t,3)]^{(1)} = \exp[-0.0250t]$,

$[S_i^2(t,4)]^{(1)} = \exp[-0.03125t]$, $i = 1,2,3,4$;

- at the operation state z_2

$[S_i^2(1)]^{(2)} = \exp[-0.0130t]$,

$[S_i^2(2)]^{(2)} = \exp[-0.0208t]$,

$[S_i^2(3)]^{(2)} = \exp[-0.0260t]$,

$[S_i^2(4)]^{(2)} = \exp[-0.0325t]$, $i = 1,2,3,4$;

- at the operation state z_3

$[S_i^2(1)]^{(3)} = \exp[-0.0135t]$,

$[S_i^2(2)]^{(3)} = \exp[-0.0216t]$,

$[S_i^2(3)]^{(3)} = \exp[-0.0270t]$,

$[S_i^2(4)]^{(3)} = \exp[-0.03375t]$, $i = 1,2,3,4$;

- at the operation state z_4

$[S_i^2(1)]^{(4)} = \exp[-0.01325t]$,

$[S_i^2(2)]^{(4)} = \exp[-0.0212\, t]$,

$[S_i^2(3)]^{(4)} = \exp[-0.0265t]$,

$[S_i^2(4)]^{(4)} = \exp[-0.033125t]$, $i = 1,2,3,4$;

- at the operation state z_5

$[S_i^2(1)]^{(5)} = \exp[-0.01375t]$,

$[S_i^2(2)]^{(5)} = \exp[-0.0220t]$,

$[S_i^2(3)]^{(5)} = \exp[-0.0275t]$,

$[S_i^2(4)]^{(5)} = \exp[-0.034375t]$, $i = 1,2,3,4$. (28)

4.4. Multistate ageing car wheel system impacted by its operation process safety characteristics

Since the car wheel system is a five-state (z = 4) homogeneous series system, then considering (28) and applying (10) and (15) from [30], its conditional safety functions in particular operation states z_b, $b = 1,2,3,4,5$, are respectively given by:

- at the operation state z_1

$$[\boldsymbol{S}^2(t,\cdot)]^{(1)} = [\ [\boldsymbol{S}^2(t,1)]^{(1)}, [\boldsymbol{S}^2(t,2)]^{(1)}, [\boldsymbol{S}^2(t,3)]^{(1)}, [\boldsymbol{S}^2(t,4)]^{(1)}],\ t \in <0,\infty),$$

where

$$[\boldsymbol{S}^2(t,1)]^{(1)} = \prod_{i=1}^{4}[S_i^2(t,1)]^{(1)} = [\exp[-0.0125t]]^4 = \exp[-0.050t],$$

$$[\boldsymbol{S}^2(t,2)]^{(1)} = \prod_{i=1}^{4}[S_i^2(t,2)]^{(1)} = [\exp[-0.0200t]]^4 = \exp[-0.080t],$$

$$[\boldsymbol{S}^2(t,3)]^{(1)} = \prod_{i=1}^{4}[S_i^2(t,3)]^{(1)} = [\exp[-0.0250t]]^4 = \exp[-0.100t],$$

$$[\boldsymbol{S}^2(t,4)]^{(1)} = \prod_{i=1}^{4}[S_i^2(t,4)]^{(1)} = [\exp[-0.03125t]]^4 = \exp[-0.125t];$$

- at the operation state z_2

$$[\boldsymbol{S}^2(t,\cdot)]^{(2)} = [1,\ [\boldsymbol{S}^2(t,1)]^{(2)}, [\boldsymbol{S}^2(t,2)]^{(2)}, [\boldsymbol{S}^2(t,3)]^{(2)},\ [\boldsymbol{S}^2(t,4)]^{(2)}], t \in <0,\infty),$$

where

$$[\boldsymbol{S}^2(t,1)]^{(2)} = \prod_{i=1}^{4}[S_i^2(t,1)]^{(2)} = [\exp[-0.0130t]]^4 = \exp[-0.0520t],$$

$$[\boldsymbol{S}^2(t,2)]^{(2)} = \prod_{i=1}^{4}[S_i^2(t,2)]^{(2)} = [\exp[-0.0208t]]^4 = \exp[-0.0832t],$$

$$[\boldsymbol{S}^2(t,3)]^{(2)} = \prod_{i=1}^{4}[S_i^2(t,3)]^{(2)} = [\exp[-0.0260t]]^4 = \exp[-0.1040t],$$

$$[\boldsymbol{S}^2(t,4)]^{(2)} = \prod_{i=1}^{4}[S_i^2(t,4)]^{(2)} = [\exp[-0.0325t]]^4 = \exp[-0.1300t];$$

- at the operation state z_3

$$[\boldsymbol{S}^2(t,\cdot)]^{(3)} = [1,\ [\boldsymbol{S}^2(t,1)]^{(3)}, [\boldsymbol{S}^2(t,2)]^{(3)},$$

$$[\boldsymbol{S}^2(t,3)]^{(3)},\ [\boldsymbol{S}^2(t,4)]^{(3)}],\ t \in <0,\infty),$$

where

$$[\boldsymbol{S}^2(t,1)]^{(3)} = \prod_{i=1}^{4}[S_i^2(t,1)]^{(3)} = [\exp[-0.0135t]]^4 = \exp[-0.0540t],$$

$$[\boldsymbol{S}^2(t,2)]^{(3)} = \prod_{i=1}^{4}[S_i^2(t,2)]^{(3)} = [\exp[-0.0216t]]^4 = \exp[-0.0864t],$$

$$[\boldsymbol{S}^2(t,3)]^{(3)} = \prod_{i=1}^{4}[S_i^2(t,3)]^{(3)} = [\exp[-0.0270t]]^4 = \exp[-0.1080t],$$

$$[\boldsymbol{S}^2(t,4)]^{(3)} = \prod_{i=1}^{4}[S_i^2(t,4)]^{(3)} = [\exp[-0.03375t]]^4 = \exp[-0.1350t];$$

- at the operation state z_4

$$[\boldsymbol{S}^2(t,\cdot)]^{(4)} = [1,\ [\boldsymbol{S}^2(t,1)]^{(4)},\ [\boldsymbol{S}^2(t,2)]^{(4)},\ [\boldsymbol{S}^2(t,3)]^{(4)},\ [\boldsymbol{S}^2(t,4)]^{(4)}],\ t \in <0,\infty),$$

where

$$[\boldsymbol{S}^2(t,1)]^{(4)} = \prod_{i=1}^{4}[S_i^2(t,1)]^{(4)} = [\exp[-0.01325t]]^4 = \exp[-0.0530t],$$

$$[\boldsymbol{S}^2(t,2)]^{(4)} = \prod_{i=1}^{4}[S_i^2(t,2)]^{(4)} = [\exp[-0.0212\ t]]^4 = \exp[-0.0848t],$$

$$[\boldsymbol{S}^2(t,3)]^{(4)} = \prod_{i=1}^{4}[S_i^2(t,3)]^{(4)} = [\exp[-0.0265t]]^4 = \exp[-0.1060t],$$

$$[\boldsymbol{S}^2(t,4)]^{(4)} = \prod_{i=1}^{4}[S_i^2(t,4)]^{(4)} = [\exp[-0.033125t]]^4 = \exp[-0.1325t];$$

- at the operation state z_5

$$[\boldsymbol{S}^2(t,\cdot)]^{(5)} = [1,\ [\boldsymbol{S}^2(t,1)]^{(5)},\ [\boldsymbol{S}^2(t,2)]^{(5)},\ [\boldsymbol{S}^2(t,3)]^{(5)},\ [\boldsymbol{S}^2(t,4)]^{(5)}],\ t \in <0,\infty),$$

where

$$[\boldsymbol{S}^2(t,1)]^{(5)} = \prod_{i=1}^{4}[S_i^2(t,1)]^{(5)} = [\exp[-0.01375t]]^4 = \exp[-0.0550t],$$

$$[\boldsymbol{S}^2(t,2)]^{(5)} = \prod_{i=1}^{4}[S_i^2(t,2)]^{(5)} = [\exp[-0.0220t]]^4 = \exp[-0.0880t],$$

$$[\boldsymbol{S}^2(t,3)]^{(5)} = \prod_{i=1}^{4}[S_i^2(t,3)]^{(5)} = [\exp[-0.0275t]]^4 = \exp[-0.1100t],$$

$$[\boldsymbol{S}^2(t,4)]^{(5)} = \prod_{i=1}^{4}[S_i^2(t,4)]^{(5)} = [\exp[-0.034375t]]^4 = \exp[-0.1375t]. \quad (29)$$

Hence, the mean values $[\boldsymbol{\mu}^1(u)]^{(b)}$, $u = 1,2,3,4$, $b = 1,2,3,4,5$, of the car wheel system lifetimes in the safety state subsets $\{u, u+1, ...4\}$, $u = 1,2,3,4$, at the operation states z_b, $b = 1,2,3,4,5$, expressed in years, respectively are:

- at the operation state z_1

$$[\boldsymbol{\mu}^2(1)]^{(1)} = \int_0^\infty [\boldsymbol{S}^2(t,1)]^{(1)} dt = \int_0^\infty \exp[-0.050t]dt = 20,$$

$$[\boldsymbol{\mu}^2(2)]^{(1)} = \int_0^\infty [\boldsymbol{S}^2(t,2)]^{(1)} dt = \int_0^\infty \exp[-0.080t]dt = 12.5,$$

$$[\boldsymbol{\mu}^2(3)]^{(1)} = \int_0^\infty [\boldsymbol{S}^2(t,3)]^{(1)} dt = \int_0^\infty \exp[-0.100t]dt = 10,$$

$$[\boldsymbol{\mu}^2(4)]^{(1)} = \int_0^\infty [\boldsymbol{S}^2(t,4)]^{(1)} dt$$

$$= \int_0^\infty \exp[-0.125t]dt = 8;$$

- at the operation state z_2

$$[\boldsymbol{\mu}^2(1)]^{(2)} = \int_0^\infty [\boldsymbol{S}^2(t,1)]^{(2)} dt$$

$$= \int_0^\infty \exp[-0.0520t]dt \cong 19.231,$$

$$[\boldsymbol{\mu}^2(2)]^{(2)} = \int_0^\infty [\boldsymbol{S}^2(t,2)]^{(2)} dt$$

$$= \int_0^\infty \exp[-0.0832t]dt \cong 12.019,$$

$$[\boldsymbol{\mu}^2(3)]^{(2)} = \int_0^\infty [\boldsymbol{S}^2(t,3)]^{(2)} dt$$

$$= \int_0^\infty \exp[-0.1040t]dt \cong 9.615,$$

$$[\boldsymbol{\mu}^2(4)]^{(2)} = \int_0^\infty [\boldsymbol{S}^2(t,4)]^{(2)} dt$$

$$= \int_0^\infty \exp[-0.1300t]dt \cong 7.692;$$

- at the operation state z_3

$$[\boldsymbol{\mu}^2(1)]^{(3)} = \int_0^\infty [\boldsymbol{S}^2(t,1)]^{(3)} dt$$

$$= \int_0^\infty \exp[-0.0540t]dt \cong 18.519,$$

$$[\boldsymbol{\mu}^2(2)]^{(3)} = \int_0^\infty [\boldsymbol{S}^2(t,2)]^{(3)} dt$$

$$= \int_0^\infty \exp[-0.0864t]dt \cong 11.574,$$

$$[\boldsymbol{\mu}^2(3)]^{(3)} = \int_0^\infty [\boldsymbol{S}^2(t,3)]^{(3)} dt$$

$$= \int_0^\infty \exp[-0.1080t]dt \cong 9.259,$$

$$[\boldsymbol{\mu}^2(4)]^{(3)} = \int_0^\infty [\boldsymbol{S}^2(t,4)]^{(3)} dt$$

$$= \int_0^\infty \exp[-0.1350t]dt \cong 7.407;$$

- at the operation state z_4

$$[\boldsymbol{\mu}^2(1)]^{(4)} = \int_0^\infty [\boldsymbol{S}^2(t,1)]^{(4)} dt$$

$$= \int_0^\infty \exp[-0.0530t]dt \cong 18.867,$$

$$[\boldsymbol{\mu}^2(2)]^{(4)} = \int_0^\infty [\boldsymbol{S}^2(t,2)]^{(4)} dt$$

$$= \int_0^\infty \exp[-0.0848t]dt \cong 11.792,$$

$$[\boldsymbol{\mu}^2(3)]^{(4)} = \int_0^\infty [\boldsymbol{S}^2(t,3)]^{(4)} dt$$

$$= \int_0^\infty \exp[-0.1060t]dt \cong 9.434,$$

$$[\boldsymbol{\mu}^2(4)]^{(4)} = \int_0^\infty [\boldsymbol{S}^2(t,4)]^{(4)} dt$$

$$= \int_0^\infty \exp[-0.1325t]dt \cong 7.547;$$

- at the operation state z_5

$$[\boldsymbol{\mu}^2(1)]^{(5)} = \int_0^\infty [\boldsymbol{S}^2(t,1)]^{(5)} dt$$

$$= \int_0^\infty \exp[-0.0550t]dt \cong 18.182,$$

$$[\boldsymbol{\mu}^2(2)]^{(5)} = \int_0^\infty [\boldsymbol{S}^2(t,2)]^{(5)} dt$$

$$= \int_0^\infty \exp[-0.0880t]dt \cong 11.364,$$

$$[\boldsymbol{\mu}^2(3)]^{(5)} = \int_0^\infty [\boldsymbol{S}^2(t,3)]^{(5)} dt$$

$$= \int_0^\infty \exp[-0.1100t]dt \cong 9.091,$$

$$[\boldsymbol{\mu}^2(4)]^{(5)} = \int_0^\infty [\boldsymbol{S}^2(t,4)]^{(5)} dt$$

$$= \int_0^\infty \exp[-0.1375t]dt \cong 7.273. \qquad (30)$$

From the results (22) and (29), applying (17), the car wheel system unconditional safety function is given by

$$S^2(t,\cdot) = [[S^2(t,1)], [S^2(t,2)], [S^2(t,3)], [S^2(t,4)]], \quad (31)$$

where

$$S^2(t,1) \cong \sum_{b=1}^{5} p_b [S^2(t,1)]^{(b)}$$
$$= 0.272\exp[-0.050t] + 0.261\exp[-0.0520t] + 0.280\exp[-0.0540t] + 0.112\exp[-0.0530t] + 0.075\exp[-0.0550t],$$

$$S^2(t,2) \cong \sum_{b=1}^{5} p_b [S^2(t,2)]^{(b)}$$
$$= 0.272\exp[-0.080t] + 0.261\exp[-0.0832t] + 0.280\exp[-0.0864t] + 0.112\exp[-0.0848t] + 0.075\exp[-0.0880t],$$

$$S^2(t,3) \cong \sum_{b=1}^{5} p_b [S^2(t,3)]^{(b)}$$
$$= 0.272\exp[-0.100t] + 0.261\exp[-0.1040t] + 0.280\exp[-0.1080t] + 0.112\exp[-0.1060t] + 0.075\exp[-0.1100t],$$

$$S^2(t,4) \cong \sum_{b=1}^{5} p_b [S^2(t,4)]^{(b)}$$
$$= 0.272\exp[-0.125t] + 0.261\exp[-0.1300t] + 0.280\exp[-0.1350t] + 0.112\exp[-0.1325t] + 0.075\exp[-0.1375t], \ t \in <0,\infty). \quad (32)$$

The graph of the car wheel system impacted by its operation process safety function is shown in *Figure 1*.

As the critical safety state is $r = 2$, then by (4) from [9] and (32), the car wheel system risk function, is given by

$$r^2(t) = 1 - S^2(t,2)$$
$$= 1 - \{0.272\exp[-0.080t] + 0.261\exp[-0.0832t] + 0.280\exp[-0.0864t] + 0.112\exp[-0.0848t] + 0.075\exp[-0.0880t]\}, \text{ for } t \geq 0. \quad (33)$$

The values of the above risk function are given in *Table 1*. Applying the formula for inverse risk function from [18] to (33) and using the values given in *Table 1*, the moment when the car wheel system impacted by its operation process risk function exceeds a permitted level $\delta = 0.05$, is

$$\tau^2 = r^{2^{-1}}(\delta) \cong 0.612 \text{ years}. \quad (34)$$

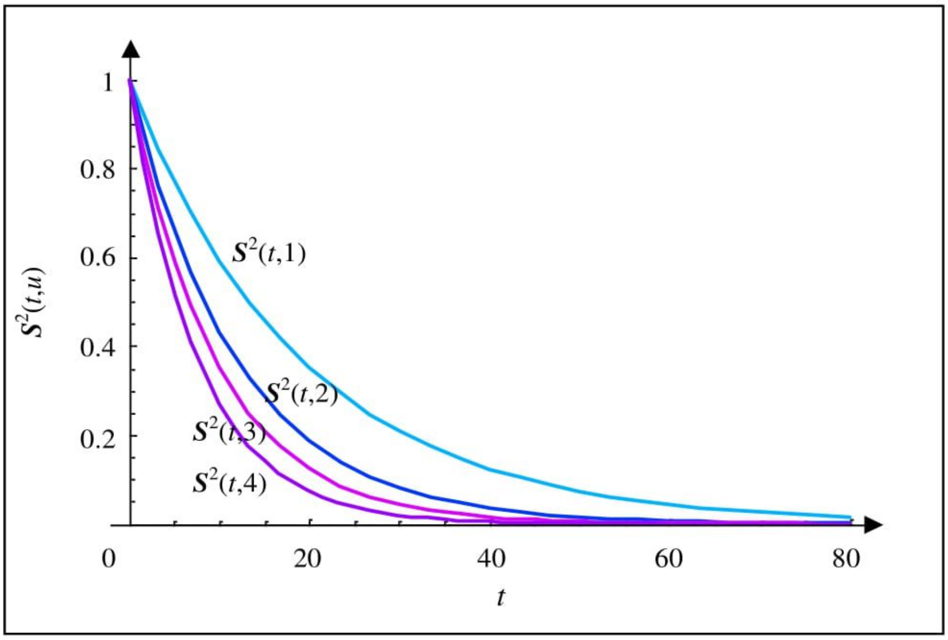

Figure 1. The graph of the car wheel system impacted by its operation process safety function $S^2(t,\cdot)$ coordinates

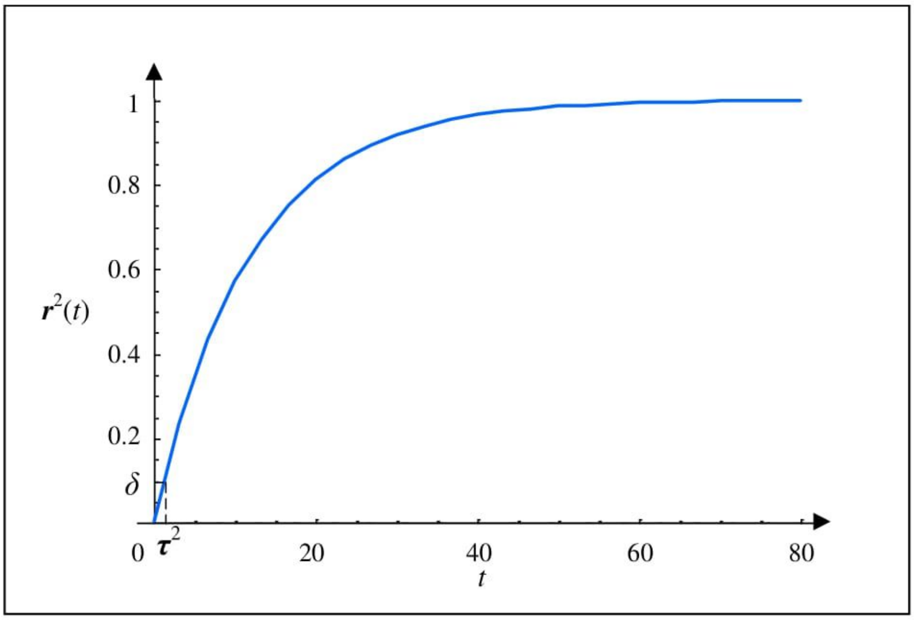

Figure 2. The graph of the risk function $\boldsymbol{r}^2(t)$ of the car wheel system impacted by its operation process

Table 1. The values of the car wheel system impacted by its operation risk function

t	$\boldsymbol{r}^2(t) = 1 - \boldsymbol{S}^2(t,2)$
0.6	0.04902
0.601	0.0491
0.602	0.04917
0.603	0.04925
0.604	0.04933
0.605	0.04941
0.606	0.04949
0.607	0.04957
0.608	0.04965
0.609	0.04973
0.61	0.04981
0.611	0.04989
0.612	**0.04997**
0.613	0.05005
0.614	0.05013
0.615	0.05021
0.616	0.05029
0.617	0.05037
0.618	0.05045
0.619	0.05053
0.62	0.05061

The graph of the risk function $\boldsymbol{r}^2(t)$ of the car wheel system impacted by its operation process, the system fragility curve, is shown in *Figure 2*.

Considering (30) and (32) and applying (17), the expected values of the car wheel system impacted by its operation process lifetimes in the safety state subsets {1,2,3,4}, {2,3,4}, {3,4}, {4}, respectively are:

$$\boldsymbol{\mu}^2(1) \cong \sum_{b=1}^{5} p_b[\mu^2(1)]^{(b)} \cong 0.272 \cdot 20 + 0.261 \cdot 19.231 + 0.280 \cdot 18.519 + 0.112 \cdot 18.867 + 0.075 \cdot 18.182 \cong 19.121 \text{ years},$$

$$\boldsymbol{\mu}^2(2) \cong \sum_{b=1}^{5} p_b[\mu^2(2)]^{(b)} \cong 0.272 \cdot 12.5 + 0.261 \cdot 12.019 + 0.280 \cdot 11.574 + 0.112 \cdot 11.792 + 0.075 \cdot 11.364 \cong 11.951 \text{ years},$$

$$\boldsymbol{\mu}^2(3) \cong \sum_{b=1}^{5} p_b[\mu^2(3)]^{(b)} \cong 0.272 \cdot 10 + 0.261 \cdot 9.615 + 0.280 \cdot 9.259 + 0.112 \cdot 9.434 + 0.075 \cdot 9.091 \cong 9.560 \text{ years},$$

$$\boldsymbol{\mu}^2(4) \cong \sum_{b=1}^{5} p_b[\mu^2(4)]^{(b)} \cong 0.272 \cdot 8 + 0.261 \cdot 7.692 + 0.280 \cdot 7.407 + 0.112 \cdot 7.547 + 0.075 \cdot 7.273 \cong 7.646 \text{ years}. \quad (35)$$

Further, it follows that the mean values of the car wheel system impacted by its operation process lifetimes in the particular safety states, according to the formula from [15], [21], are:

$$\bar{\mu}^2(1) = \mu^2(1) - \mu^2(2) \cong 7.170,$$
$$\bar{\mu}^2(2) = \mu^2(2) - \mu^2(3) \cong 2.391,$$
$$\bar{\mu}^2(3) = \mu^2(3) - \mu^2(4) \cong 3.27,$$
$$\bar{\mu}^2(4) = \mu^2(4) \cong 7.646 \text{ years.} \quad (36)$$

The intensities

$$\lambda^1(t,1),\ \lambda^1(t,2),\ \lambda^1(t,3),\ \lambda^1(t,4),\ t \in <0,\infty),$$

of degradation (ageing) of the car wheel system impacted by its operation process, i.e. the intensities of the car wheel system impacted by its operation process departure from the safety state subsets {1,2,3,4}, {2,3,4}, {3,4}, {4}, can be determined according to the formula [21]

$$\lambda^2(t,u) = -\frac{d\mathbf{S}^2(t,u)}{dt} / \mathbf{S}^2(t,u),\ t \in <0,\infty),$$
$$u = 1,2,3,4, \quad (37)$$

where $\mathbf{S}^2(t,u)$, $t \in <0,\infty)$, u = 1,2,3,4, are given by (32).

The graphs of the intensities of ageing (37) of the car wheel system impacted by its operation process are shown in *Figure 3*.

After applying the formula (37) to the system safety function coordinates given by (32), the car wheel system impacted by its operation process limit intensities of ageing are:

$$\lambda^2(1) = \lim_{t\to\infty} \lambda^2(t,1)$$

$$= \lim_{t\to\infty} \frac{0.272 \cdot 0.050\exp[-0.050t][1+o(1)]}{0.272\exp[-0.050t][1+o(1)]} = 0.050,$$

$$\lambda^2(2) = \lim_{t\to\infty} \lambda^2(t,2)$$

$$= \lim_{t\to\infty} \frac{0.272 \cdot 0.080\exp[-0.080t][1+o(1)]}{0.272\exp[-0.080t][1+o(1)]} = 0.080,$$

$$\lambda^2(3) = \lim_{t\to\infty} \lambda^2(t,3)$$

$$= \lim_{t\to\infty} \frac{0.272 \cdot 0.100\exp[-0.100t][1+o(1)]}{0.272\exp[-0.100t][1+o(1)]} = 0.100,$$

$$\lambda^2(4) = \lim_{t\to\infty} \lambda^2(t,4)$$

$$= \lim_{t\to\infty} \frac{0.272 \cdot 0.125\exp[-0.0125t][1+o(1)]}{0.272\exp[-0.0125t][1+o(1)]} = 0.125. \quad (38)$$

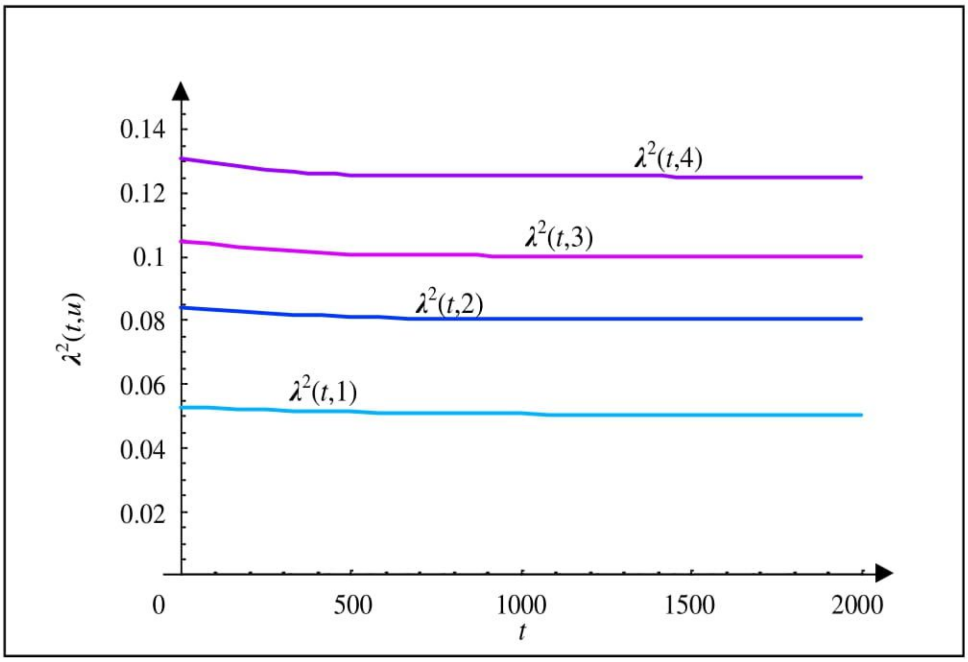

Figure 3. The graphs of the intensities of ageing of the car wheel system impacted by its operation process

By (35), applying formula from [21], the car wheel system impacted by its operation process approximate mean intensities of ageing are:

$$\lambda^2(1) = \frac{1}{\mu^2(1)} = \frac{1}{19.121} \cong 0.0523,$$

$$\lambda^2(2) = \frac{1}{\mu^2(2)} = \frac{1}{11.951} \cong 0.0837,$$

$$\lambda^2(3) = \frac{1}{\mu^2(3)} = \frac{1}{9.56} \cong 0.1046,$$

$$\lambda^2(1) = \frac{1}{\mu^2(4)} = \frac{1}{7.646} \cong 0.1308. \quad (39)$$

The values of the car wheel system impacted by its operation process intensities of ageing given by (39) are more sensible in this system resilience evaluation than those given by (38). Thus, considering (39) and the values of the car wheel system intensities of ageing without of operation impact, determined by (29) in [9] and applying formula from [21], the coefficients of the operation process impact on the car wheel system intensities of ageing (the resilience indicators to its operation process) are given respectively by:

$$\rho^2(t,1) = \frac{\lambda^2(1)}{\lambda^0(1)} \cong \frac{0.0523}{0.050} \cong 1.046,$$

$$\rho^2(t,2) = \frac{\lambda^2(2)}{\lambda^0(2)} \cong \frac{0.0837}{0.080} \cong 1.046,$$

$$\rho^2(t,3) = \frac{\lambda^2(3)}{\lambda^0(3)} \cong \frac{0.1046}{0.100} \cong 1.046,$$

$$\rho^2(t,4) = \frac{\lambda^2(4)}{\lambda^0(4)} \cong \frac{0.1308}{0.125} \cong 1.046. \quad (40)$$

Finally, by (40), the car wheel system resilience indicator to its operation process impact [21], i.e. the coefficient of the car wheel system resilience to operation process impact is

$$\boldsymbol{RI}^2(t,2) = \frac{1}{\rho^2(t,2)} \cong 0.956 = 95.6\%. \ t \in <0,+\infty). \quad (41)$$

The comparison of safety indicators (16)–(25) from [9] and (31)–(39) proves a noticeable influence of the operation process on the car wheel system safety what is also clearly expressed in the resilience indicators to operation process impact (40) and (41).

5. Conclusion

In the chapter, the approach to the safety analysis of multistate ageing complex systems that considers their operation processes impact is presented. The proposed approach to safety analysis of a multistate ageing system impacted by its operation process, allows us to determine the safety indicators, practically important and useful for the users of the complex multistate ageing systems changing their components safety parameters during their operation [15].

Combining the results of the safety analysis of multistate ageing systems with the results of the safety analysis of systems impacted by their operation processes, the joint safety analysis of a complex multistate ageing system considering simultaneously its ageing and external operation impacts is performed and the new results that improve significantly the accuracy of the real system safety examination are found.

The generalization of the obtained new results to safety analysis of multistate ageing complex systems at their operation conditions through considering additionally their inside dependences [9] seems to be a very important task for practice.

Thus, as a consequence of the above analysis, the further research could be focused on safety analysis of complex systems, considering their ageing [15], inside dependencies [9] and outside impacts [15] simultaneously [10], and the use of the achieved results to improve their safety [15], strengthen their resilience and mitigate [3] the effects of their degradation and failures.

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